Multiplicity and concentration of solutions for elliptic systems with vanishing potentials

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In this talk we use variational methods to study the strongly coupled elliptic system

$$\begin{cases} -\Delta u + \lambda a(x)u = \frac{p}{p+q} |u|^{p-2} u|v|^q, \\ -\Delta v + \lambda b(x)v = \frac{q}{p+q} |u|^p |v|^{q-2}v, \\ u, v \in \mathcal{D}^{1,2}(\mathbb{R}^N), \end{cases}$$

where $N \geq 3$, $\lambda > 0$ is a parameter, p, q > 1 and $p + q < 2^* := 2N/(N-2)$. We suppose that the potentials are nonnegative and the intersection of the sets where they vanish has positive measure. A technical condition, imposed on the product of the potentials, allows us to consider a setting where we do not assume any positive lower bound for the potentials. Considering the associated functional, defined on an appropriated subspace of $\mathcal{D}^{1,2}(\mathbb{R}^N) \times \mathcal{D}^{1,2}(\mathbb{R}^N)$, we are able to establish results on the existence and multiplicity of solutions for the system when the parameter λ is sufficiently large. We also study the asymptotic behavior of these solutions when $\lambda \to \infty$.

This is a jointly work with E.A.B. Silva and M. Xavier.