Orthogonal Double Covers of Complete Bipartite Graphs by Symmetric Starters

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Abstract

Let H be a graph on n vertices and $\mathcal{G}$ a collection of n subgraphs of H, one for each vertex. Then $\mathcal{G}$ is an orthogonal double cover (ODC) of H if every edge of H occurs in exactly two members of $\mathcal{G}$ and any two members of $\mathcal{G}$ share exactly an edge whenever the corresponding vertices are adjacent in H. If all subgraphs in $\mathcal{G}$ are isomorphic to a given graph G, then $\mathcal{G}$ is said to be an ODC of H by G.

We construct the *ODC*s of $H = K_{n,n}$ by $G = P_{m+1} \cup^{v} S_{n-m}$ (union of a path P_{m+1}, and a star S_{n-m} where the center v of the star is a one of the path ends, $m = 5,6,7,8,9,10$). In all cases, G is a symmetric starter of the cyclic group of order n.

Keywords: Orthogonal double cover; ODC; Graph decompositions; Symmetric starter.
AMS Subject Classification: 05C70, 05B30

1. Introduction

An *orthogonal double cover* (ODC) of the complete graph K_n is a collection $\mathcal{G}$ of n spanning subgraphs (called *pages*) such that

(i) every edge of K_n is an edge in exactly two of the pages,

(ii) any two pages share exactly one edge.

If all pages in $\mathcal{G}$ are isomorphic to a given graph G then $\mathcal{G}$ is said to be an ODC of K_n by G.

There is an extensive literature on *ODCs* of K_n by G, see e.g. [2,4,6,8,9,10]. A survey on the topic is given in [5] .

Recently, this concept has been generalized replacing K_n by an arbitrary graph H as follows. Let H be an arbitrary graph with n vertices and let $\mathcal{G} = \{G_0,\ldots,G_{n\text{-}1}\}$ be a collection of n spanning subgraphs of H (called pages). $\mathcal{G}$ is called an ODC of H if there exists a bijective mapping $\varphi : V(H) \to \mathcal{G}$ such that:

(i) every edge of H is contained in exactly two of the graphs $G_0,\ldots,G_{n\text{-}1}$.

(ii) for every choice of different vertices a,b of H,

$$| E(\varphi(a)) \cap E(\varphi(b)) | = \begin{cases} 1 & \textit{if } \{a,b\} \in E(H), \text{ or} \\ 0 & \text{otherwise.} \end{cases}$$

If all pages in $\mathcal{G}$ are isomorphic to a given graph G, then $\mathcal{G}$ is said to be an ODC of H by G. Note that in this case H is necessarily a regular graph of degree $|E(G)|$. Moreover, if H is not complete, G must be disconnected.

While in principle any regular graph H is worth considering (e.g., the remarkable case of hypercubes has been investigated in [7]), the choice of $H = K_{n,n}$ is quite natural, also in view of a technical motivation: ODCs in such graphs are of help in order to construct ODCs of K_n (see [1], p. 48).

An algebraic construction of ODCs via "*symmetric starters*" (see Section 2) has been exploited to get a complete classification of ODCs of $K_{n,n}$ by G for $n \le 9$: a few exceptions apart, all graphs G are found this way (see [1], Table 1). This method has been applied in both [3] and [1] to detect some infinite classes of graphs G for which there is an ODC of $K_{n,n}$ by G.

In particular, let G be the graph $(P_{m+1} \cup^v S_{n-m}) \cup (n-1)K_1$, where $\cup^v$ denotes the union of a path of length m and a $(n-m)$-star, attached by a vertex v that is both an end-vertex of P_{m+1} and the center of S_{n-m}, as shown in Figure 1.

For all m and n such that $2 \le m \le 6$ and $m \le n$ it was established in [3] that there is an ODC of $K_{n,n}$ by G as described above.

Our goal here is to improve this result, by showing that the same is true for $2 \le m \le 10$ and $m \le n$. Namely, we shall prove the following.

Theorem 1.1. *Let n and m be integers such that $2 \le m \le 10$ and $m \le n$. Then there is an ODC of $K_{n,n}$ by $G = (P_{m+1} \cup^v S_{n-m}) \cup (n-1)K_1$.*

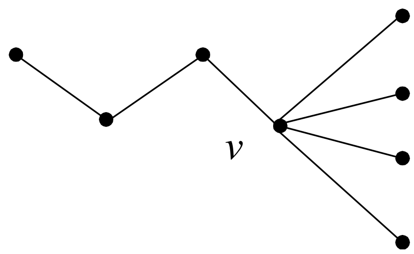

Figure 1: The graph $P_4 \cup^v S_4$.

Clearly, the above G is a subgraph of $K_{n,n}$ if and only if $m \le n$. Besides, for $m = 1$ we have $P_2 \cup^v S_{n-1} = S_n$, a trivial case. This explains the inequalities appearing in the above statement of Theorem 1.1.

Preliminaries are to be exposed in Section 2, while Section 3 will contain the results that lead to the proof of Theorem 1.1.

2. ODC of $K_{n,n}$ by symmetric starters

All graphs here are finite, simple and undirected. For all integers $n \ge 2$, we will denote by P_n the path of length $n-1$ and by S_n the *n*-star (that is, the complete bipartite graph $K_{1,n}$). Moreover, K_1 is the graph consisting of only one vertex.

Let $\Gamma = \{\gamma_0, \ldots, \gamma_{n\text{-}1}\}$ be an (additive) abelian group of order *n*. The vertices of $K_{n,n}$ will be labeled by the elements of $\Gamma \times \mathbb{Z}_2$. Namely, for $(v,i) \in \Gamma \times \mathbb{Z}_2$ we will write v_i for the corresponding vertex and define $\{w_i, u_j\} \in E(K_{n,n})$ if and only if $i \ne j$, for all $w, u \in \Gamma$ and $i, j \in \mathbb{Z}_2$.

Let G be a spanning subgraph of $K_{n,n}$ and let $a \in \Gamma$. Then the graph G with $E(G+a) = \{(u+a, v+a) : (u,v) \in E(G)\}$ is called the *a-translate* of G. The length of an edge $e = (u,v) \in E(G)$ is defined by $d(e) = v - u$. As an example, Figure 2 shows the edges of G_{0_0} labeled by their lengths.

G is called a half starter with respect to Γ *if* $|E(G)| = n$ and the lengths of all edges in G are pairwise mutually different, i.e. $\{d(e) : e \in E(G)\} = \Gamma$. The following three results were established in [1].

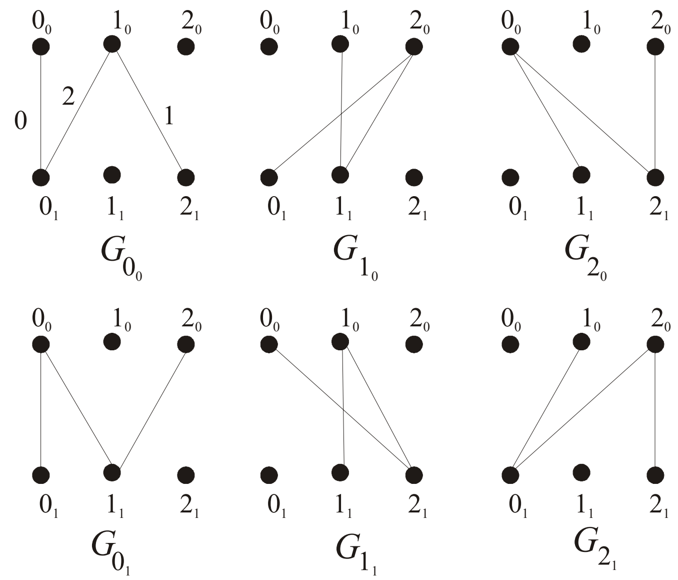

Figure 2: *ODC* of $K_{3,3}$ by $G = P_4$ with $\Gamma = \mathbb{Z}_3$.

Theorem 2.1. *If* G is a *half starter, then the union of all translates of* G *forms an edge decomposition of* $K_{n,n}$, i.e. $\bigcup_{a\in\Gamma} E(G+a) = E(K_{n,n})$.

Here, the half starter will be represented by the vector: $v(G) = (v_{\gamma_0}, v_{\gamma_1}, \dots, v_{\gamma_{n-1}})$. Where $v_{\gamma_i} \in \Gamma$ and $(v_{\gamma_i})_0$ is the unique vertex $((v_{\gamma_i}, 0) \in \Gamma \times \{0\})$ that belongs to the unique edge of length γ_i. For example, in Figure 2 the graph G_{0_0} is a half starter with respect to $\mathbb{Z}_3$ represented by (0,1,1) (e.g. $\{1_0, 2_1\}$ is the unique edge of length 1, thus $v_1 = 1$).

Two half starter vectors $v(G_0)$ and $v(G_1)$ are said to be orthogonal if $\{v_\gamma(G_0) - v_\gamma(G_1) : \gamma \in \Gamma\} = \Gamma$.

Theorem 2.2. *If two half starters* $v(G_0)$ *and* $v(G_1)$ *are orthogonal, then* $G = \{G_{a,\,i} \,:\, (a,i) \in \Gamma \times \mathbb{Z}_2\}$ *with* $G_{a,\,i} = G_i + a$ *is an ODC of* $K_{n,n}$.

The subgraph G_s of $K_{n,n}$ with $E(G_s) = \{\{u_0, v_1\} : \{v_0, u_1\} \in E(G)\}$ is called the symmetric graph of G. Note that if G is a half starter, then G_s is also a half starter .

A half starter G is called a symmetric starter with respect Γ if $v(G)$ and $v(G_s)$ are orthogonal.

Theorem 2.3. *Let* n *be a positive integer and let* G be *a half starter represented by* $v(G) = (v_{\gamma_0}, v_{\gamma_1}, \dots, v_{\gamma_{n-1}})$. *Then* G *is symmetric starter if and only if*

$$\{v_\gamma - v_{-\gamma} + \gamma : \gamma \in \Gamma\} = \Gamma.$$

3. The main results

In view of Section 2, all we need is to find suitable symmetric starters for the cases under studyEach of these will be dealt with in a lemma.

Lemma 3.1. *For all integers* $n \geq 5$ *the vector* $v(G) = (0,2,0,2,2,2,\ldots,2,2,2,4,4)$ *is a symmetric starter of* $\mathbb{Z}_n$, *isomorphic to* $(P_6 \cup^{(2,0)} S_{n-5}) \cup (n-1)K_1$.

Proof. For any integer $i \in \mathbb{Z}_n$, we can define the vector $v(G) = (0,2,0,2,2,2,\ldots,2,2,2,4,4)$ as follows.

$$v_i(G) = \begin{cases} 0 & i = 0,\ 2, \text{ or} \\ 4 & i = n-2,\ n-1, \text{or} \\ 2 & \text{otherwise.} \end{cases}$$

Therefore we find

$$v_{-i}(G) = \begin{cases} 0 & i = 0,\ n-2, \text{ or} \\ 4 & i = 1,\ 2, \text{ or} \\ 2 & \text{otherwise.} \end{cases}$$

Then we have

$$v_i(G) - v_{-i}(G) + i = \begin{cases} 0 & i = 0, \text{ or} \\ n-1 & i = 1, \text{ or} \\ n-2 & i = 2, \text{ or} \\ 1 & i = n-1, \text{ or} \\ 2 & i = n-2, \text{ or} \\ i & \text{otherwise.} \end{cases}$$

It is easily checked that $\{v_i(G) - v_{-i}(G) + i : i \in \mathbb{Z}_n\} = \mathbb{Z}_n$, hence it is a symmetric starter by Theorem 2.3. □

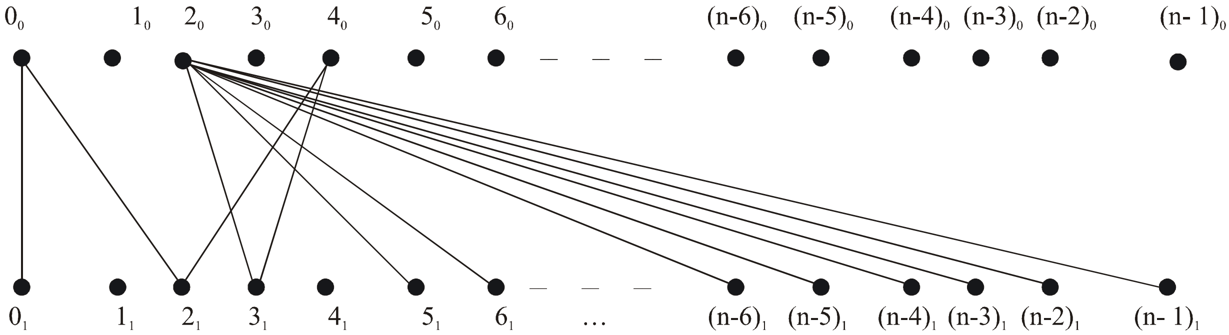

Figure 3: A symmetric starter of $\mathbb{Z}_n$ for an ODC of $K_{n,n}$ by $(P_6 \cup^{(2,0)} S_{n-5}) \cup (n-1)K_1$.

From Figure 3, and for $i \in \mathbb{Z}_n$, the i^{th} graph isomorphic to the symmetric starter $(P_6 \cup^{(2,0)} S_{n-5}) \cup (n-1)K_1$ has the edges:

$$E(G_i) = \{i_1, i_0, (i+2)_1, (i+4)_0, (i+3)_1, (i+2)_0\} \cup \{\{(i+2)_0, (i+j)_1\} : 5 \le j \le n-1\} \ .$$

Lemma 3.2. *For all integers $n \ge 6$ the vector $v(G) = (2, n-1, 0, n-4, n-5, n-6, \ldots, 6, 5, 4, 3, 2, 0, n-1)$ is a symmetric starter of $\mathbb{Z}_n$ isomorphic to $(P_7 \cup^{(n-1,1)} S_{n-6}) \cup (n-1)K_1$.*

Proof. For any integer $i \in \mathbb{Z}_n$, we can define the vector

$v(G) = (2, n-1, 0, n-4, n-5, n-6, \ldots, 6, 5, 4, 3, 2, 0, n-1)$ as follows.

$$v_i(G) = \begin{cases} 2 & i = 0, \text{ or} \\ n-1 & i = 1,\ n-1, \text{ or} \\ 0 & i = 2,\ n-2, \text{ or} \\ n-i-1 & \text{otherwise.} \end{cases}$$

Therefore we find

$$v_{-i}(G) = \begin{cases} 2 & i = 0, \text{ or} \\ n-1 & i = 1,\ n-1, \text{ or} \\ 0 & i = 2,\ n-2, \text{ or} \\ i-1 & \text{otherwise.} \end{cases}$$

Then we have

$$v_i(G) - v_{-i}(G) + i = \begin{cases} 0 & i = 0, \text{ or} \\ i & i = 1,\ 2,\ n-2,\ n-1, \text{ or} \\ -i & \text{otherwise.} \end{cases}$$

It is easily checked that $\{v_i(G) - v_{-i}(G) + i : i \in \mathbb{Z}_n\} = \mathbb{Z}_n$, hence it is a symmetric starter by Theorem 2.3. □

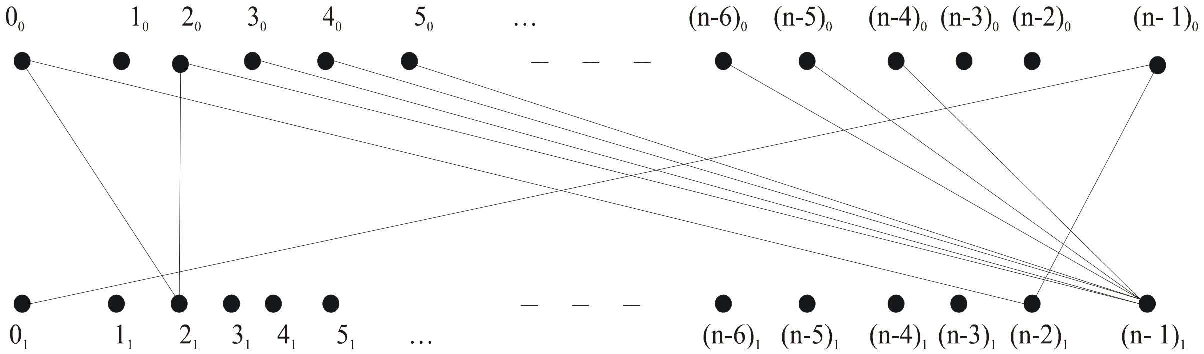

Figure 4: A symmetric starter of $\mathbb{Z}_n$ for an ODC of $K_{n,n}$ by $(P_7 \cup^{(n-1,1)} S_{n-6}) \cup (n-1)K_1$

From Figure 4, and for all $i \in \mathbb{Z}_n$, the i^{th} graph isomorphic to the symmetric starter $(P_7 \cup^{(n-1,1)} S_{n-6}) \cup (n-1)K_1$ has the edges:

$$E(G) = \{i_{1,} (i+n-1)_0, (i+n-2)_1, i_0, (i+2)_1, (i+2)_0, (i+n-1)_1\} \cup \{\{(i+j)_0, (i+n-1)_1\} : 3 \le j \le n-4\}.$$

Lemma 3.3. *For all integers* $n \ge 7$ *the vector* $v(G) = (0,1,2,0,2,2,2,\dots,2,2,2,6,6,1)$ *is a symmetric starter of* $\mathbb{Z}_n$ *isomorphic to* $(P_8 \cup^{(2,0)} S_{n-7}) \cup (n-1)K_1$.

Proof. For any integer $i \in \mathbb{Z}_n$, we can define the vector $v(G) = (0,1,2,0,2,2,2,\dots,2,2,2,6,6,1)$ as follows**.**

$$v_i(G) = \begin{cases} 0 & i = 0,\ 3,\ \text{or} \\ 1 & i = 1,\ n-1,\ \text{or} \\ 6 & i = n-3,\ n-2,\ \text{or} \\ 2 & \text{otherwise.} \end{cases}$$

Therefore we find

$$v_{-i}(G) = \begin{cases} 0 & i = 0,\ n-3,\ \text{or} \\ 1 & i = 1,\ n-1,\ \text{or} \\ 6 & i = 2,\ 3,\ \text{or} \\ 2 & \text{otherwise.} \end{cases}$$

Then we have

$$v_i(G) - v_{-i}(G) + i = \begin{cases} -i & i = 2,\ 3,\ n-2,\ n-3,\ \text{or} \\ i & \text{otherwise.} \end{cases}$$

It is easily checked that $\{v_i(G) - v_{-i}(G) + i : i \in \mathbb{Z}_n\} = \mathbb{Z}_n$ hence it is a symmetric starter by Theorem 2.3. □

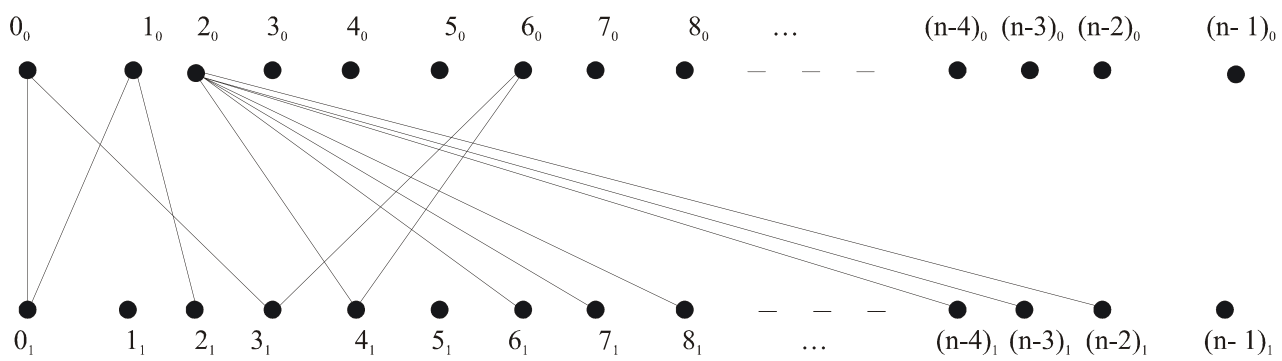

Figure 5: A symmetric starter of $\mathbb{Z}_n$ for an ODC of $K_{n,n}$ by $(P_8 \cup^{(2,0)} S_{n-7}) \cup (n-1)K_1$.

From Figure 5, and for all $i \in \mathbb{Z}_n$, the i^{th} graph isomorphic to the symmetric starter $(P_8 \cup^{(2,0)} S_{n-7}) \cup (n-1)K_1$ has the edges:

$$E(G) = \{(i+2)_1, (i+1)_0, i_1, i_0, (i+3)_1, (i+6)_0, (i+4)_1, (i+2)_0\} \cup \{\{(i+2)_0, (i+j)_1\} : 6 \le j \le n-2\}.$$

Lemma 3.4. *For all integers* $n \ge 8$ *the vector* $v(G) = (0,4,0,3,2,2,2,2,\ldots,2,2,2,3,4,6)$ *is a symmetric starter of* $\mathbb{Z}_n$ *isomorphic to* $(P_9 \cup^{(2,0)} S_{n-8}) \cup (n-1)K_1$.

Proof. For any integer $i \in \mathbb{Z}_n$, we can define the vector $v(G) = (0,4,0,3,2,2,2,2,\ldots,2,2,2,3,4,6)$ as follows.

$$v_i(G) = \begin{cases} 0 & i = 0,\ 2, \text{ or} \\ 4 & i = 1,\ n-2, \text{ or} \\ 3 & i = 3,\ n-3, \text{ or} \\ 6 & i = n-1, \text{ or} \\ 2 & \text{otherwise.} \end{cases}$$

Therefore we find

$$v_{-i}(G) = \begin{cases} 0 & i = 0,\ n-2, \text{ or} \\ 4 & i = 2,\ n-1, \text{ or} \\ 3 & i = 3,\ n-3, \text{ or} \\ 6 & i = 1, \text{ or} \\ 2 & \text{otherwise.} \end{cases}$$

Then we have

$$v_i(G) - v_{-i}(G) + i = \begin{cases} -i & i = 1,\ 2,\ n-2,\ n-1, \text{ or} \\ i & \text{otherwise.} \end{cases}$$

It is easily checked that $\{v_i(G) - v_{-i}(G) + i : i \in \mathbb{Z}_n\} = \mathbb{Z}_n$ hence it is a symmetric starter by Theorem 2.3. □

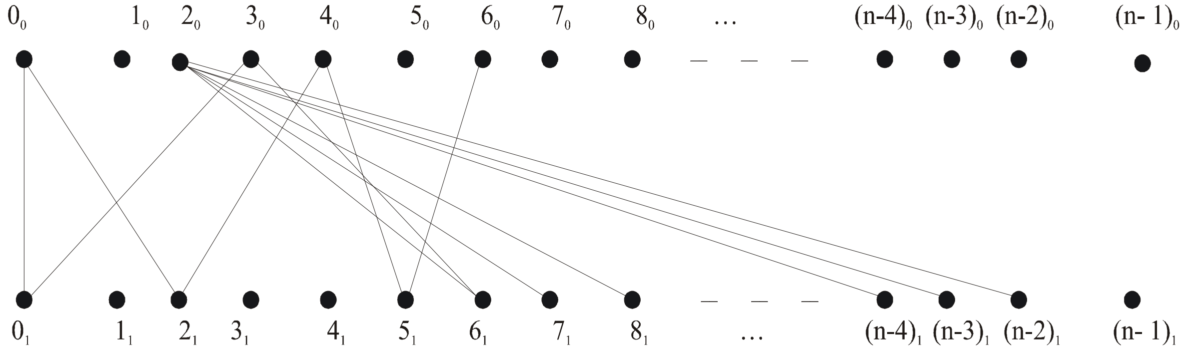

Figure 6: A symmetric starter of $\mathbb{Z}_n$ for an ODC of $K_{n,n}$ by $(P_9 \cup^{(2,0)} S_{n-8}) \cup (n-1)K_1$.

From Figure 6, and for all $i \in \mathbb{Z}_n$, the i^{th} graph isomorphic to the symmetric starter $(P_9 \cup^{(2,0)} S_{n-8}) \cup (n-1)K_1$ has the edges:

$$E(G) = \{(i+6)_0, (i+5)_1, (i+4)_0, (i+2)_1, i_0, i_1, (i+3)_0, (i+6)_1, (i+2)_0\} \cup \{\{(i+2)_0, (i+j)_1\} : 7 \le j \le n-2\}.$$

Lemma 3.5. *For all integers* $n \ge 9$ *the vector* $v(G) = (0,1,4,2,0,2,2,2,\ldots,2,2,2,8,8,4,1)$ *is a symmetric starter of* $\mathbb{Z}_n$ *isomorphic to* $(P_{10} \cup^{(2,0)} S_{n-9}) \cup (n-1)K_1$.

Proof. For any integer $i \in \mathbb{Z}_n$, we can define the vector $v(G) = (0,1,4,2,0,2,2,2,\ldots,2,2,2,8,8,4,1)$ as follows.

$$v_i(G) = \begin{cases} 0 & i = 0,\ 4, \text{ or} \\ 1 & i = 1,\ n-1, \text{ or} \\ 4 & i = 2,\ n-2, \text{ or} \\ 8 & i = n-3,\ n-4, \text{ or} \\ 2 & \text{otherwise.} \end{cases}$$

Therefore we find

$$v_{-i}(G) = \begin{cases} 0 & i = 0,\ n-4, \text{ or} \\ 1 & i = 1,\ n-1, \text{ or} \\ 4 & i = 2,\ n-2, \text{ or} \\ 8 & i = 3,\ 4, \text{ or} \\ 2 & \text{otherwise.} \end{cases}$$

Then we have

$$v_i(G) - v_{-i}(G) + i = \begin{cases} -i & i = 3,\ 4,\ n-3,\ n-4, \text{ or} \\ i & \text{otherwise.} \end{cases}$$

It is easily checked that $\{v_i(G) - v_{-i}(G) + i : i \in \mathbb{Z}_n\} = \mathbb{Z}_n$ hence it is a symmetric starter by Theorem 2.3. □

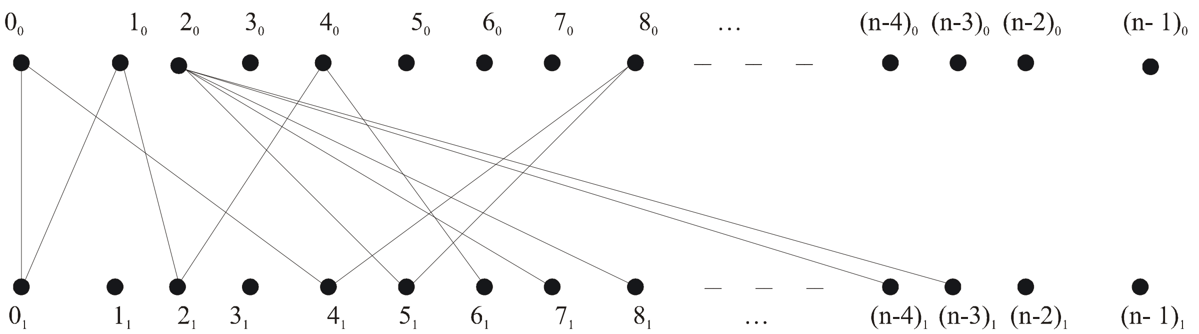

Figure 7: A symmetric starter of $\mathbb{Z}_n$ for an ODC of $K_{n,n}$ by $(P_{10} \cup^{(2,0)} S_{n-9}) \cup (n-1)K_1$.

From Figure 7, and for all $i \in \mathbb{Z}_n$, the i^{th} graph isomorphic to the symmetric starter $(P_{10} \cup^{(2,0)} S_{n-9}) \cup (n-1)K_1$ has the edges:

$$E(G) = \{(i+6)_{1,}(i+4)_0,(i+2)_1,(i+1)_0,i_1,i_0,(i+4)_1,(i+8)_0,(i+5)_1,(i+2)_0\} \cup \{\{(i+2)_0,(i+j)_1\}: 7 \le j \le n-3\}.$$

Lemma 3.6. *For all integers $n \ge 10$ the vector $v(G) = (0,1,4,5,0,3,3,3,3,\ldots,3,3,3,3,8,5,8,1)$ is a symmetric starter of $\mathbb{Z}_n$ isomorphic to $(P_{11} \cup^{(3,0)} S_{n-10}) \cup (n-1)K_1$.*

Proof. For any integer $i \in \mathbb{Z}_n$, we can define the vector $v(G) = (0,1,4,5,0,3,3,3,3,\ldots,3,3,3,3,8,5,8,1)$ as follows.

$$v_i(G) = \begin{cases} 0 & i = 0,\ 4, \text{ or} \\ 1 & i = 1,\ n-1, \text{ or} \\ 4 & i = 2, \text{ or} \\ 5 & i = 3,\ n-3, \text{ or} \\ 8 & i = n-2,\ n-4, \text{ or} \\ 3 & \text{otherwise.} \end{cases}$$

Therefore we find

$$v_{-i}(G) = \begin{cases} 0 & i = 0,\ n-4, \text{ or} \\ 1 & i = 1,\ n-1, \text{ or} \\ 4 & i = n-2, \text{ or} \\ 5 & i = 3,\ n-3, \text{ or} \\ 8 & i = 2,\ 4, \text{ or} \\ 3 & \text{otherwise.} \end{cases}$$

Then we have

$$v_i(G) - v_{-i}(G) + i = \begin{cases} -i & i = 2,\ 4,\ n-2,\ n-4, \text{ or} \\ i & \text{otherwise.} \end{cases}$$

It is easily checked that $\{v_i(G) - v_{-i}(G) + i : i \in \mathbb{Z}_n\} = \mathbb{Z}_n$ hence it is a symmetric starter by Theorem 2.3. □

From Figure 8, and for all $i \in \Gamma$, the i^{th} graph isomorphic to the symmetric starter $(P_{11} \cup^{(3,0)} S_{n-10}) \cup (n-1)K_1$ has the edges:

$$E(G) = \{(i+4)_{0,}(i+6)_1,(i+8)_0,(i+4)_1,i_0,i_1,(i+1)_0,(i+2)_1,(i+5)_0,(i+8)_1,(i+3)_0\} \cup \{\{(i+3)_0,(i+j)_1\}: 9 \le j \le n-2\}.$$

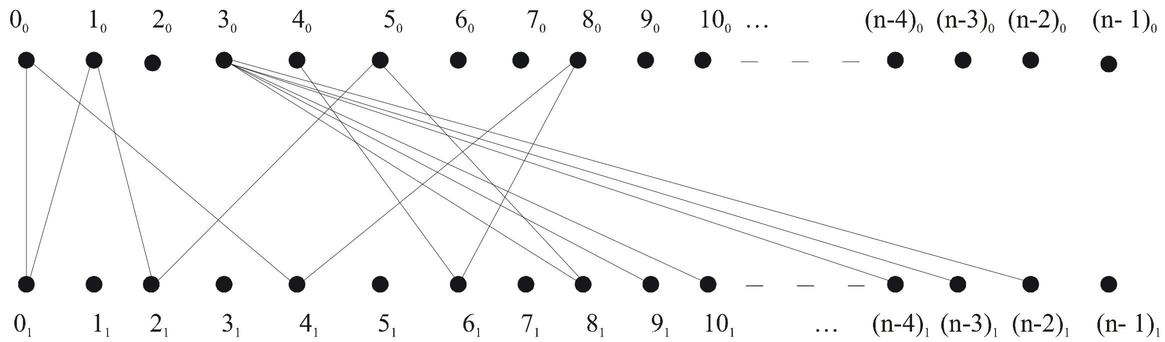

Figure 8: A symmetric starter of $\mathbb{Z}_n$ for an ODC of $K_{n,n}$ by $(P_{11} \cup^{(3,0)} S_{n-10}) \cup (n-1)K_1$

Proof of Theorem 1.1. For $m \leq 4$ the statement was already proved in [3]. For each $m \geq 5$, Lemmas 3.1 to 3.5 provide a symmetric starter of $\mathbb{Z}_n$ with the appropriate graph G. In view of Theorem 2.2, the translates of G form an ODC of $K_{n,n}$. □

Note that ODCs for cases $n = 5$ and $n = 6$ were found already in [3], but not via symmetric starters.

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