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Analysing transportation system reliability: the case study of the metro system of Milan

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Abstract

This paper introduces a methodology to monitor the passenger flow in a subway transport system and analyse the system reliability under different offer and demand scenarios. Motivated by a collaboration with ATM - the company responsible for the management of the public transport in Milan - we focus on the subway system of Milan with the aim of helping operation managers to handle the daily access of travellers to the train stations during Covid-19 pandemic. In details, we first apply a calibration procedure to estimate a reliable OD matrices; then, a model able to monitor the passenger flow by estimating, for each train, the number of passengers getting on and off at each station, along with the load factor of the train along the line. Results highlight the subway sections and the stations most at risk of congestion under different offer and demand scenarios; moreover, eventual queues at each station are estimated. The proposed approach develops a flexible and scalable method to analyse and monitor any urban railway system in any city.

Keywords: OD Matrix, Network Analysis, System Reliability, Metro System, Covid-19.

1 Introduction

In the last decades, due to the increasing in population in urban centers, the travel demand on public transports has rapidly increased. In many metropolitan areas, the metro has become the preferred mode of public transport, due to the low travel times and high accessibility [1]. The rapid increasing in passenger flow has made transport companies to face several challenges for maintaining a high quality level service. Congestion situations can occur, especially in the peak hours, slowing the trains travelling times and thus reducing the passengers quality perception of the service and the travel security. For this reason, many efforts have been put in developing services for passenger flow monitoring, thus obtaining a timely warning on the state of crowds in public places (e.g. [2], [3], [4]).

The COVID-19 pandemic has had a great impact on contemporary public transportation worldwide, leading to an unprecedented decline in travel demand due to the imposed mobility restrictions ([5]). However, despite the decreasing in demand, the International Association of Public Transport (UITP) considers the maintenance of high levels of service to ensure safe distancing as one of the main challenges associated with resuming public transport operations ([6]). The risk of contagion must be minimized by public transport operators both on-board and during passenger waiting time; for this reason, in addition to personal protective equipment and new system of air renovation and filtering, limitations must also be placed on occupation rates to avoid crowding situations and ensure physical distancing ([7]). All major countries around the world have imposed several bus and train capacity constraints (reducing the maximum allowed number of passengers per vehicle) in order to fight the spread of the virus. This is also the case of Italy, where the authorised vehicle capacity has been reduced up to 27% in April 2020. Hence, public transport companies are facing the problem of providing a high quality service respecting the imposed capacity constraints. Therefore, despite the great decrease in travel demand on public transport, with the weaknesses of mobility restrictions, the returning of passengers to public transport systems is imposing to transport companies a great challenge, making the problem of monitoring passenger flow and avoid congestion situations of prior importance ([8]).

Motivated by a collaboration with ATM, the company responsible for the management of the public transport in Milan, the second biggest city in Italy, we analysed the metro system of Milan with the aim of monitoring the passenger flow in the transport system and analyse the system reliability under different offer and demand scenarios. In detail, starting from Origin-Destination matrices, we developed a model able to estimate, for each train, the number of passengers departing and arriving at each station, along with the load factor of the train along the line. By doing so, we were able to extract useful insights for the operation managers, highlighting the subway sections and the stations most at risk of congestion, according to different offer and demand scenarios. Therefore, differently from other works whose objective is to set the optimal frequency of public transport lines in order to conform with the pandemic imposed capacity constraints (e.g., [9] and [10]), we focus on the problem of monitoring passenger flow and system reliability.

The paper is structures as follow: in Section 2 the data at hand are introduced; the used methodology and the related literature review are presented in Section 3; obtained results are shown in Section 4. Conclusions are presented in Section

5.

2 The Milan Metro system

The company responsible for the management of public transport in Milan is ATM (Azienda Trasporti Milanesi), which manages both the surface transport and the underground transport of the municipality. The Milan Metro is currently the largest underground system in Italy for length (96,8 km), number of lines (4), number of stations (113) and ridership (about 1.4 mln of trips per day during working days before COVID-19 pandemic and about 365 mln per year). In Figure 1 the whole infrastructure is represented: the four different lines (M1, M2, M3 and M5) are highlighted and seven interchange stations can be identified; note that each of these stations is considered, both in the data and in the metro system modeling explained in the next paragraphs, as two different stations, one for each line crossing it. In these stations a walking transfer is present to connect the interchanging lines.

Figure 1: The metro infrastructure of Milan, highlighting the four different lines and showing the interchange stations with a white square.

2.1 Available data

To analyse the passenger flow across the metro network, we rely on a set of data collected by the company itself. Three different data sources have been analysed:

- the "turnstiled" data;
- the OD matrices;

- the timetable data.

The "turnstiled" data consists of a counting of turnstiles movements during the day. In details, for each station and each time interval of 10 minutes, the number of passengers crossing the turnstiles is collected, i.e. we know the number of passengers arriving and departing from each station during the day. The OD matrices contain the number of trips between the 113 stations of the metro system collected through the tickets tapping system. Each OD matrix reports the number of trips departing from one station to another in a time interval of one hour during the day (00:00-00:59, 01:00-01:59, ..., 23:00-23:59). Notice that, even if from OD matrices it should be possible to reconstruct information contained in the "turnstiled" data, due to the different systems employed for the data collection process, these two information are not always aligned. This is due to the fact that tickets tapping is not always mandatory when exiting a station, especially for big and congested stations during peak hours to avoid crowding situation around the turnstiles, thus not recording some OD trips; therefore, as shown in the next section, OD matrices results to be biased with respect to "turnstiled" data in some stations and some hours of the day. Finally, the timetable data consists of the train service timetable information provided by the company. Note that, to describe tha data in the next section, we will refer to the 8^{th} of October 2020.

2.2 Modeling of the metro network

The metro network can be modeled as a directed network where the nodes are the stations and the edges are the connections among stations. In detail, let $G(V, E)$ be the network modelling the system, with V the set of nodes, representing the 113 stations, and $E \subset (V \times V)$ the set of edges, representing the 232 connections among stations. Two different types of connections could be identified in the system: *standard edges* - edges modelling the rail line connecting two consecutive stations belonging to the same line - and *interchange edges*, edges modelling the underground walking transfer connecting the two interchanging lines in interchange stations; the obtained network is composed from 218 *standard edges* and 14 *interchange edges*.
Moreover, to model the system, different attributes on both edges and nodes of the network should be considered. Indeed, even if the topology of the network remains the same during the whole day, some features, as for instance the travelling times of trains, vary according to the hour of the day according to the timetable. To take into account the within day variability, we decide to focus on time intervals of one hour, which is a reliable time scale for urban mobility systems ([11]) and it is also the thinnest time scale on which the OD matrices are available. Hence, setting $H = \{00:00 - 00:59, ..., 23:00 - 23:59\}$ as the set of observed time intervals, we construct a directed network with different node/edge attributes. In details, for each edge $(i, j) \in E$, we define:

- $\boldsymbol{TT_{ij}}$, the vector of the edges *Travelling Times* for each hour of the day $h \in H$, which is equal to the average hourly train travelling time for *standard edges*, or equal to the average hourly walking time for *interchange edges*.

Note that, while the average hourly train travelling time between two consecutive stations is obtained looking at the timetable, instead, the average hourly walking time is a data obtained by the company through handheld stopwatches. For each node $i \in V$, we define an attribute related to the *Waiting Time* in that station in the different hours of the day due to the timetable; note that the *Waiting Time* does not only depend on the departing station but also on the arrival one, this because not all trains travelling on a line can join all couple of stations in that lines due to the presence of bifurcations in some of the lines (see Figure 1). We thus evaluate the *Waiting Time* to reach a node j from i as the average time elapsed between two subsequent trains connecting these two stations in a specific hour. Hence, for each node $i \in V$, the following attribute is defined:

- $\boldsymbol{WT_i}$, a matrix containing for each hour of the day $h \in H$ the hourly average Waiting Times to go from node i to all the nodes j, with $j \in V$, which can be directly reached from i taking one train.

Once the modeling of the network is built, we also consider other information, coming from the different available data sources, related to the network. In details, considering the OD matrices, we define as:

- $OD_{ij}(h)$, the number of detected trips from node $i \in V$ to node $j \in V$ for each hour of the day $h \in H$.

To be consistent with the time scale on which the OD matrices are collected, the "turnstiled" data are analysed at hourly interval as:

- $d_i(h)$, the total number of departures from node $i \in V$ for each hour of the day $h \in H$;

- $a_i(h)$, the total number of arrivals from node $i \in V$ for each hour of the day $h \in H$.

As already mentioned in previous paragraphs, even if the two datasets potentially contain the same information about the number of passengers departing and arriving at each station in a specific time interval, the data differ due to the differences in data collection modes. It appears that the information collected in the OD matrices result to be incomplete with respect to the total number of "turnstiled" passengers in each station. In details, considering the data at hand, it can be observed that the two following conditions are not always satisfied as it should be in real situations:

$$\sum_{j \in V} OD_{ij}(h) = d_i(h) \qquad \forall i \in V \qquad \forall h \in H$$

and

$$\sum_{i \in V} OD_{ij}(h) = a_j(h) \qquad \forall j \in V \qquad \forall h \in H.$$

Checking the two previous formula, and taking for example the data related to the 8^{th} of October 2020, we observe that the OD matrices cover around the 85% of departures trips contained in the "turnstiled". In details, the number of trips recorded from "turnstiled" data is about 765 K while the number of trips recorded from OD matrices is about 658 K. To verify the spatio-temporal coverage of OD matrices with respect to "turnstiled" data (i.e. to verify if there is any biased structure in the coverage), we evaluate for each hour h and each station the following values:

$$Arrivals\,coverage\,percentage = \frac{\sum_{j \in V} OD_{ij}(h)}{d_i(h)} \tag{1}$$

and

$$Departures\,coverage\,percentage = \frac{\sum_{i \in V} OD_{ij}(h)}{a_j(h)}. \tag{2}$$

The obtained results shows how OD matrices have a good coverage in almost all the stations, both in terms of departures and arrivals with respect to "turnstiled" data, but a low coverage (e.g., sometimes low than 50% when looking at arriving passengers) in few stations. In Figure 2 and 3, we report as example the obtained values for line M3. Observing the Figures, we notice that looking at departures the coverage is higher than 80% in almost all the stations, with the exception of the central part of the line whose stations reveal a coverage between 60%-80% in the afternoon; focusing, instead, on arrivals, a very low coverage, lower than 50%, is highlighted looking at Centrale and Rogoredo stations during the whole day. Hence, making evident that the lack of departures/arrivals in the OD matrices is time and station specif, the OD matrices appear to be inaccurate and unreliable.
Due to the inaccuracy of the available OD matrices, in the next chapters, we will have to deal with the problem of estimating a proper OD matrix so to correctly analyse the passenger flow across the metro system.

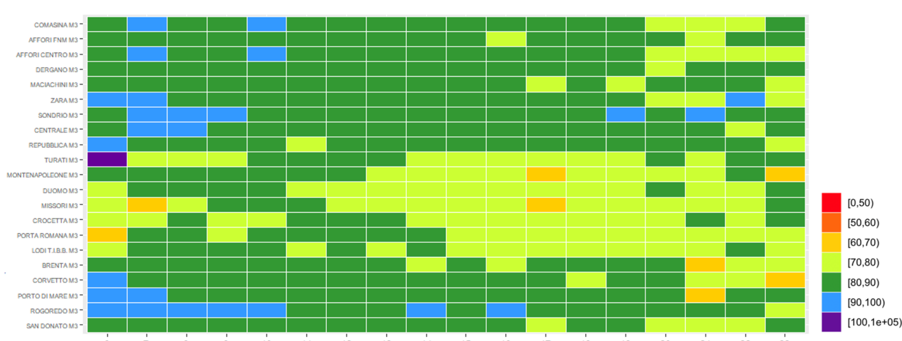

Figure 2: Departures coverage percentage of OD matrices with respect to "turnstiled" data for stations belonging to line M3 during the day.

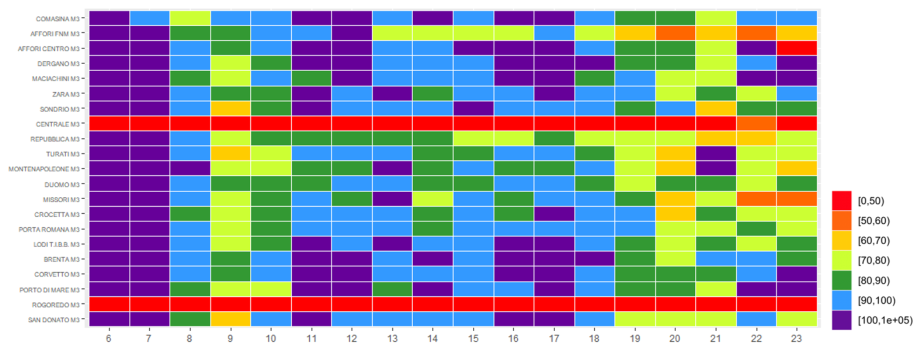

Figure 3: Arrivals coverage percentage of OD matrices with respect to "turnstiled" data for stations belonging to line M3 during the day.

3 Methods

To monitor the reliability of the Milan subway system under different offer and demand scenarios, we need to develop a traffic assignment simulator able to highlight the dynamic of each train, in terms of getting on and off passengers and trains load factor. To do so, accurate OD matrices are required along with a model able to define the route choices for each OD couple and assign the different passengers at each train.

3.1 OD Matrix calibration

OD matrices are fundamental to understand the dynamic of traffic demand as they allow to observe where passengers are coming from and going to. For this reason, as these data are not always available or reliable, a wide literature on the OD matrix estimation and calibration problem has been developed in the last decades.
A broad research area encompasses models which estimate OD matrices using data about zone potential attraction, land use and travel costs; this is for example the case of gravity models, [12] and [13]. Other methods employ a wide range of data source, e.g. traffic counts and assignment of OD matrices to actual service, to asses OD matrices; among these works [14] and [15] employ an information minimisation and entropy maximisation principles and a generalized least square method, respectively, to estimate OD matrices from traffic counts. More recent works (e.g., [16], [17], [18] and [19]) exploit these models to obtain solutions which better fit the available dynamic traffic counts and overcome computational problems related to optimization problems. In addition, the OD matrix estimation optimization problem is severely under-determined ([19], [20]). Another set of techniques employed to calibrate OD matrices are growth models (e.g., [21],[22]), these methods are well suited when an OD matrix is already present and the aim is to calibrate or predict a new OD matrix

knowing the generation and attraction flow of each zone, assuming no changes in travel patterns due to changes in spatial accessibility or supply.
Due to the nature of the problem and the available data, we decide to rely on Growth Models to obtain accurate OD matrices starting from the ones already available in the data according to arrivals and departures in the "turnstiled" data; in details, we make use of the Furness Method, introduced in [22], which is well suited with our data sources; the goodness of this method is discussed in [23].
Let $G(V, E)$ be the directed network modeling the metro system, with V the set of nodes and $E \subset V \times V$ the set of edges, as defined in Section 2.
Recalling Section 2, for each hour of the day $h \in H$, the following data are known:

- $OD_{ij}(h)$: the number of detected trips from node i to node j, with $(i, j) \in V \times V$ in the time interval $h \in H$;
- $d_i(h)$: the total number of departures from node $i \in V$ in the time interval $h \in H$;
- $a_j(h)$: the total number of arrivals in node $j \in J$ in the time interval $h \in H$.

We define $X_{ij}(h)$, with $(i, j) \in V \times V$ and $h \in H$, as the unknown variables which represent the true number of trips from node i to node j at hour h. From now on, until the end of this section, we drop for simplicity of notation the hourly dependence from our data, e.g. referring to $OD_{ij}(h)$ as OD_{ij}. Following the Furness procedure, at each iteration, for each couple of origin i and destination j, a growth factor (τ_{ij}) is applied to obtain a calibrated OD matrix as follow:

$$X_{ij}^{(1)} = \tau_{ij}^{(1)} OD_{ij}$$

$$X_{ij}^{(n)} = \tau_{ij}^{(n)} X_{ij}^{(n-1)}$$

$$X_{ij}^{(n)} = \prod_{k=1}^{n} \tau_{ij}^{(k)} OD_{ij}$$

The growth factors are estimated as to respect the constraints on departures and arrivals at each node:

$$\sum_{j \in V} X_{ij}^{(2n+1)} = d_i \qquad \forall i \in V$$

$$\sum_{i \in V} X_{ij}^{(2n)} = a_j \qquad \forall j \in V$$

At each iteration the row and column growth factors, respectively α_i and β_j, are iteratively evaluated as:

$$\alpha_i^{(n+1)} = \frac{d_i}{\sum_{j \in V} X_{ij}^{(2n)}} \qquad \forall i \in V \qquad n \geq 1$$

$$\beta_j^{(n)} = \frac{a_i}{\sum_{i \in V} X_{ij}^{(2n-1)}} \qquad \forall j \in V \qquad n \geq 1$$

In details the algorithm works as in Algorithm 3.1.

Algorithm 1 Furness Method

Data: Original OD matrix OD, arrival for each node i, departures for each node j, $error_{max}$, $iter_{max}$
Result: New calibrated OD matrix X
Set:
$error = error(OD)$;
$X^{(0)} = OD$;
while $iter < iter_{max}$ *and* $error > error_{max}$ **do**
  Evaluate α_i and multiply each row for α_i;
  Evaluate β_j and multiply each column for β_j;
  $error = error(X^{(iter)})$
end

The error at each iteration is estimated as the squared errors between the total arrivals and departures obtained summing the OD trips and the detected arrivals and departures for each node from "turnstiled" data as:

$$error = \sqrt{\frac{\sum_{i \in V}(\sum_{j \in V} X_{ij} - d_i)^2}{|V|} + \frac{\sum_{j \in V}(\sum_{i \in V} X_{ij} - a_j)^2}{|V|}}$$

At the end of the algorithm a calibrated OD matrices which minimizes the deviation from "turnstiled" data is obtained.

3.2 Route choice selection and passenger flow simulation

To model the dynamics of each train across the network, in terms of both getting on and off passengers and trains load factor, we need to develop a traffic assignment simulator. In the last decades, several methods have been proposed in the literature regarding traffic assignment procedures. The majority of available works deals with the problem of traffic assignment for road traffic networks (see [24] to have a review of the models evolution in this regard), while the literature related to the analysis of subway and/or railway transit systems is less extensive. [8] develops a mathematical passenger route choice and train scheduling model to asses the maximum transport capacity of the Dutch railway network. [25] introduces a schedule-based loading model capable of distribute passengers over the network and validates it with data from the Mass Transit Railway network in Hong Kong. [26] proposes a scheduled-based dynamic traffic assignment model, whose performance are illustrated through a large-scale network of Beijing (Peking) subway. [27] employs a logit multinomial model to study how different covariates (e.g. travel time, waiting time, network topology

and passengers preferences) could influence the estimation of the probability of selecting one route in the Santiago metro for each OD pair; this last work relies on the wide literature of route assignment employing a discrete choice model to estimate the probability of choosing a specific route among a set of selected ones (e.g. [28], [29], [30]).
In our work, we develop a deterministic assignment model in which travel choices are known a priori and passengers do not change their established route if they are in a queue but they wait for the following available train. As alternative, we could have estimated the travel choices through user equilibrium criteria, therefore relying on passengers past experience (passengers memory and habit); however, when the interest is in the monitoring of the system on a particular day or scenario, finding an equilibrium solution is not actually applicable ([25]). Moreover, the existence of a unique equilibrium is just one of the possible cases and a day-to-day process may oscillate among different equilibria or even show a chaotic pattern ([31]). Therefore, since we are interested in investigating and monitoring the performance of the network under different offer and demand scenarios, we stick with the a priori estimation of the route choices.
[32] divides the route choice procedure into two steps: generate a set of possible alternative routes and calculate the probability a given route is selected by passengers. Following this idea, we employ a route choice method which assumes that passengers tend to select the shortest path in terms of needed time considering both waiting, transfer and travelling times; this approach is consistent with classical route choice model [33], as we do not have additional information about passengers preferences (e.g. comfort preferences) to be included in the model. In addition, no data about past passengers route choice are available, making difficult to employ procedures from the discrete choice model literature.
To select the set of possible alternative routes, given an (O,D) pair, all simple paths to reach the destination D from origin O are evaluated on the directed network of the metro system and then the admissible ones are selected following some criteria. Let $G(V, E)$ be a directed network representing a metro system in a fixed time interval, with V the set of nodes representing the stations of the network and $E \subset (V \times V)$ the set of edges representing the connections between stations; each edge, which can be a *standard edges* or *interchange edge*, as specified in Section 2, has an associated weight given by the travelling time or transfer time on that connection in the observed time interval. We have to consider that, as already mentioned, the Milan Metro system is composed by four lines connecting the city center of Milan, the second biggest city in Italy, with the hinterland areas. Therefore, passengers use the metro line both for short trips (e.g. few stops in the center of the city) and for long ones (e.g. to reach the city center from the hinterland). For this reason, as (O,D) pairs include both short and long trips, we decide to not include, in the admissible set of paths for each OD, the k-shortest paths ([34]) as often done in literature (e.g. [35], [36]), as this choice could be unfeasible for short travels. In details, first all the simplest paths between the couple (O,D), coupled with their associated cost in terms of needed times, are computed and then only a subset of admissible ones is selected. The set of admissible paths is selected, for each (O,D) pair,

considering the shortest one and all the paths with a cost lower that the cost of the shortest one plus at maximum a 15% of it. The probability of using a specific path is then evaluated for each OD trip as the weighted inverse of the cost of each admissible path. Note that, as the paths costs change according to the hour of the day, different admissible paths can be identified during the day considering the same OD pair.
After estimating the admissible paths for each couple OD at any hour, the number of OD trips employing a specific path is evaluated considering the number of passengers going to D from O and the probability of using a specific path, i.e. allowing to know, for each hour, the number of passengers travelling on a specific path. In this way, for each hour, we obtain a set of OD trips with associated path which is needed to assign the different passengers at each train of the metro system. Moreover, to assign passengers to relevant trains, departure time for each OD trip on each path is evaluated at thinner time intervals. First, for each hour, the OD trips are distributed on time intervals of 10 minutes considering the percentage distribution of departing passengers from each station and each time interval of 10 minutes in that hour, as observed in the "turnstiled" data. Then, for each time interval of 10 minutes, the OD trips are uniformly distributed on intervals hh of one minute. Finally, at each departure time, we add 30 seconds as the time needed to reach the platform from turnstiles.
To simulate the passenger flow on the metro system, a simulator, which works separately for each line, of the system is implemented. Therefore, all OD trips travelling on paths involving more than one line should be considered as different trips, one for each line, on different sub-paths with different departure times. Lets take, for example, an OD trip, with departure time hh, related to one path going from station $O = i$ to $D = j$ and including an *interchange edge* connecting station k and q belonging to two different lines. In this case, the OD trip is re-allocated into two sub-paths with two different departure times: the first sub-path going from station i until the interchange station k with departure time equal to hh; the second one going from the interchange station q until station j with departure time equal to $hh + WT_{i,k}(\cdot) + TT_{i,q}(\cdot)$, where $WT_{i,k}(\cdot)$ is the *Waiting Time* to go from i to k and $TT_{i,q}(\cdot)$ is the *Travelling Time* to go from i to q, estimated summing the *Travelling Time* of all the crossed edges to go from i to q, evaluated as defined in Section 2 in the hourly time interval to which hh belongs. The same process is adopted for all OD trips on paths covering more than two lines simply obtaining a number of sub-paths equal to the number of crossed lines. Once each OD trip is related to a sub-path covering a single line, we can simulate the passenger flow on the different lines of the metro system. In detail, given a direction l on a specific line, we consider as Z_l the set of trains travelling on l from the timetable and V_l the set of stations in l. To simulate the passenger flow on the metro system, for each train $z \in T_l$ and each station $i \in V_l$:

1. the set of passengers travelling from i to another station $j \in V_l$ with departure time hh before the arrival time of the train z in i which are not already assigned to any previous train are getting on z;

2. the set of passengers which has already got on the train z in the previous station with destination i is getting off;

3. in case some passengers could not get on the train z due to train capacity limitation they are marked as queuing passengers at station i waiting for the next train.

As the assignment model works separately for each line and direction, since the queuing passengers can effect the other lines delaying departure times of some passengers, an iterative procedure should be develop. In detail, if some passengers could not get on a train, thus delaying their arrival on other stations, then the departure time of OD trips on subsequent sub-paths on the other lines should be updated before simulating the passenger flow on the other lines and directions. The algorithm works updating the departures times of each OD trip on each line any time a delay is identified in some OD trips in a line, see Algorithm 2.

Algorithm 2 Assignment model algorithm

```
while iter < iter_max do
    for line and direction l do
        Passenger flow simulation for each train z ∈ Z_l and station i ∈ V_l;
        if ∑_{i∈V_l,t∈Z_l} queue_{z,i} > 0 then
            Update departing times on other lines and directions;
        end
    end
    if ∑_l ∑_{i∈V_l,z∈Z_l} queue_{z,i} = 0 Or the system has converged
    then
        STOP
    else
        iter=iter+1
    end
end
```

To evaluate the algorithm convergence we need to estimate if results are not changing in one iteration with respect to the previous one. To do so, at each iteration *iter* the identified queues for each train z in each station i, $queue_{z,i}$, are compared to the ones at the previous iteration $iter - 1$ and the mean absolute difference (MD) is evaluated as:

$$MD(iter) = \frac{1}{\sum_l |V_l||Z_l|} \sum_l \sum_{i \in V_l, z \in Z_l} (|queue_{z,i}(iter) - queue_{z,i}(iter-1)|) \quad (3)$$

When the difference is zero it means that the model has converged and the system is stable.

4 Analyses and results

In this Section, we display the performance of our model by applying the methods presented in Section 3 on a specific scenario of offer and demand. In details, we simulate the behavior of the metro system in two scenarios that we call scenario A and scenario B. In both cases the demand scenario is the mobility demand occurred the 8^{th} of October 2020; the offer scenario is the timetable effective on that day, but we consider different train capacity offers in the two considered scenarios: in scenario A the train capacity is fixed to the 80% of train capacity under normal condition, while in scenario B the train capacity is fixed to the 27% of train capacity under normal condition; notice that scenario A is the real scenario occurred at the 8^{th} of October 2020, while scenario B is a simulated scenario to observe what would have been happen with more restrictive policies.

4.1 Calibrated OD matrices

First of all, the hourly OD matrices have been calibrated by means of the Furness method, as explained in Section 3.1. To show the accuracy of the obtained calibrated OD matrices, we evaluate the *Departures coverage percentage* and *Arrival coverage percentage* (respectively defined as in (1) and (2)) of these matrices with respect to the data collected by "turnstiled". Notice that the calibrated OD matrices cover the 99% of the "turnstiled" passengers entering in the metro system. In Figure 4 and 5, we report as example the obtained coverage per station and hour for line M3. By comparing these outputs with the ones obtained from the initial OD matrices (Figure 2 and 3), it is trivial to observe that the resulting OD matrices do not show any evident lack of departures/arrivals with respect to "turnstiled" data both for departures and arrivals.

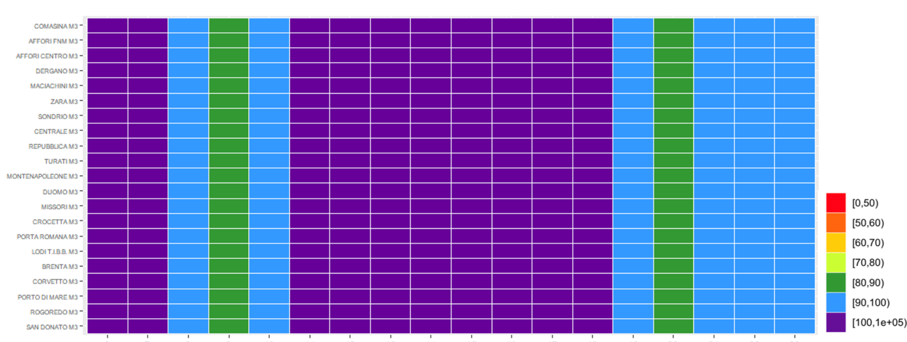

Figure 4: Departures coverage percentage of the calibrated OD matrices with respect to "turnstiled" data for stations belonging to line M3 during the day.

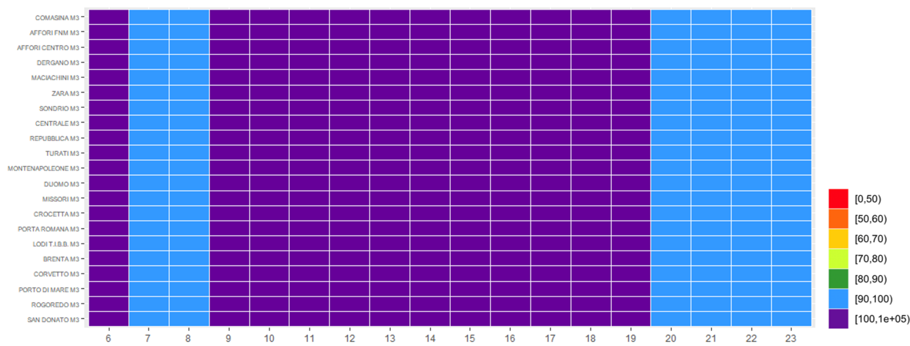

Figure 5: Arrivals coverage percentage of the calibrated OD matrices with respect to "turnstiled" data for stations belonging to line M3 during the day.

4.2 Passengers flow simulation

Then, the admissible travel routes for each couple Origin-Destination in a specific hour are identified. In details, for each OD pair, the admissible paths are selected following the procedure explained in Section 3.2. In Figure 6 the distribution of (O,D) pairs with respect to the number of selected paths at h =08:00-08:59 is reported.

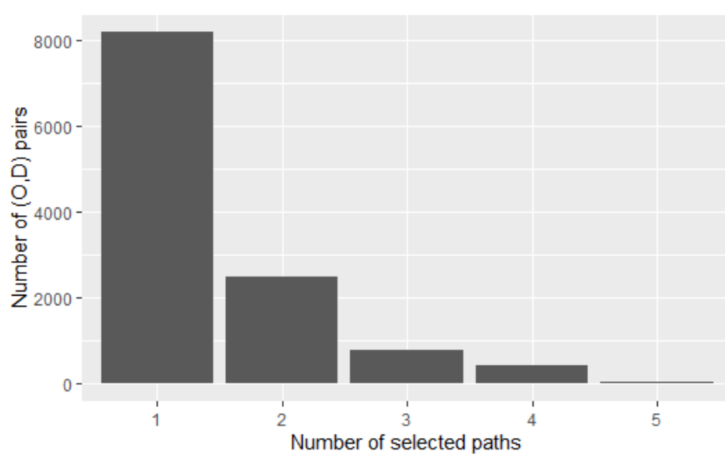

Figure 6: Distribution of (O,D) pairs with respect to the number of selected paths at h =08:00-08:59.

After identifying the paths employed by passengers for each (O,D) pair at each time interval $h \in H$, the assignment model, presented in Section 3.2, is applied. For each train in each line the passenger flow getting on and off and the load factor are evaluated in each station. The eventual forming of queue is estimated when passengers can not get on the train due to capacity constraints.

Simulation with no queues

When considering scenario A, no congestion situations are identified, thus only one iteration in the model is required. In details, the higher value in term of load factor is reached on the M1 line in the railroad from Cadorna M1 to Cairoli M1 at 8:40 and it is equal to 61%.

In Figure 7 and 8, we report some examples of obtained results for some trains in different hours of the day: the first one in the morning rush hour and the second one in the afternoon rush hour, respectively. In details, at each stop of the line, Figure 7 and 8 display the number of passengers getting on and off, the people queuing and the load factor of the train with respect to the capacity under normal condition highlighting with a red line the allowed capacity in the considered scenario (80%). Figure 7 displays results for a train in the morning rush our: a train belonging to line M1, departing from Bisceglie at 08:22 with final destination Sesto F.S. at 09:08. Figure 8 displays results for a train in the afternoon rush our: a train belonging to line M3, departing from San Donato at 18:05 with final destination Comasina at 18:36.

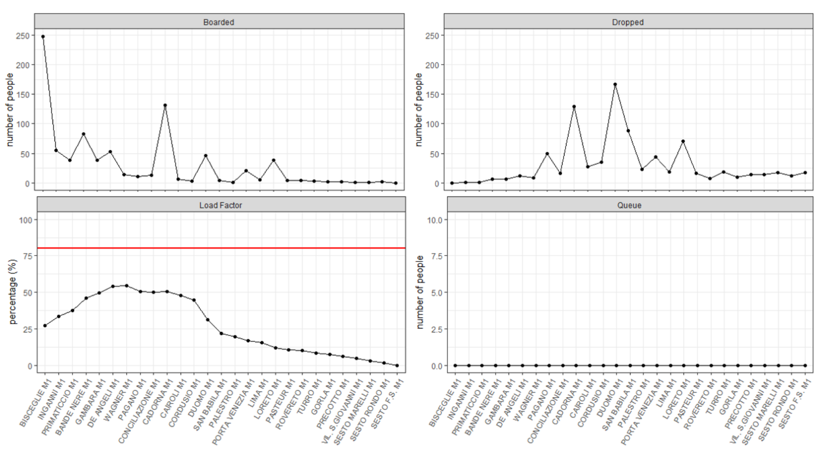

Figure 7: Results for a morning rush hour train under scenario A.

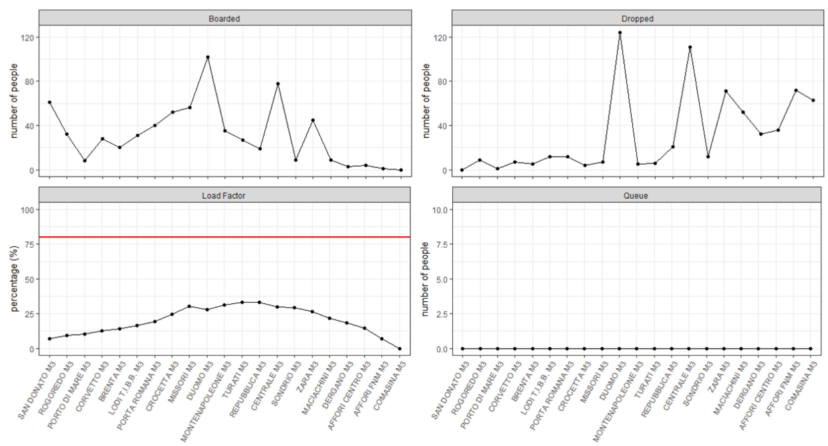

Figure 8: Results for a afternoon rush hour train under scenario A.

Simulation with queues

When considering scenario B, crowding situations happen in many stations generating queues. For this reason the model needs to be iterated until convergence as presented in Section 3.2. In Figure 9 the Mean Absolute Difference, estimated as in (3), of queues between two consecutive iterations is reported showing that convergence is reached at iteration 2.

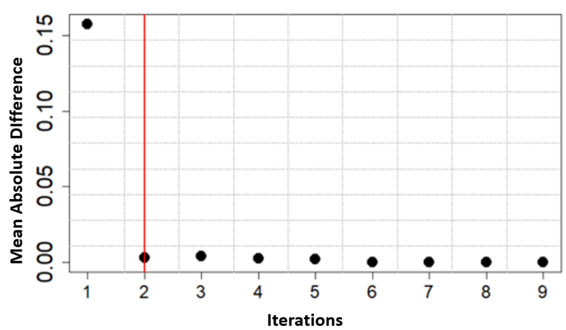

Figure 9: MD of queues between two consecutive iterations highlighting in red when the algorithm converges making $MD(iter) = 0$.

Results reveal that among the monitored 2307 trains, about 55% of them reaches the maximum allowed value of capacity and produce queues in at least one station. In details, looking at the number of stations characterised from queues for each train, we observe a minimum of one and a maximum of eleven stations. For instance, the train departing from Rho at 08:26 with final destination Sesto F.S. at 09:13 (belonging to line M1) generate queues in eleven of the

31 crossed stations, forcing almost 3000 passengers to wait for the subsequent trains.
In Figure 10 and 11, we report some examples of obtained results, exactly as done for scenario A: at each stop of the line, the Figures display the number of passengers getting on and off, the people queuing and the load factor of the train with respect to the capacity under normal condition highlighting with a red line the allowed capacity in the considered scenario (27%). In Figure 10 the obtained results for the same train showed in Figure 7 (a train departing from Bisceglie at 08:22 with final destination Sesto F.S. at 09:08) are reported, showing how in this scenario, due to the congestion on the train, queuing passengers are present in some stations generating crowding situation both on trains and platforms. In details, in the first part of the line (between Bisceglie and Cairoli) the train is almost always full (the maximum capacity is reached) and queues of the order of magnitude of almost one thousand are generated. Figure 11 displays the results obtained for the same train showed in Figure 8 (a train departing from San Donato at 18:05 with final destination Comasina at 18:36) under the reduction of capacity constraints of scenario B. Even in this case, queuing passengers and crowding situations are present. In details, the worst station is Missori where about 1250 passengers can not board and have to wait for the subsequent trains.

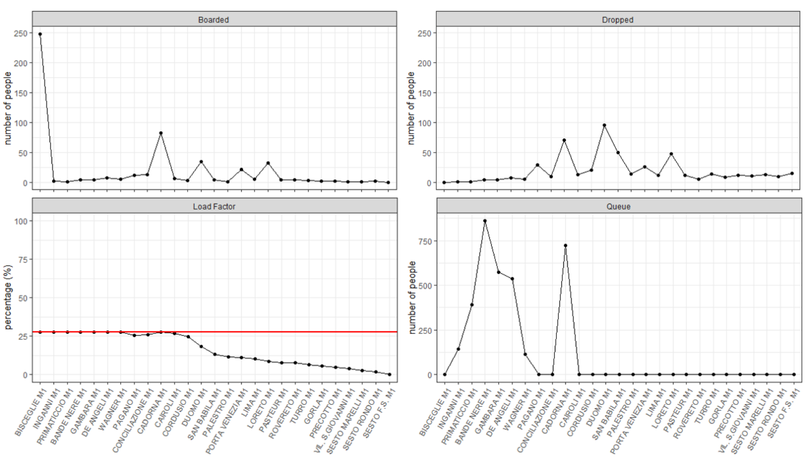

Figure 10: Results for a morning rush hour train under scenario B.

In conclusion, in Figure 12 we highlight the train stations in which a queue is present under scenario B, respectively, in the morning (7:00-8:00) and the afternoon (18:00-19:00) rush hours. Focusing on the morning rush our, there are 58 stations generating queues, which cover all the four lines with the exception of line M5 (the only line with no queues). Notice that, among these 59 stations, all of them generate a queue only on one train platform of the two directions of the line; in details, all the queues are mainly generated in stations in the suburb from trains directed to the city center. Focusing on the afternoon rush

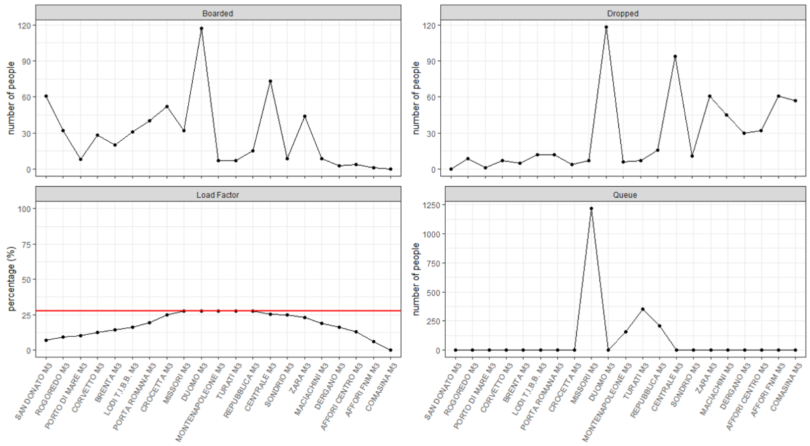

Figure 11: Results for a afternoon rush hour train under scenario B.

our, there are 32 stations generating queues, 17 of which only on one platform (i.e. direction) and 15 of which on both platforms. It can be noticed how these stations are mainly in the city center and are, for the most, not the same stations which are characterised from queues in the morning. All these results suggest that the subway system, as well know from ATM, is mainly used from commuters travelling to the city center in the morning and going outside of the city center in the afternoon.

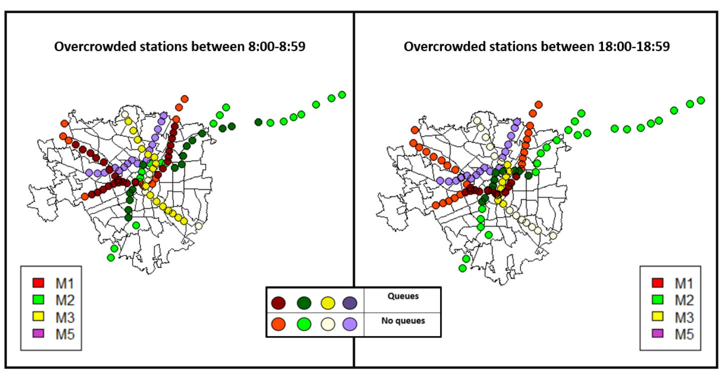

Figure 12: Metro system of Milan highlighting with darker colours the stations with a queue under scenario B, respectively, between 8:00-8:59 and 18:00-18:59.

5 Conclusion

In this work we developed a model able to monitor the passenger flow in a urban railway system, estimating, for each train, the number of passengers getting on and off at each station, along with the load factor of the train along the line. In details, we first apply a calibration procedure to estimate a reliable OD matrix starting from a prior OD matrix coupled with accurate data related to the departures and arrivals at each station. Then, the route choice for each OD pair is estimated by properly taking into account the costs for each path in terms of needed time, considering both waiting, transfer and travelling times. Finally, a simulator able to assign the different passengers at each train, monitoring the load factor of the train and therefore handling potential queues is implemented. The developed model has been tested on the metro system of Milan, in Italy, to monitor the reliability of the network under different offer and demand scenarios. Results provide useful insights which have been shared with ATM, the company responsible for the management of the public transport in Milan, allowing them to best handle the management of the service. In detail, the provided results highlight the subway sections and the stations most at risk of congestion, showing the crowding of each train and station platform at any hour of the day. It is obvious that these type of information turn out to be more than ever essential in the present worldwide scenario in which public transport companies, due to the spread of COVID-19, are facing the problem of providing a high quality service respecting several imposed capacity constraints, thus minimizing crowding situations.
In conclusion, despite this work focuses on the metro system of Milan in Italy, the math and the methodology developed easily allow to analyse and monitor any larger urban railway system in different cities, therefore obtaining a flexible and scalable approach.

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