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Multiscale Modelling with Application to Paediatric Cardiac Surgery

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***Abstract*— The study of the haemodynamics of 3D models of vascular districts with complex anatomy with the aid of numerical models requires to prescribe correct conditions at the boundary of the district of interest. The cardiovascular system is a highly integrated circuit and global systemic effects cannot be neglected even when the interest lays on a specific local area. To this aim, a novel approach, called geometrical multiscale, has been devised where models of different level of detail (and computational cost) are coupled together. In particular, we focus in this work on an application where the 3D Navier–Stokes equations are coupled with a non-linear system of ordinary differential equations governing the systemic circulation. The application is the simulation of different procedures for paediatric cardiac surgery.**

I. Introduction

Surgery for the treatment of congenital heart diseases has been improving very rapidly in recent years. The developed surgical repairs often impose major reconstructive procedures, creating a totally new circulation. The so-called ‘Fontan circulation’ refers to a cardiovascular configuration resulting from a group of operations used to by-pass the non-functional right heart. In such a univentricular circulation, blood returning from the body reaches the lungs via direct blood vessel connections without a pumping chamber. Different surgical procedures have been developed to create the Fontan circulation.

One of these procedures is the Total CavoPulmonary Connection (TCPC). In the TCPC the superior and the inferior venae cavae (SVC and IVC, respectively) are directly connected to the right pulmonary artery, the latter by means of an intra-atrial or extra cardiac tunnel. This final configuration is often obtained through an intermediate stage, the Bidirectional CavoPulmonary Anastomosis (BCPA), where only the SVC is connected to the right pulmonary artery. The two surgical stages (BCPA and TCPC) lead to peculiar geometries (T-shaped and cross-shaped, respectively) which are associated with unusual fluid-dynamics. Major problems possibly impacting the univentricular circulation are related to energy losses and blood distribution from the body into the lungs. The absence of a functioning right ventricle limits the energy available for pulmonary blood flow.

Recent advancements in numerical simulation techniques make it possible to set up models of the preoperative situation in a specific patients and to reproduce in silico different techniques (virtual surgery). However, although current medical imaging provide very detailed information on the patient anatomy, it is impractical - if not impossible - to simulate the whole cardiovascular system at the same level of details. Reduced models can be used instead, from one dimensional models able to accurately describe pulse wave propagation to lumped parameter models describing the general behaviour of the whole cardiovascular system. Since these models operate at different space dimensions their coupling calls for an appropriate mathematical analysis and the development of suitable numerical techniques (Formaggia et al., 2009). In fact, the coupling of different models serves two purposes. On the one hand it allows to account for the local/systemic interactions. On the other hand it provides a less invasive and more physically sound way of imposing boundary conditions to the three dimensional problem, based on averaged flux or pressures rather than point-wise values.

II. Methods

A. The mathematical models

The lumped parameter model is based on the decomposition of the cardiovascular system in several components, usually called compartments, describing the evolution of blood flow and pressure. Any spatial distribution is lost and the mathematical model of each compartment reduces a (possibly non-linear) system of ordinary differential equations, as it is shown in Fig. 1 for the case of a model of the left ventricle.

In this case, the corresponding differential model is given by

$$\frac{dV}{dt} = Q = \frac{dC}{dt}P + C\frac{dP}{dt} + M_Q(t)$$

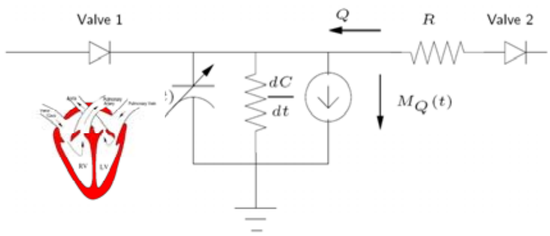

Fig. 1. An example of a lumped parameter model for the ventricle.

where V and P denote the ventricle volume and average pressure, respectively, while C is a time dependent compliance and M_Q is a forcing term that accounts for the contraction of the muscle fibres. All different compartments are then joined in a network by imposing continuity of pressure and conservation of mass at each junction. This gives rise eventually to a system of non-linear ordinary differential-algebraic equations which may be written synthetically as

$$\mathbf{f}(\dot{\mathbf{y}}(t), \mathbf{y}(t), t, \mathbf{g}) = \mathbf{0}$$

where $\mathbf{y}$ indicates the vector of flux and pressure in each compartment that forms the model, and $\mathbf{g}$ the set of given input data (the ones modelling heart action, for instance). The components of $\mathbf{y}$ corresponding to data at the interface between models will be coupled to the corresponding quantities calculated with the three dimensional model. The latter are determined by solving the well-known time dependent Navier-Stokes equations numerically. To this aim, we have relied to the finite volume scheme implemented in the commercial software Fluent by Ansys Inc. The vessel walls have been kept fixed, so the conditions at the wall boundary prescribe a zero velocity.

Different techniques may be devised for the coupling at the interface between models (Formaggia et al., 2009). A stable coupling is obtained if conservation of mass flux together with the continuity of total pressure is imposed (Quarteroni et al., 2003)

$$\mathbf{p}_{NS} + \frac{1}{2}||\mathbf{u}_{NS}||^2 = p_{1D} \qquad \int_\Gamma \mathbf{u}_{NS} d\gamma = Q_{1D}$$

where $\mathbf{p}_{NS}$ and $\mathbf{u}_{NS}$ are pressure and velocity in the Navier-Stokes model, respectively, while p_{1D} and Q_{1D} are the average pressure and flux provided by the reduced model. With Γ we have indicated the interface section.

A staggered scheme has been adopted, where the two models are advanced sequentially at each time step. Thus, the coupling conditions may be split as boundary conditions for the Navier-Stokes problem and forcing terms for the lumped parameter one. In particular, in the Navier-Stokes model averaged fluxes may be imposed on the proximal section, while pressure boundary conditions may be adopted on distal sections. One of the advantages of using a staggered procedure is the possibility of employing an available and well tested flow solver as “black box”. Indeed, in this work the lumped parameter model has been implemented as a user defined function (UDF) of Fluent.

The staggering procedure introduces an additional approximation error in time, of order O(Δt), which is however of the same order of that of the adopted scheme, since both Navier-Stokes and lumped parameter models are advanced in time by a backward Euler method. For the Navier-Stokes equations a segregated solver is used, where velocity and pressure calculations are split using a Chorin-Temam type technique. This is a rather standard procedure which allows to reduce computational costs, again at the price of a O(Δt) error.

Other coupling techniques suitable also for higher order methods, based on Lagrange multipliers, control theory or variational methods, as well as the extension to fluid/structure interaction models have been studied and some are reported in Formaggia et al. (2010). We mention that boundary conditions on average flux or pressure are not standard for the Navier-Stokes problem, whose classic formulation would require point-wise boundary data instead (see Veneziani et al., 2005 and 2007).

A graphical illustration of the adopted multiscale approach is shown in Fig. 2.

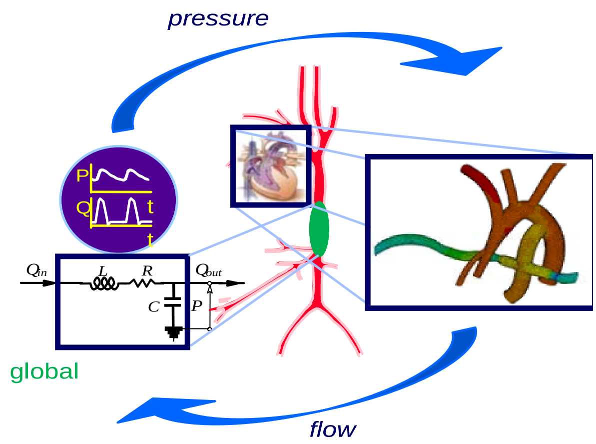

Fig. 2. A graphical illustration of the multiscale model adopted.

B. The application to the hypoplastic left heart syndrome

We illustrate an application of geometrical multiscale modelling to the study of the haemodynamics of different surgical procedures for the treatment of hypoplastic left heart syndrome (Migliavacca et al., 2006, Pennati et al., 2010). They are depicted in Fig. 3.

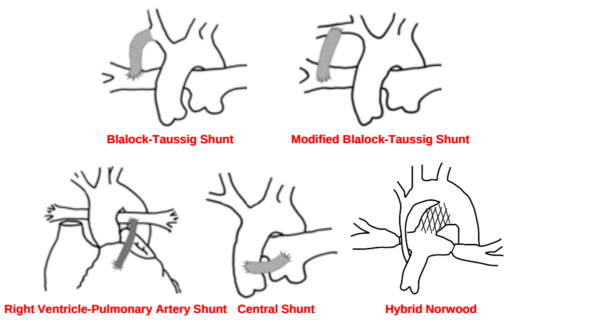

Fig. 3 Five different surgical techniques to treat the hypoplastic left heart.

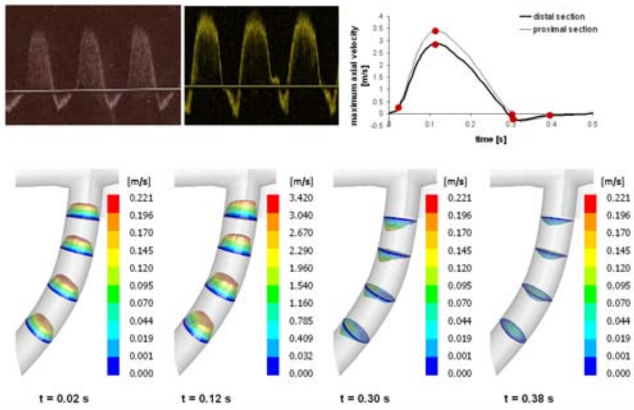

Fig. 4. Doppler tracing in a right ventricle-pulmonary artery shunt and velocity profiles in the main pulmonary artery. The numerical and clinical results are in good agreement in terms of backward flow.

Application to modelling of congenital heart diseases has been so far the most significant one. In fact prescription of known flows and pressures at the inlets/outlets of a 3D, ‘stand-alone’ model does not allow one to determine realistic haemodynamics.

In the right ventricle – pulmonary artery shunt (Sano operation) backward flow in the shunt is the major concern for the clinicians and its preoperative quantification was possible with the multiscale approach. Figure 4 reports some of the results obtained from the simulations which demonstrate antegrade flow through the conduit only during systole as observed with Doppler analysis performed in patients.

Another example is related to the most recent correction for the treatment of hypoplastic left heart syndrome: the so-called hybrid Norwood. This operation was introduced as an alternative strategy with a less invasive initial procedure, combining surgical techniques (branch pulmonary artery banding) and interventional cardiology techniques (stenting of the ductus arteriosus and balloon atrial septostomy). Result of the hybrid Norwood circulation is a parallel between (i) the banded pulmonary arteries in series with the pulmonary vascular bed and (ii) the stented ductus arteriosus in series with the systemic vascular bed. To avoid cardiac overload and an uneven distribution of the cardiac output to the pulmonary and systemic circulations, a proper balance between the two systems should be achieved. Since in the first period of life the pulmonary vascular resistances (PVR) are approximately ten times lower than the systemic vascular resistances (SVR), they should be increased by an appropriate narrowing of the pulmonary arteries to obtain comparable resistances in the two parallel branches.

The results from the multiscale simulations (Fig. 5) show how, by varying the pulmonary bandings diameter and the ductal diameter, the major impact on the variables of interest is due to the banding diameter of the pulmonary arteries. This has to be ascribed to the presence of SVR and PVR: the former is much bigger than the resistance represented by the stented ductus, while the latter is comparable to the resistances given by the bandings. The best performances, i.e. balanced flow ratios, occur when 2 mm bandings are used. Narrower bandings lead to an inadequate pulmonary flow while larger bandings do not assure an appropriate systemic flow and diminish the amount of oxygen available to tissues.

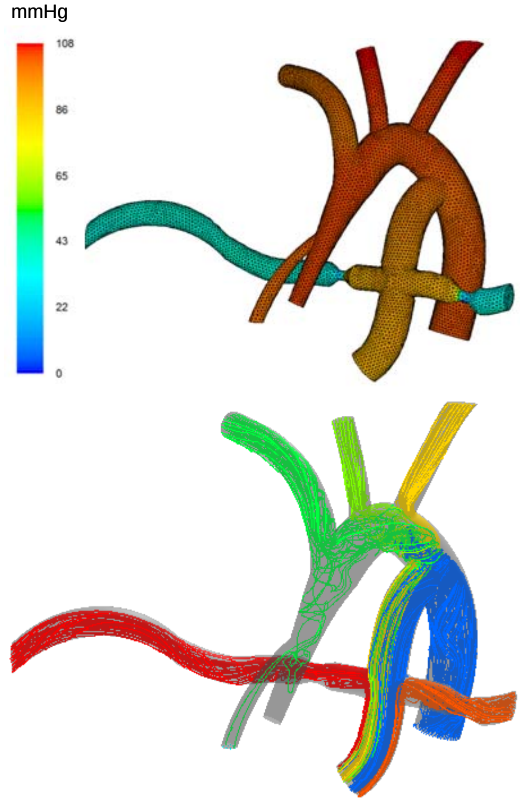

Fig. 5. Hybrid Norwood: pressure contours (upper panel) and pathlines coloured on the basis of the outlet vessels at systolic peak (lower panel) (Corsini et al., 2010).

III. Conclusion

With the aid of the current imaging techniques it is nowadays possible to reconstruct three-dimensional models which reproduce both the complex anatomy of the investigated region and the details of surgical reconstructions. This fact enables to perform patient specific simulations where, on the basis of computational fluid dynamic simulations, the surgical correction efficacy in terms of haemodynamics can be quantitatively evaluated. In this context, it is important to take into consideration the possible interactions with the global circulation. The proposed multiscale technique allows the integration in the analysis of aspect of the physiology of the global circulation at a reasonable computational cost.

Acknowledgment

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