Overdetermined problems with possibly degenerate ellipticity, a geometric approach

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Abstract. Given an open bounded connected subset Ω of \mathbb{R}^n , we consider the overdetermined boundary value problem obtained by adding both zero Dirichlet and constant Neumann boundary data to the elliptic equation $-\operatorname{div}(A(|\nabla u|)\nabla u) = 1$ in Ω . We prove that, if this problem admits a solution in a suitable weak sense, then Ω is a ball. This is obtained under fairly general assumptions on Ω and A. In particular, Amay be degenerate and no growth condition is required. Our method of proof is quite simple. It relies on a maximum principle for a suitable P-function, combined with some geometric arguments involving the mean curvature of $\partial\Omega$.