THE THREE-DIMENSIONAL FLOW IN A CHANNEL PAST A SEMISUBMERGED OBSTACLE: SOLVABILITY OF A LINEARIZED PROBLEM

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Abstract

We consider the three-dimensional problem of the steady flow of a heavy ideal fluid past a surface-piercing obstacle in a rectangular channel of constant depth. The flow is parallel at infinity upstream, with constant velocity c. We discuss an approximate linear problem obtained in the limit of a "flat obstacle". This is a boundary value problem for the Laplace equation in a three-dimensional unbounded domain, with a second order condition on part of the boundary (the Neumann-Kelvin condition). By a Fourier expansion of the potential function, we reduce the three-dimensional problem to a sequence of elliptic problems in a two-dimensional strip for the Fourier coefficients. A unified treatment of the two dimensional problems can be achieved by a special variational approach, relying on some a priori properties of finite energy solutions; as a result, we prove unique solvability for $c \neq c_{m,k}$, where $c_{m,k}$ is a known sequence of values depending on the dimensions of the channel and on the limit length of the obstacle. Accordingly, we can prove the existence of a solution of the three-dimensional problem; the related flow has in general a non trivial wave pattern at infinity downstream. We also investigate the regularity of the solution in a neighborhood of the obstacle. The meaning of the singular values $c_{m,k}$ is discussed from the point of view of the nonlinear theory.

AMS subject classifications: 35J20, 35J25,76B15.

Keywords: Three dimensional flow, Neumann-Kelvin condition, variational problems.