Regularity and nonexistence results for anisotropic quasilinear elliptic equations in convex domains

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Let $\Omega \subset \mathbf{R}^n$ $(n \geq 3)$ be a smooth bounded domain, consider n numbers $m_i \geq 2$ for all $i = 1, ..., n$, take $\lambda > 0$ and $p > 1$. In a recent paper [1], it was shown that existence and nonexistence results for nontrivial solutions to the following anisotropic quasilinear elliptic problem

$$\begin{cases} -\displaystyle\sum_{i=1}^{n} \partial_i \left(|\partial_i u|^{m_i - 2} \partial_i u \right) = \lambda u^{p-1} & \text{in } \Omega \\ u \geq 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{1}$$

are in fact related to the regularity of the solutions to the following "coercive regularized" problem

$$\begin{cases} -\displaystyle\sum_{i=1}^{n} \partial_i \left[(|\partial_i w|^{m_i - 2} + \varepsilon (1 + |Dw|^2)^{(m_- - 2)/2}) \partial_i w \right] + \lambda |w|^{p-2} w = f & \text{in } \Omega \\ w = 0 & \text{on } \partial\Omega \end{cases} \tag{2}$$

where $\varepsilon > 0$, $m_- := \min\{m_1, \ldots, m_n\}$, f is a smooth function and $\partial_i = \partial/\partial x_i$ for $i = 1, ..., n$. For (1) and (2) we show that the convexity of Ω plays an important role in regularity and nonexistence results: using recent results in [2] we improve the statements in [1].

References

[1] I. Fragalà, F. Gazzola, B. Kawohl, *Existence and nonexistence results for anisotropic quasilinear elliptic equations*, Ann. Inst. H. Poincaré A.N.L. 21 (2004), 715–734.

[2] G.M. Lieberman, *Gradient estimates for anisotropic elliptic equations*, preprint (2004).