Intertwining semiclassical solutions to a Schrödinger-Newton system

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We consider a Schrödinger-Newton system in $\mathbb{R}^3$, obtained by coupling together a magnetic linear Schrödinger equation with Newton's gravitational law, which was proposed by Penrose in 1998. It leads to the problem

$$\begin{cases} \left(-\hbar i\nabla + A\right)^2 u + V(x)u = \hbar^{-2}\left(W * |u|^2\right) u, \\ u \in L^2(\mathbb{R}^3, \mathbb{C}), \\ \hbar\nabla u + iAu \in L^2(\mathbb{R}^3, \mathbb{C}^3), \end{cases}$$

where $\hbar > 0$ is Planck's constant, $A : \mathbb{R}^3 \to \mathbb{R}^3$ is a magnetic potential, $V : \mathbb{R}^3 \to \mathbb{R}$ is an electric potential, and the convolution kernel is the Coulomb potential $W(x) = |x|^{-1}$. We assume that A and V are compatible with the action of a group G of linear isometries of $\mathbb{R}^3$. Given a group homomorphism $\tau : G \to \mathbb{S}^1$ into the unit complex numbers, we prove the existence of semiclassical solutions $u_\hbar : \mathbb{R}^3 \to \mathbb{C}$ which satisfy

$$u_\hbar(gx) = \tau(g)u_\hbar(x)$$

for all $g \in G$, $x \in \mathbb{R}^3$. We show that there is a combined effect of the symmetries and the electric potential on the number of solutions of this type.

This is joint work with Silvia Cingolani and Simone Secchi.