Existence and equilibration of global weak solutions to Navier–Stokes–Fokker–Planck systems

Endre Süli
Mathematical Institute, University of Oxford
endre.suli@maths.ox.ac.uk

We show the existence of global-in-time weak solutions to a general class of coupled FENE (Finitely Extensible Nonlinear Elastic) type and Hookean type bead-spring chain models that arise from the kinetic theory of dilute solutions of polymeric liquids with noninteracting polymer chains. The class of models involves the unsteady incompressible Navier–Stokes equations in a bounded domain in $\mathbb{R}^d$, $d = 2$ or 3, for the velocity and the pressure of the fluid, with an elastic extra-stress tensor appearing on the right-hand side in the momentum equation. The extra-stress tensor stems from the random movement of the polymer chains and is defined by the Kramers expression through the associated probability density function that satisfies a Fokker–Planck-type parabolic equation, a crucial feature of which is the presence of a centre-of-mass diffusion term. We require no structural assumptions on the drag term in the Fokker–Planck equation; in particular, the drag term need not be corotational. With a square-integrable and divergence-free initial velocity datum $\mathbf{u}_0$ for the Navier–Stokes equation and a nonnegative initial probability density function ψ_0 for the Fokker–Planck equation, which has finite relative entropy with respect to the Maxwellian M, we prove, *via* a limiting procedure on certain regularization parameters, the existence of a global-in-time weak solution $t \mapsto (\mathbf{u}(t), \psi(t))$ to the coupled Navier–Stokes–Fokker–Planck system, satisfying the initial condition $(\mathbf{u}(0), \psi(0)) = (\mathbf{u}_0, \psi_0)$, such that $t \mapsto \mathbf{u}(t)$ belongs to the classical Leray space and $t \mapsto \psi(t)$ has bounded relative entropy with respect to M and $t \mapsto \psi(t)/M$ has integrable Fisher information (w.r.t. the measure $\mathrm{d}\mu := M(\mathbf{q})\,\mathrm{d}\mathbf{q}\,\mathrm{d}\mathbf{x}$) over any time interval $[0, T]$, $T > 0$. If the density of body forces $\mathbf{f}$ on the right-hand side of the Navier–Stokes momentum equation vanishes, then a weak solution constructed as above is such that $t \mapsto (\mathbf{u}(t), \psi(t))$ decays exponentially in time to $(\mathbf{0}, M)$ in the $\mathbf{L}^2 \times L^1$ norm, at a rate that is independent of $(\mathbf{u}_0, \psi_0)$.

The talk is based on a series of recent papers [1–3] with John W. Barrett (Department of Mathematics, Imperial College London).

References

[1] J W Barrett and E Süli. Existence and equilibration of global weak solutions to kinetic models for dilute polymers I: finitely extensible nonlinear bead-spring chains. M3AS 21 (6), 1-79, 2011. DOI: 10.1142/S0218202511005313. Available from http://www.worldscinet.com/m3as/00/0001/S0218202511005313.html

[2] J W Barrett and E Süli. Existence and equilibration of global weak solutions to kinetic models for dilute polymers II: Hookean-type bead-spring chains. M3AS, 22(5), 2012. To appear. Available from http://arxiv.org/abs/1008.3052

[3] J W Barrett and E Süli. Reflections on Dubinskii's nonlinear compact embedding theorem. Publ. Inst. Math. (Belgrade). To appear. Available from http://arxiv.org/abs/1101.1990