Traces of Sobolev functions with one square integrable directional derivative

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Abstract

We consider the Sobolev spaces of square integrable functions v, from $\mathbb{R}^n$ or from one of its hyperquadrants Q, into a complex separable Hilbert space, with square integrable sum of derivatives $\sum_{\ell=1}^n \partial_\ell v$. In these spaces we define closed trace operators on the boundaries ∂Q and on the hyperplanes $\{r_\ell = z\}$, $z \in \mathbb{R}\backslash\{0\}$, which turn out to be possibly unbounded with respect to the usual L^2-norm for the image. Therefore we introduce also bigger trace spaces with weaker norms which allow to get bounded trace operators, and, even if these traces are not L^2, we prove an integration by parts formula on each hyperquadrant Q. Then we discuss surjectivity of our trace operators and we establish the relation between the regularity properties of a function on $\mathbb{R}^n$ and the regularity properties of its restrictions to the hyperquadrants Q.

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