Superlinear elliptic problems with sign changing coefficients

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Abstract: Via variational methods, we study multiplicity of solutions for the problem

$$\begin{cases} -\Delta u = \lambda b(x)|u|^{q-2}u + a\,u + g(x,u) & \text{in} \quad \Omega\,, \\ u = 0 & \text{on} \quad \partial\Omega\,. \end{cases}$$

where a simple example for $g(x,u)$ is $|u|^{p-2}u$; here a, λ are real parameters, $1 < q < 2 < p \leq 2^*$ and $b(x)$ is a function in a suitable space L^σ.

We obtain a class of sign changing coefficients $b(x)$ for which two non-negative solutions exist for any $\lambda > 0$, and a total of five nontrivial solutions are obtained when λ is small and $a \geq \lambda_1$.

Note that this type of results are valid even in the critical case.