Workshop on Semi-implicit and semi-Lagrangian methods for hyperbolic problems

Sponsored by INDAM-GNCS, local organizer: Luca Bonaventura

MOX-Dipartimento di Matematica
Politecnico di Milano
Sala di Consiglio del Dipartimento di Matematica
Edificio 14 'La Nave'

March 1 13.45 Opening and welcome

14.00-14.40

Gabriella Puppo (Università dell'Insubria)
Conservative-non conservative schemes for nonlinear conservation laws.

In this work, we study the possibility of computing the numerical solution of hyperbolic conservation laws, using different formulations for the same problem. We show that the Lax Wendroff theorem, which guarantees that weak solutions travel with the correct speeds, requires only the consistency of the final update. Thus we exploit the freedom allowed by the theorem to improve the accuracy of the numerical solutions using convenient not necessarily conservative formulations of the system of PDEs. In particular, we compute the stage values of a Runge Kutta update of the solution using primitive variables, while enforcing the conservative form only on the final update of each time step. The resulting schemes provide solutions which can be less polluted by spurious oscillations than standard finite volume schemes, and, in some cases, the computational times are noticeably reduced.

14.40-15.20

Matteo Semplice (Università di Torino)
Very high order CWENO reconstructions for finite volume schemes.

An high order finite-volume scheme puts the following strain on the reconstruction procedure: it should be able to compute accurate and non-oscillatory point values of the variables at many locations on the cell boundary, starting from the cell averages on the given cell and on its neighbours. This difficulty is increased if the scheme employs h-adaptivity (wide variety in the topology of neighbours, need to compute subcell averages in refinement), well-balancing (reconstruction points also inside the cells), error indicators that require numerical quadrature on the cell. In such situations the WENO technique suffers from the difficulty of computing suitable sets of weights for each reconstruction point (existence, optimality, non-negativity, ...). However the CWENO construction is much more flexible since the linear weights need not satify accuracy requirements. An additional advantage is that CWENO computes the nonlinear weights for a reconstruction polynomial to be later evaluated at reconstruction points and the computation of nonlinear weights need not be repeated for each reconstruction point. Many CWENO reconstructions have been considered in the literature, up to order 4 in two space dimensions and order 5 in one space dimension. In this talk I will focus on CWENO reconstructions for 1D grids of order 5 and above, introducing an hierarchic construction that in principle may be extended to arbitrary order.

15.20-16.00

Emiliano Cristiani (IAC-CNR)
Blended numerical schemes for the advection equation.

In this talk we propose a method to couple two or more explicit numerical schemes approximating the same time-dependent PDE, in order to create new schemes which inherit advantages (and drawbacks) of the original ones. We consider both advection equations and fully nonlinear conservation laws. By coupling a macroscopic (Eulerian) scheme with a microscopic (Lagrangian) scheme, we get a new kind of multiscale numerical method.

16.00-16.40

Coffee break

16.40-17.20

Simone Cacace (Università Sapienza, Roma)
A new approach to the numerical solution of ergodic problems for Hamilton-Jacobi equations.

We propose a new approach to the numerical solution of ergodic problems arising in the homogenization of Hamilton-Jacobi (HJ) equations. It is based on a Newton-like method for solving inconsistent systems of nonlinear equations, coming from the discretization of the corresponding ergodic HJ equations. We show that our method is able to solve efficiently cell problems in very general contexts, e.g., for first and second order scalar convex and nonconvex Hamiltonians, weakly coupled systems, dislocation dynamics and mean field games, also in the case of more competing populations. We present a large collection of numerical tests in dimension one and two, showing the performance of the proposed method, both in terms of accuracy and computational time.

17.20-18.00

Giuseppe Visconti (Università dell'Insubria)
Kinetic models for traffic flow with multivalued diagrams.

Experimental studies on vehicular traffic provide data on quantities like density, flux, and mean speed of the vehicles. However, the diagrams relating these variables (the fundamental and speed diagrams) show some peculiarities not yet fully reproduced nor explained by mathematical models. For this reason, we study new traffic models which allow one to reproduce the experimentally observed scattering of fundamental diagrams in the phase of congested traffic and to predict a sharp phase transition between the free and the congested phases of traffic, with a sharp capacity drop across the phase transition. Resting on the methods of kinetic theory, first we introduce a Boltzmann-type model characterized by simple binary interaction rules and by a steady-state solution whose structure allows one to reduce the computational cost of the time evolution towards the equilibrium. Although the diagrams provided by the model show the capacity drop, they do not reproduce the scattering of data. Therefore, we extend the construction of the model to the case of more than one type of vehicles having different microscopic characteristics and in this case the scattered behavior observed in the congested phase results as consequence of the fact that a given road occupancy can be obtained with different compositions of the mixtures. Finally, we propose a Fokker-Planck type kinetic model based on taking a suitable asymptotic limit of a single-population Boltzmann model in which the binary interactions are replaced by assuming that the drivers react to the mean speed. The Fokker-Planck approximation allows one to compute easily the steady-state and the resulting diagrams provide both the capacity drop and the scattering of data without using the multi-population framework.

18.00-18.40

Roberto Ferretti (Università di Roma 3)
Recent results and open issues in the SL treatment of nonlinear conservation law.

Semi-Lagrangian (SL) schemes are usually applied to problems concerning environmental fluid dynamics, plasma physics and the Hamilton-Jacobi equations, but their application to (especially hyperbolic) nonlinear conservation laws has not been studied in equal depth. In this talk, I will present a review of recent result in this direction, both theoretical and numerical. A special emphasis will be given to conservative SL schemes and to the treatment of 'small' viscous terms added to hyperbolic conservation laws.

March 2

9.20-10.00

Elisabetta Carlini (Università Sapienza, Roma)
A Semi-Lagrangian scheme for the Fokker-Planck equation.

We propose a Semi-Lagrangian scheme for the Fokker-Planck equation. The scheme is first order, explicit, preserves non-negativity, conserves the mass and allows large time steps. We present numerical simulations in one and two dimensions and applications to Mean Field Game Problem and Hughes model for pedestrian flow.

10.00-10.40

Raffaele D'Ambrosio (Università di Salerno)
Modified collocation methods for stiff problems.

The talk is focused on the numerical solution of stiff problems by collocation based numerical methods. While classical methods, e.g. Runge-Kutta methods, suffer from order reduction phenomenon when applied to stiff problems, the methods we present here are free from order reduction, since their order of convergence remains uniformly constant overall the integration interval. The employed technique mainly relies on suitable modifications of the classical collocation technique, without heightening the computational cost. A-stable and L-stable methods with uniform high order of convergence will be presented, also in comparison with classical collocation based Runge-Kutta methods. Implementation issues in a variable stepsize environment are also given and a selection of numerical tests will be presented.

10.40-11.20

Coffee break

11.20-12.00

Leonardo Scandurra (Università di Catania)
Semi-Implicit Method for All Mach Number Flow for the Euler Equations of Gas Dynamics on Staggered Grids.

An original numerical method to solve the all-Mach number flow for the Euler equations of gas dynamics on staggered grids is presented. The system is discretized to second order in space on staggered grid, in a fashion similar to the Nessyahu-Tadmor central scheme for 1D models and Jang-Tadmor central scheme for 2D modes, thus simplifying the flux computation. This approach turns out to be extremely simple, since it requires no equation splitting. We consider the isentropic case and the general case. Both approaches are based on IMEX strategy, in which some term is treated explicitly, while other terms are treated implicitly, thus avoiding the classical CFL restriction due to acoustic waves. The schemes are implemented to second order accuracy in time. Suitably well-prepared initial conditions are considered, which depend on the Mach number. In one space dimension ,we obtain the same profiles found in the literature for the isentropic case and for the general Euler system at all Mach numbers. Current work is related on the development of second order accurate schemes for 2D problems and higher order accurate schemes for 1D and 2D problems.

12.00-12.40

Giovanni Tumolo (ICTP Abus Salam, Trieste)
Recent progresses in the development of an adaptive DG dynamical core.

Some new progresses are presented on an adaptive discretization approach for NWP model equations, which combines the semi-Lagrangian technique with a TR-BDF2 semi-implicit time discretization and with a discontinuous Galerkin spatial discretization with (arbitrarily high) variable and dynamically adaptive element degree. These new developments include a more quantitative description of the efficiency gain given by the p-adaptivity and by the novel TRBDF2 based semi-Lagrangian time integrator compared with the off-centered trapezoidal rule, the introduction of the orography representation in z-coordinate and the development of a mass conservative formulation on distorted meshes.

12.40-13.30

Francis X. Giraldo (Naval Postgraduate School, Monterey)
The role of Time-Integration for Operational Geophysical Fluid Dynamics Models.

In this talk, I will describe the issues that face time-integration methods for production-type geophysical fluid dynamics models, such as weather prediction models. My group at the Naval Postgraduate School has been developing a new class of weather model for the U.S. Navy. This new model, called NUMA, uses advances numerical methods such as spectral elements and discontinuous Galerkin methods with adaptive mesh refinement and has been shown to scale to millions of Message-Passing-Interface (MPI) ranks. NUMA also scales quite well (up to 90% efficiency) on Graphics-Processing-Units (GPUs). However, much of the success of these models rely on both the accuracy and efficiency of time-integrators. Although explicit time-integrators scale well on these systems, they require too small a time-step to be able to run a real atmospheric simulation. Therefore, we rely on both Implicit-Explicit (IMEX) and fully-implicit methods but these methods pose other challenges such as scalability on both MPI and GPU systems. In this talk, I will describe our work in this particular area but will also give an overview of the general NUMA model and its application to weather prediction.