Existence results for quasilinear elliptic exterior problems involving convection term with nonlinear Robin boundary conditions

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Abstract

In this paper, we show a result of the existence of solutions for elliptic exterior problems involving convection term and nonlinear Robin boundary conditions of the type

$$(P_\lambda) \qquad \begin{cases} -div(\xi(x)\nabla u) + u & = & \lambda f(x,u,\nabla u) & \text{in} & \Omega \\ \xi(x)\partial_\nu u + \alpha(x,u)u & = & 0 & \text{on} & \partial\Omega, \end{cases}$$

where Ω is a smooth exterior domain in $\mathbb{R}^N (N \geq 3)$, ν is the unit vector of the outward normal on $\partial\Omega$, ξ is a positive continuous function, λ is a real parameter, nonlinearity $f(x,u,\nabla u) = h(x,u) + g(x,\nabla u)$ is such that h is a sublinear function and g is bounded from above by a gradient term (or convection term) of the type $|\nabla u|^\beta$ with $0 < \beta < 1$, and $\alpha(x,\mu)\mu$ is a continuous function such that $0 \leq \alpha(x,\mu) \leq b(x)|\mu|^{p-2}, \quad \forall\ (x,\mu) \in \Omega \times \mathbb{R}^*$ and $\alpha(x,0)=0$, where $1 < p < \frac{2(N-1)}{N-2}$.

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