Jeff Cheeger: *Curvature bounds and applications to* 4*-dimensional Einstein metrics*

Abstract: We will discuss some ideas concerning the structure of Riemannian manifolds which have been developed over roughly the last 40 years, with contributions from a number of mathematicians.

In the first lecture, we will describe the structure of balls of a small but definite size, in complete Riemannian manifolds with bounded sectional curvature, say $|K_{M^n}| \leq 1$. This leads to a generalization for such Riemnnian manifolds, of the "thick-thin" decomposition for Riemann surfaces with a metric of constant curvature -1. We will also discuss the "good chopping", technique, which enables one to surround an arbitrary subset, by an n-dimensional submanifold with boundary, for which the area and the second fundamental form of the boundary are controlled.

As examined in the next two lectures, these ideas gain considerable scope when combined with scaling. They then provide a structure theory on a suitable intrinsic scale, for arbitrary complete Riemannian manifolds, whose sectional curvature might not be bounded. When specialized to 4-dimensional Einstein manifolds, which might have arbitrarily thin parts, the theory can be combined with more standard techniques and used to obtain bounds on the sectional curvature and in certain cases, the existence of thick parts of a definite size. Eventually, the well known positivity of the Gauss-Bonnet integrand for 4-dimensional Einstein manifolds enters this part of the discussion. The structure theory, together with chopping, enables the positivity to be exploited in local form, via the Gauss-Bonnet formula for manifolds with boundary.