Localization of P-S and Cerami sequences in a mountain pass geometry.

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Abstract Let X be a real Banach space. For $\Phi \in C^1(X, R)$ and $e \in X\backslash\{0\}$, set $\Gamma = \{f \in C([0,1], X) : f(0) = 0 \text{ and } f(1) = e\}$ and then $c = \inf_{f\in\Gamma} \max_{t\in[0,1]} \Phi(f(t))$.
Suppose also that there exists $r \in (0, \|e\|)$ such that $\inf_{\|u\|=r} \Phi(u) \geq \max\{\Phi(0), \Phi(e)\}$, a situation which is usually referred to as a mountain pass geometry for Φ.

Under these hypotheses, it is well-known that there exists a Palais-Smale sequence of approximate critical points of Φ converging to the level c:

$$\text{there exists } \{u_n\} \subset X \text{ such that } \Phi(u_n) \to c \text{ and } \Phi'(u_n) \to 0.$$

Indeed, there is even a Cerami sequence:

$$\text{there exists } \{v_n\} \subset X \text{ such that } \Phi(v_n) \to c \text{ and } (1 + \|v_n\|)\Phi'(v_n) \to 0.$$

Let S be a closed subset of X such that $\{0, e\} \in S$ and $c_S = c$ where $c_S = \inf_{f\in\Gamma_S} \max_{t\in[0,1]} \Phi(f(t))$ and $\Gamma_S = \{f \in \Gamma : f(t) \in S \text{ for all } t \in [0,1]\}$. In this case, one might expect to find approximate critical points converging to level c close to S.

We show that this is true for P-S sequences, but not necessarily for Cerami sequences

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