CONSTRUCTION OF ENERGIES ON COMPACT SUBSETS OF $\mathbb{R}^N$

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ABSTRACT. We study the possibility to construct quadratic forms on compact subsets of $\mathbb{R}^N$. The method we use is the following: if $K \subseteq \mathbb{R}^N$ is compact, $r > 0$, $\alpha \geq 0$ and u is a continuous function, let $f_r^{(\alpha)}$ be the Dirichlet integral on the r-neighborhood of K , normalized by a factor $r^{\alpha-N}$; we take the Γ-limit, with respect to the uniform convergence, of $f_r^{(\alpha)}(u)$, as $r \to 0$.

For regular compact sets we obtain the integral on K of the square of the tangent gradient of u . For totally disconnected compact sets we always obtain zero. Finally we give an example of compact set for which the upper and lower Γ-limits are different.

We also define a dimension of the compact set K to be the infimum of the numbers α so that $f_r^{(\alpha)}$ Γ-converges to 0 . We call it capacity dimension and prove that it is less than or equal to the box dimension.

The capacity dimension of regular sets equals the topologic one. Totally disconnected sets have capacity dimension equal to 0. Finally we give an example in which it is grater than the Hausdorff one.

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