Parsimonious HJM Modelling for Multiple Yield-Curve Dynamics

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Talk Outline

1. The Raising of the Basis
2. Multiple-Curve Modelling
3. Model Calibration and Numerical Examples
4. Conclusions and Future Developments
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The Raising of the Basis

1. The Raising of the Basis

2. Multiple-Curve Modelling

3. Model Calibration and Numerical Examples

4. Conclusions and Future Developments
The Raising of the Basis

The Credit Crunch – I

- Classical interest-rate models were formulated to satisfy by construction no-arbitrage relationships, which allow to hedge forward-rate agreements in terms of zero-coupon bonds.

- Starting from summer 2007, with the spreading of the credit crunch, market quotes of forward rates and zero-coupon bonds began to violate the usual no-arbitrage relationships under
  - the pressure of a liquidity crisis reducing the credit lines, and
  - the possibility of a systemic break-down suggesting that counterparty risk could not be considered negligible any more.
The resulting picture, see Henrard (2007), describes a money market where forward rates of different tenors seem to act as a different underlying assets.

Indeed, complex dynamical patterns seem to drive market data.

→ There is evidence of a large, persistent and time-varying component of the Euribor-Eurepo spread that cannot not be explained only by counterparty credit risk.

→ The sharp rise in the Euribor-Eurepo spread of September 2008 is only found three-four months later in the CDS spread series, confirming that a liquidity crisis needs time to evolve as credit crisis, see Eisenschmidt and Tapking (2009).
The Credit Crunch – III

Historical series of Euribor-1y minus Eurepo-1y spread (black line) and a synthetic index formed by senior one-year CDS of a basket of twelve representative European banks (red line). Values are in basis points.
Money-Market Data – I

- Many authors extended yield-curve bootstrapping to a multi-curve setting eventually resulting in new pricing models, where each curve belongs to a different underlying asset.
  
  Among them we cite: Bianchetti (2009), Chibane and Sheldon (2009), Kijima et al. (2009), Martínez (2009), Mercurio (2009, 2010), Fujii et al. (2010), Kenyon (2010).

- As shown in Pallavicini and Tarenghi (2010) there are evidences that the money market for Euro area has moved to a multi-curve setting for what concerns the pricing of plain-vanilla instruments like interest-rate swaps, and some derivative contracts.

- On the other hand, the money market for Euro area does not quote options on all rate tenors.
In Euro area only options on the six-months tenor are widely listed, while the three-months tenor is present only in few quotes.

- Swaptions entering a swap of a tenor of one-year and cap/floors with a maturity less than two years are quoted on three-month tenor.
- Some broker, since few months, start contributing cap/floors on three- and six-month tenor for all maturities.
- All other derivative contracts are quoted on six-month tenor.

Thus, any model which requires a different dynamics for each term-structure, has the problem that market quotes cannot be found to fix all its degrees of freedom.

- Hence, the requirement of using a parsimonious model, since the hypothesis of introducing different underlying assets may lead to over-parametrization issues that affect the calibration procedure.
Risk-Free Rates – I

- We found on the money market Eonia swap contracts (OIS) up to thirty years, while Eurepo is quoted only up to one year.
  - Longer maturities are only indexed on Euribor and Eonia indices.
- Because of the plurality of available OIS instruments and of the reduced credit/liquidity exposure on overnight deposits, to many extent Eonia rates are the best available proxy for risk-free rates.
  - This point has been stressed by many authors, and we refer to Fujii et al. (2010) for more detailed arguments.
  - We refer to Brigo et al. (2011) for the impact of collateralization (and counterparty risk) on discounting.
Risk-Free Rates – II

- We assume that the market is arbitrage free, hence postulating the existence of a risk-neutral measure.
  - Under risk-neutral measure every (risk-free) tradable asset instantaneously increases its value at the risk-free rate $r_t$, which we identify with the Eonia rate.

- We introduce (risk-free) zero-coupon bond prices and instantaneous forward rates as

$$P_t(T) := \mathbb{E}_t \left[ - \int_t^T du \, r_u \right], \quad f_t(T) := \mathbb{E}_t^T [ r_T ]$$

where the first expectation is taken under risk-neutral measure, and the last expectation is taken under a measure whose numeraire is $P_t(T)$ (hereafter simply $T$-forward measure).
A whole risk-free curve can be bootstrapped by usual methods starting from Eonia fixing and OIS market quotes.

For future use we define risk-free linearly compounding forward rates

\[ E_t(T, x) := \frac{1}{x} \left( \exp \left\{ \int_{T-x}^{T} du f_t(u) \right\} - 1 \right) \]
Euribor Rates – I

- Euribor rates \( (L_t(T)) \) are the indices used as reference rate for deposits in the Euro area. Further they are used as underlying asset for many interest-rate derivatives (IRS, basis swaps, ...).
- As usual we can introduce forward rates \( (F_t(T, x)) \) defined as
  \[
  F_t(T, x) := \mathbb{E}_t^T [ L_{T-x}(T) ]
  \]
  which are by construction martingales under the \( T \)-forward measure.
- We consider one-day Euribor rates as being risk-free, so that we identify one-day Euribor rates with Eonia rates.
- By pushing this analogy further we interpret Euribor rates as microscopic rates at the same level of the short-rate, and we write
  \[
  r_t = \lim_{x \to 0} L_t(t + x), \quad f_t(T) = \lim_{x \to 0} F_t(T, x)
  \]
The usual no-arbitrage relationship between (risk-free) zero-coupon bond prices and Euribor rates holds only for non-defaultable counterparties and instruments without liquidity risk. Hence, for a generic tenor $x > 0$ we get

$$L_t(t + x) \neq \frac{1}{x} \left( \frac{1}{P_t(t + x)} - 1 \right)$$

Hence, when the presence of credit and liquidity risks invalidate the possibility of replicating Euribor indexed deposits with non-risky bonds $P_t(T)$.

Indeed, interest-rate modelling should consider Euribor rates of different tenors as different assets.
Euribor Rates – III

- Forward rates bootstrapped from the market according to Pallavicini and Tarenghi (2010). On the $x$-axis we have the rate start-dates, while on the $y$-axis we have the value of the forward rates. Market data observed on 15 February 2011.
Talk Outline

1. The Raising of the Basis

2. Multiple-Curve Modelling
   - Extending the HJM Framework
   - Recovering Market Rates
   - Volatility Dynamics

3. Model Calibration and Numerical Examples

4. Conclusions and Future Developments
Our proposal is to extend the logic of the HJM framework to describe with a single family of Markov processes all the yield curves we are interested in.

In the literature many authors proposed generalizations of the HJM framework starting, see for instance Cheyette (2001), Andersen and Andreasen (2002), Carmona (2004), Andreasen (2006), or Chiarella (2010).

We refer to such papers for a complete list of references on HJM related topics.

In recent papers Martínez (2009) and Fujii (2010) extended the HJM framework to deal with multiple-yield curves.
Models Requirements – II

- Let us summarize the basic requirements the model must fulfill:
  1. existence of a risk-free curve, with instantaneous forward rates $f_t(T)$
  2. existence of Euribor rates, typical underlying of traded derivatives, with associated forwards $F_t(T, x)$
  3. no-arbitrage dynamics of the $f_t(T)$ and the $F_t(T, x)$ (both being $T$-forward measure martingales) ensuring the limit case

$$f_t(T) = \lim_{x \to 0} F_t(T, x) \quad (1)$$

- possibility of writing both the $f_t(T)$ and the $F_t(T, x)$ as functions of a common family of Markov processes, so that we are able to build parsimonious yet flexible models.

- We stress that our approach models only observed rates as in Libor Market Model approaches.
Generalized HJM Dynamics – I

- We choose, under the $T$-forward measure, the following dynamics.

$$
\begin{align*}
    df_t(T) &= \sigma_t^*(T) \cdot dW_t \\
    d(\kappa(T,x) + F_t(T,x)) &= \Sigma_t^*(T,x) \cdot dW_t
\end{align*}
$$

$$
\frac{\kappa(T,x) + F_t(T,x)}{\kappa(T,x) + F_t(T,x)} = \Sigma_t^*(T,x) \cdot dW_t
$$

with $f_0(T)$ and $F_0(T,x)$ bootstrapped from market quotes, and

$$
\sigma_t(T) := \sigma_t(T; T, 0) , \quad \Sigma_t(T,x) := \int_{T-x}^T du \sigma_t(u; T, x)
$$

where $\sigma_t(u; T, x)$ is a (row) volatility vector process, and $W_t$ is a (row) vector of independent Brownian motions.

- The set of shifts $\kappa(T,x)$ must satisfy: $\kappa(T,x) \approx 1/x$ if $x \approx 0$. 
The particular choice of a shifted forward Euribor dynamics ensures the limit $f_t(T) = \lim_{x \to 0} F_t(T, x)$, and is formally equivalent to the evolution of risk-free simple rates $E_t(T, x)$.

In literature, direct modelling of shifted forward rates is also considered in Eberlein and Kluge (2007) (see also references therein), and in Papapantaleon (2010).

Our approach focuses on market quantities and leaves us the freedom of choosing the $\sigma_t(u; T, x)$ and the $\kappa(T, x)$ such as to exactly calibrate level and skew of Euribor rates of maturity $T$ and tenor $x$.

Thus, sensitivity computation may be easily performed.

See also stochastic local volatility model presented by Torrealba (2010), where, after having calibrated the parameters of the volatility process, the local term allows for an exact calibration to some relevant market quotes.
**Constraints on the Volatility Process – I**

- Let us analyse more in detail the dynamics of the shifted forward Libors under risk-neutral measure. By integrating the SDE over the time period \([0, t]\) we get

\[
\ln \left( \frac{\kappa(T, x) + F_t(T, x)}{\kappa(T, x) + F_0(T, x)} \right) = \int_0^t \Sigma^*(T, x) \cdot \left( dW_s - \frac{1}{2} \Sigma_s(T, x) \, ds + \int_s^T du \, \sigma_s(u; u, 0) \, ds \right)
\]

- Our goal is substituting the right-hand side with a tractable formula expressed in term of a family of Markov processes.

  \[\text{This is a requirement similar to the one adopted in the HJM framework when zero-coupon bond pricing formula is considered.}\]
To ensure the tractability and a Markovian specification of the model, we extend the single-curve HJM approach by Cheyette (1992) or Ritchken and Sankarasubramanian (1995), by setting

$$\sigma_t(u; T, x) := h_t \cdot (q(u; T, x)g(t, u))$$  \hspace{1cm} (3)

where $h_t$ is a matrix adapted process, $q$ is a deterministic vector function, and $g$ is defined as

$$g(t, u) := \exp \left\{ - \int_t^u dv \lambda(v) \right\}$$

with $\lambda$ a deterministic vector function.

Further, we add the condition $q(u; u, 0) = 1$ to ensure that in the limit case $x \to 0$ we recover the standard HJM separability condition.
Hence, by plugging the expression for the volatility, we get

\[
\ln \left( \frac{\kappa(T, x) + F_t(T, x)}{\kappa(T, x) + F_0(T, x)} \right) = (4)
\]

\[
G^*(t, T - x, T; T, x) \cdot \left( X_t + Y_t \cdot \left( G_0(t, t, T) - \frac{1}{2} G(t, T - x, T; T, x) \right) \right)
\]

where the vectorial deterministic functions \( G_0 \) and \( G \) are defined as

\[
G_0(t, T_0, T_1) := \int_{T_0}^{T_1} dv \, g(t, v)
\]

\[
G(t, T_0, T_1; T, x) := \int_{T_0}^{T_1} dv \, q(v; T, x)g(t, v)
\]
Emerging Driving Markov Processes – II

The vector process $X_t$ and the matrix process $Y_t$ are defined as

$$X_t := \sum_{k=1}^{N} \int_0^t (h_sg(s, t))^* \cdot \left( dW_s + h_s \int_s^t dv g(s, v) \, ds \right)$$

$$Y_t := \int_0^t ds (h_sg(s, t))^* \cdot (h_sg(s, t))$$

Thanks to our volatility assumption they result to be Markov, and their dynamics is given by

$$dX_t = (Y_t^* \cdot 1 - \lambda(t)X_t) \, dt + h_t^* \cdot dW_t$$

$$dY_t = (h_t^* \cdot h_t - (\lambda^*(t)Y_t + Y_t\lambda(t))) \, dt$$

with $X_0 = 0, Y_0 = 0$. 
OIS Par-Rates

- For sake of completeness we may also compute Eonia simple rates $E_t(T, x)$ by plugging the separable volatility form within the relationship

  $$1 + xE_t(T, x) = \exp \left\{ \int_{T-x}^{T} dy f_t(y) \right\}$$

- Thus, we get a standard HJM result:

  $$\ln \left( \frac{1 + xE_t(T, x)}{1 + xE_0(T, x)} \right) = G_0^*(t, T - x, T) \cdot \left( X_t + Y_t \cdot \left( G_0(t, t, T) - \frac{1}{2} G_0(t, T - x, T) \right) \right)$$  \hspace{1cm} (5)

- OIS par-rates may be constructed by composing Eonia simple rates.
IRS Par-Rates

- Our framework also allows us to derive an (approximated) expression for swap rates dynamics.
- Let us consider a swap with
  - a floating leg of tenor $x$ paying Euribor at times $\{T_{a+1}, \ldots, T_b\}$, and
  - a fixed leg of tenor $\bar{x}$ paying at times $\{T_{\bar{a}+1}, \ldots, T_{\bar{b}} = T_b\}$.
- The par rate $S_{t}^{ab}(x)$ is given by
  \[
  S_{t}^{ab}(x) := \frac{\sum_{k=a+1}^{b} \alpha_k P_t(T_k) F_t(T_k, x)}{\sum_{k=\bar{a}+1}^{\bar{b}} \bar{\alpha}_k P_t(T_k)}
  \]
  where $\alpha$ and $\bar{\alpha}$ are the year fractions, respectively of the floating and the fixed leg.
It is possible to derive also an approximated dynamics for IRS par-rates, by freezing rates’ drift (see Schrager and Pelsser (2004)).

We get under the swap measure

\[ dS_{ab}^t(x) \approx (S_{ab}^t(x) + \psi_{ab}^t(x)) \sum_{k=a+1}^{b} \delta_{k}^{ab}(x) \Sigma^*(T_k, x) \cdot dW_t \quad (6) \]

where

\[ \psi_{ab}^t(x) := \frac{\sum_{k=a+1}^{b} \alpha_k P_0(T_k) \kappa(T_k, x)}{\sum_{j=\bar{a}+1}^{\bar{b}} \bar{\alpha}_j P_0(T_j)} \]

\[ \delta_{k}^{ab}(x) := \frac{\alpha_k P_0(T_k) (\kappa(T_k, x) + F_0(T_k, x))}{\sum_{j=a+1}^{b} \alpha_j P_0(T_j) (\kappa(T_j, x) + F_0(T_j, x))} \]
Let us consider a swap with
- a floating leg of tenor $x$ paying Euribor at times $\{T_{a+1}, \ldots, T_b\}$, and
- a floating leg of tenor $\bar{x} < x$ paying Euribor plus spread at times $\{T_{a+1} = T_{\bar{a}+1}, \ldots, T_{\bar{b}} = T_b\}$.

The par spread $B_{t}^{ab}(x, \bar{x})$ is given by

$$B_{t}^{ab}(x, \bar{x}) := S_{t}^{ab}(x) - S_{t}^{ab}(\bar{x})$$

where $S_{t}^{ab}(x)$ and $S_{t}^{ab}(\bar{x})$ are IRS par-rates as defined before.
In order to model implied volatility smiles, we can add a stochastic volatility process to our model, as usually done also in the single-curve HJM framework, by extending the filtration to include also the information generated by the volatility process.

A popular choice is to model the matrix process $h_t$ by means of a square-root process (see for instance Trolle and Schwartz (2009) and reference therein).

With this choice we get a shifted Heston dynamics for market rates, so that we can calculate option pricing with usual Fourier transform techniques (see Lewis (2001)).
Volatility Dynamics – II

In practice we replace the $h_t$ process by

$$h_t := \sqrt{v_t} R$$

where $R$ is a upper triangular matrix, while the variance $v_t$ is a vector process whose dynamics under risk neutral measure is given by

$$dv_t = \kappa (\theta - v_t) \, dt + \nu \sqrt{v_t} \, dZ_t \, , \quad v_0 = \bar{v}$$

where $\kappa, \theta, \nu, \bar{v}$ are constant deterministic vectors, and $Z_t$ is a vector of independent Brownian motions correlated to the $W_t$ processes as given by

$$\rho_{ij} \, dt := d\langle Z_i, W_j \rangle_t$$

where $\rho$ is a diagonal deterministic correlation matrix.
Talk Outline

1. The Raising of the Basis

2. Multiple-Curve Modelling

3. Model Calibration and Numerical Examples
   - The Weighted Gaussian Model
   - Calibration to Market Data
   - Capped Basis Swaps

4. Conclusions and Future Developments
Numerical Investigations

- Here, we select a simple but realistic volatility specification for the multi-curve HJM framework.
  - In practice we consider a generalization of a shifted $n$-factor Hull and White model associated to risk-free rates.
  - We adopt as benchmark for our model two HJM-like models, all with two driving factors and time-dependent volatilities.

- We consider two relevant numerical examples
  1. First, we focus on a simple calibration data-set based on atm-swaptions to investigate implied volatilities, see Moreni and Pallavicini (2010) for details.
  2. Then, we discuss how to price traded interest-rate derivatives depending on the volatility of Euribor rates of different tenors.
The Weighted Gaussian Model

- We consider the volatility process $h_t$ to be in the form
  \[ h_t := \varepsilon(t) h R, \quad \rho := R^* R \]
  where $h$ is a constant vector, $R$ is an upper triangular matrix, and we allow for a time varying common volatility shape
  \[ \varepsilon(t) := 1 + (\beta_0 - 1 + \beta_1 t)e^{-\beta_2 t} \]
  where $\beta_0$, $\beta_1$, $\beta_2$ are three positive constants.
- As for the tenor-maturity factors $q$ and $\kappa$, we chose a maturity independent form of the type
  \[ q_i(u; T, x) := \exp\{-x\eta_i\}, \quad \kappa(T, x) := 1/x \]
- Numerical tests are done with $n = 2$, hence with ten parameters:
  \[ \{\lambda_1, \lambda_2; h_1, h_2; \eta_1, \eta_2; \rho_{12}; \beta_0, \beta_1, \beta_2\} \]
Benchmark Models – I

  - This is a single-curve (old-style) model which we extend to incorporate time-dependent volatilities via the common time-dependent factor $\varepsilon(t)$.
  - It is obtained from our model by setting $\eta := 0$, and $F_0(T, x) := E_0(T, x)$.
  - For this model we use, as discounting and forwarding curve, a term structure obtained with old-style standard techniques from deposits, futures and swap rates.

- It has eight free parameters: $\{\lambda_1, \lambda_2; h_1, h_2; \rho_{12}; \beta_0, \beta_1, \beta_2\}$. 
The MMG model of Pallavicini and Tarenghi (2010).

- This is an uncertain parameter multi-curve model which we restrict to have only one scenario.
- It is obtained from our model by setting $\eta := 0$, it uses separate forwarding and discounting curves, and it reduces to Henrard (2009) static correction model.
- It uses the same curves as the Weighted Gaussian.

It has eight free parameters: $\{\lambda_1, \lambda_2; h_1, h_2; \rho_{12}; \beta_0, \beta_1, \beta_2\}$. 
Model Parameters

- Model parameters obtained from the calibration procedure. The last row shows the calibration error normalized to the one obtained with the G2++ model.

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<th>MMG</th>
<th>WG2++</th>
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Calibration Errors – I

- Differences in basis points between market and model-implied volatilities for G2++ model, namely calibration error in term of implied swaption volatilities. On the left axis the underlying swap’s tenor, while on the right axis its starting time.
Calibration Errors – II

- Differences in basis points between market and model-implied volatilities for MMG model, namely calibration error in term of implied swaption volatilities. On the left axis the underlying swap’s tenor, while on the right axis its starting time.
Calibration Errors – III

- Differences in basis points between market and model-implied volatilities for WG2++ model, namely calibration error in term of implied swaption volatilities. On the left axis the underlying swap’s tenor, while on the right axis its starting time.
Now, we compare the implied volatility for swaptions of different tenors and expiries as predicted by the WG2++ model and by the benchmark models.

We show both claims to enter a swap whose floating leg is indexed with the three-month Euribor rate and the ones referring to the six-month Euribor rate.

Notice that only the WG2++ model and the MMG model are able to differentiate between the two types of swaptions.

In particular, we observe that only the WG2++ model is able to preserve such difference also for longer swaption tenors.

Indeed, the MMG model produces the split because of different initial yield curves, while the WG2++ model relies also on a dynamical mechanism.
Implied volatilities by changing the underlying swap’s starting time. Underlying IRS contract of two-year (left panel) and ten-year tenor (right panel).
Let us consider a swap with

- a floating leg of tenor $\bar{x} < x$ paying Euribor at times 
  \begin{align*}
  \{ T_{a+1} = T_{\bar{a}+1}, \ldots, T_{\bar{b}} = T_{b} \}
  \end{align*}
  capped at strike $K$, and
- a floating leg of tenor $x$ paying Euribor at times 
  \begin{align*}
  \{ T_{a+1}, \ldots, T_{b} \}
  \end{align*}
  capped at the same strike $K$.

The contract’s payoff of capped basis-swaps is given by

\[ \Pi^{ab}_{\text{CBS}}(x, \bar{x}; K) := \sum_{k=a+1}^{b} \alpha_k P_t(T_k)(F_t(T_k, x) - K)^+ \]
\[ - \sum_{k=\bar{a}+1}^{\bar{b}} \bar{\alpha}_k P_t(T_k)(F_t(T_k, \bar{x}) - K)^+ \]

where $\alpha$ and $\bar{\alpha}$ are the year fractions, respectively of $x$-tenor and $\bar{x}$-tenor leg.
Often such contracts are sold at par on the market by a suitable choice for the strike $K$, namely

$$K : E_t \left[ \Pi_{CBS}^{ab}(x, \bar{x}; K) \right] = 0$$

Pricing capped basis-swaps depend mainly on

- basis-swap spread $B_{t}^{ab}(x, \bar{x})$, and
- cap volatilities $\sigma_{t}^{ab}(x; K)$ and $\sigma_{t}^{\bar{a}\bar{b}}(\bar{x}; K)$.

Are there arbitrage opportunities between these market quotes?

Consider that cap volatilities on one-month rate are not actively traded.
We start by calibrating a mixture of two WG2++ model to at- and out-the-money cap volatilities.

We need a mixture of models to catch volatility smile, otherwise we could resort to a full stochastic volatility model.

The money market quotes cap volatilities on three-month and six-month Euribor rates.

In order to get a more stable calibration we add also at-the-money swaption volatilities to feed the model with more data on both the three- and six-month tenors.

We extrapolate cap volatilities for one-month Euribor rates from the model to price CBS.
Pricing CBS with the Weighted Gaussian Model – II

- At-the-money cap volatilities implied by the WG2++ mixture model for 1m, 3m, 6m and 12m rate tenors. Market quotes as big dots. On the x-axis we have the cap maturity, while on the y-axis we have the implied volatility. Market data observed on 15 February 2011.
Pricing CBS with the Weighted Gaussian Model – III

- Price of a CBS receiving a cap on the six-month tenor and paying a cap on the one-month tenor. On the left-axis we have the cap strike, while on the right-axis we have cap maturity. Prices in basis points.
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Conclusions – I

- We described some stylized facts about the money market after the credit crunch:
  - Euribor rates of different tenors start behaving as different asset classes;
  - non-arbitrage relationships between forward rates and zero-coupon bonds fail to hold;
  - both liquidity and credit risks have the responsibility for the transformations undergone by the money-market.

- We hinted at the similarity of our framework with respect to the market models recently proposed in the literature.
We proposed a new way of modelling interest-rates by extending the (single-curve) HJM framework to the new money market scenario. Our model has the following property:

- few parameters to control all the market rates to allow calibrations in presence of only few market quotes;
- no-arbitrage dynamics for all the forward rates which collapses to standard single-curve HJM dynamics for vanishing rate tenors;
- possibility of writing all the forward rates as functions of a common family of Markov processes;
- a (closed-form) reconstruction formula for all the forward rates.

We hinted at the possibility of exact calibration of level and skew of caplet volatilities by a proper choice of model parameters.

Further, we showed some numerical examples for the calibration procedure and for the pricing of relevant multi-curve payoffs.
Further Developments

- A full implementation of a multi-curve extensions of a Gaussian model with stochastic volatility.
- Extending the model to be multi-currency by coupling different single-currency multi-curve models with a proper forex dynamics.
- Designing of more realistic volatility functional forms to improve the calibration procedure as soon as new market quotes on different Euribor rate tenors appear in the market.
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