How to rebuild a binary image from its multi-level description based on generalized salient pixels

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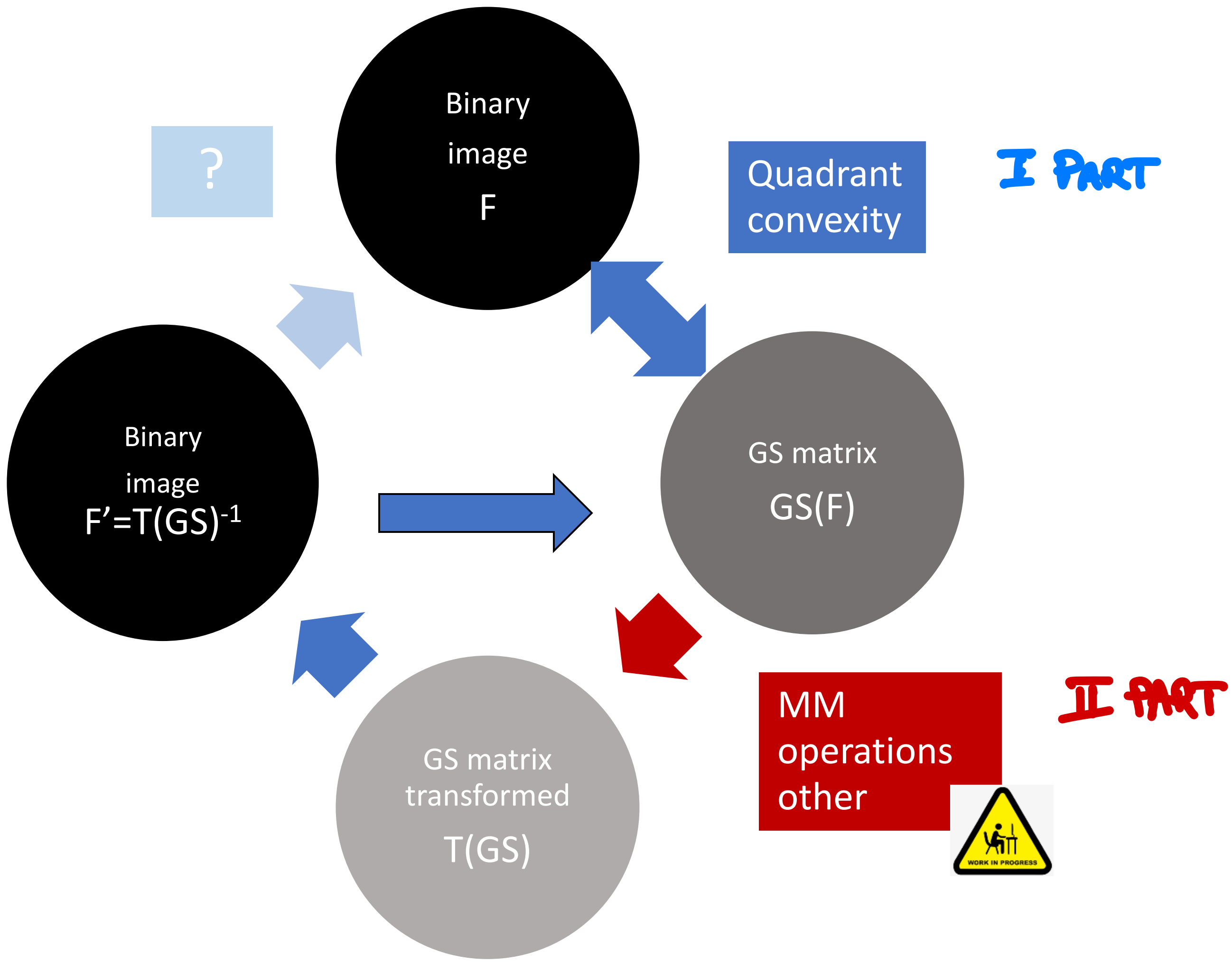
Outline



Binary
image
F
?
Quadrant
convexity
I PART
Binary
image
F'=T(GS)-1
GS matrix
GS(F)
MM
operations
other
II PART
WORK IN PROGRESS
GS matrix
transformed
T(GS)

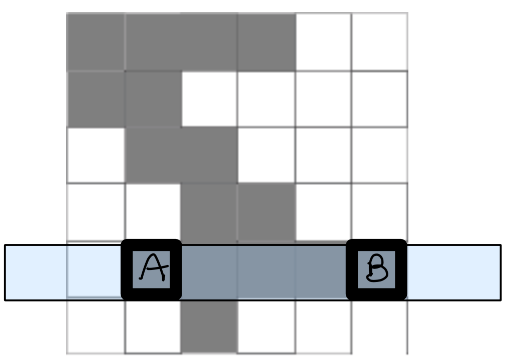
Basics :

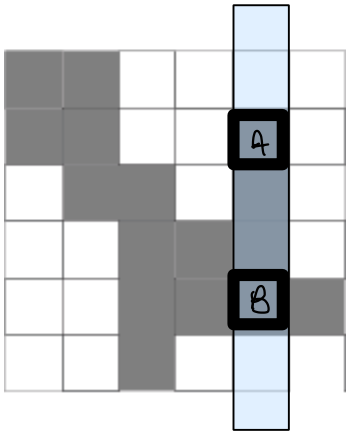
Line convexity

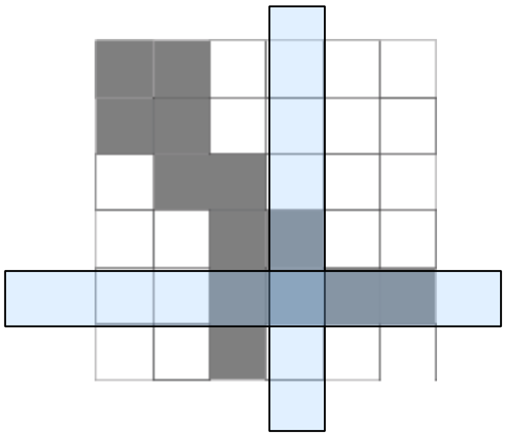
A binary image F is **row-convex** if for every pair of pixels A and B in the same row of F, all the pixels from [AB] are in F

A binary image F is **column-convex** if for every pair of pixels A and B in the same column of F, all the pixels from [AB] are in F

A binary image F is **hv-convex** if it is both **row and column-convex**

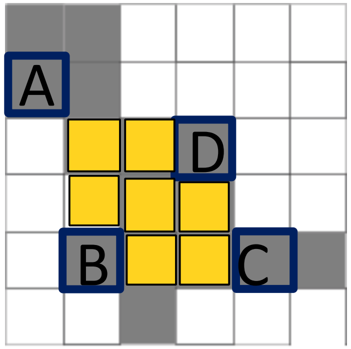


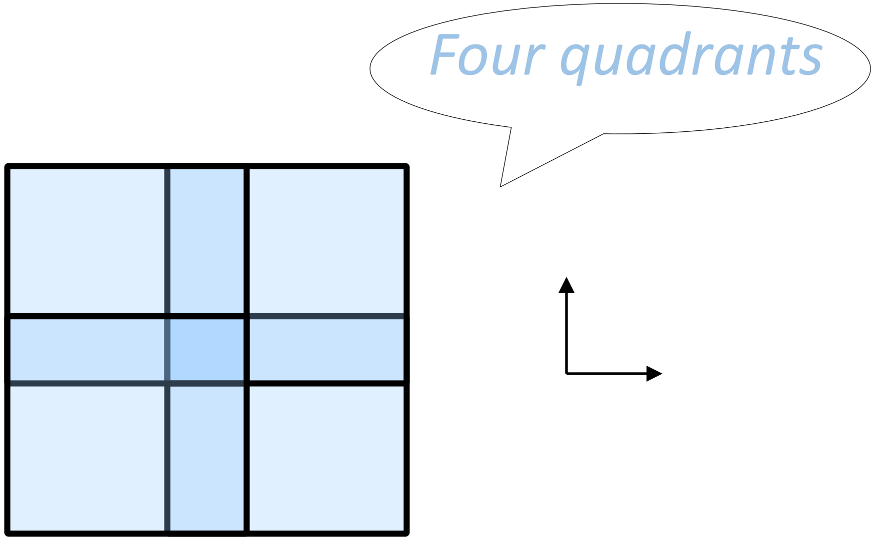




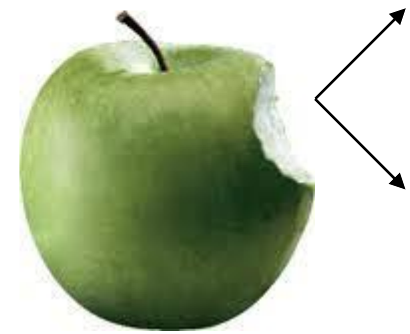
Q-convex binary image

A binary image F is **quadrant-convex** if for every quadruple of pixels A,B,C,D, each in a different quadrant, all the (yellow) pixels "inbetween" are in F

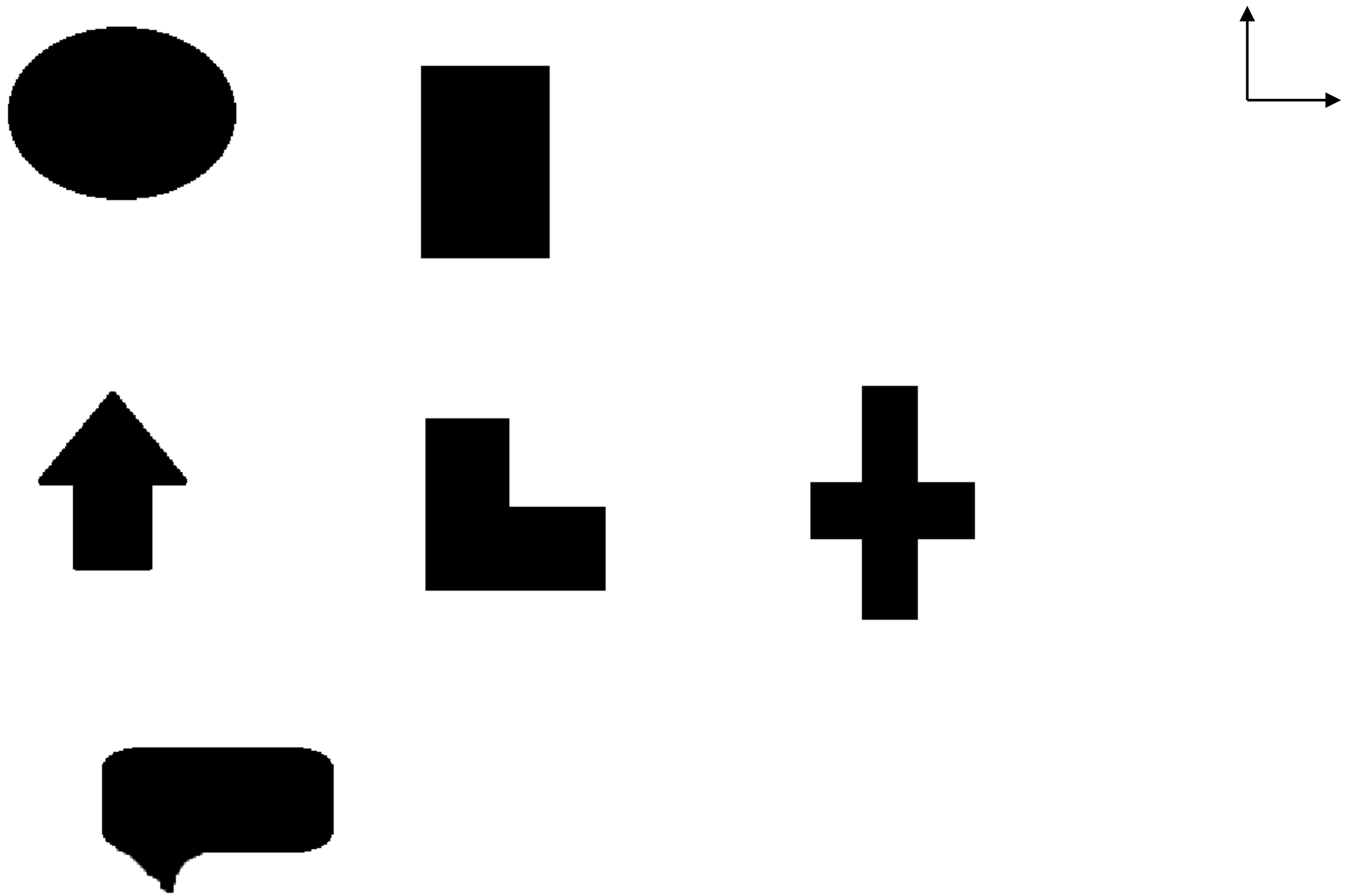




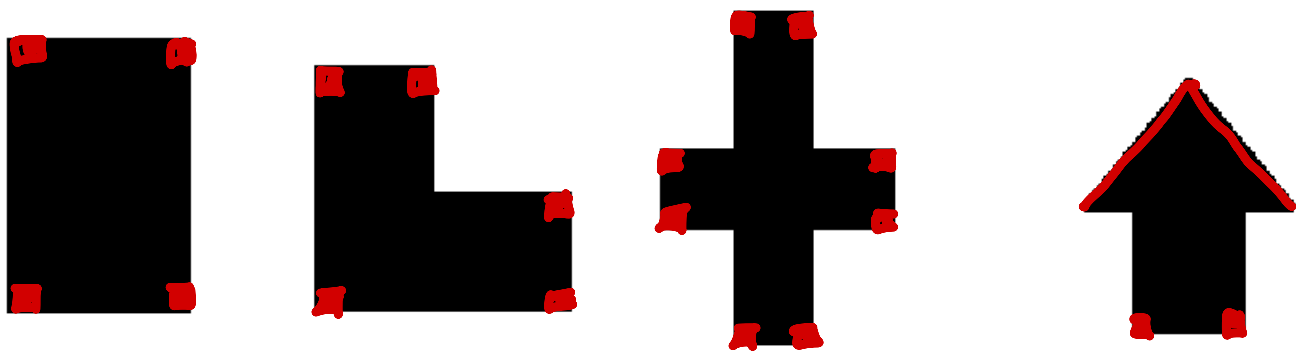
How does it look like?



How does it look like?



Salient pixels: S(F)



x of F such that there is a background quadrant in x for F\{x}

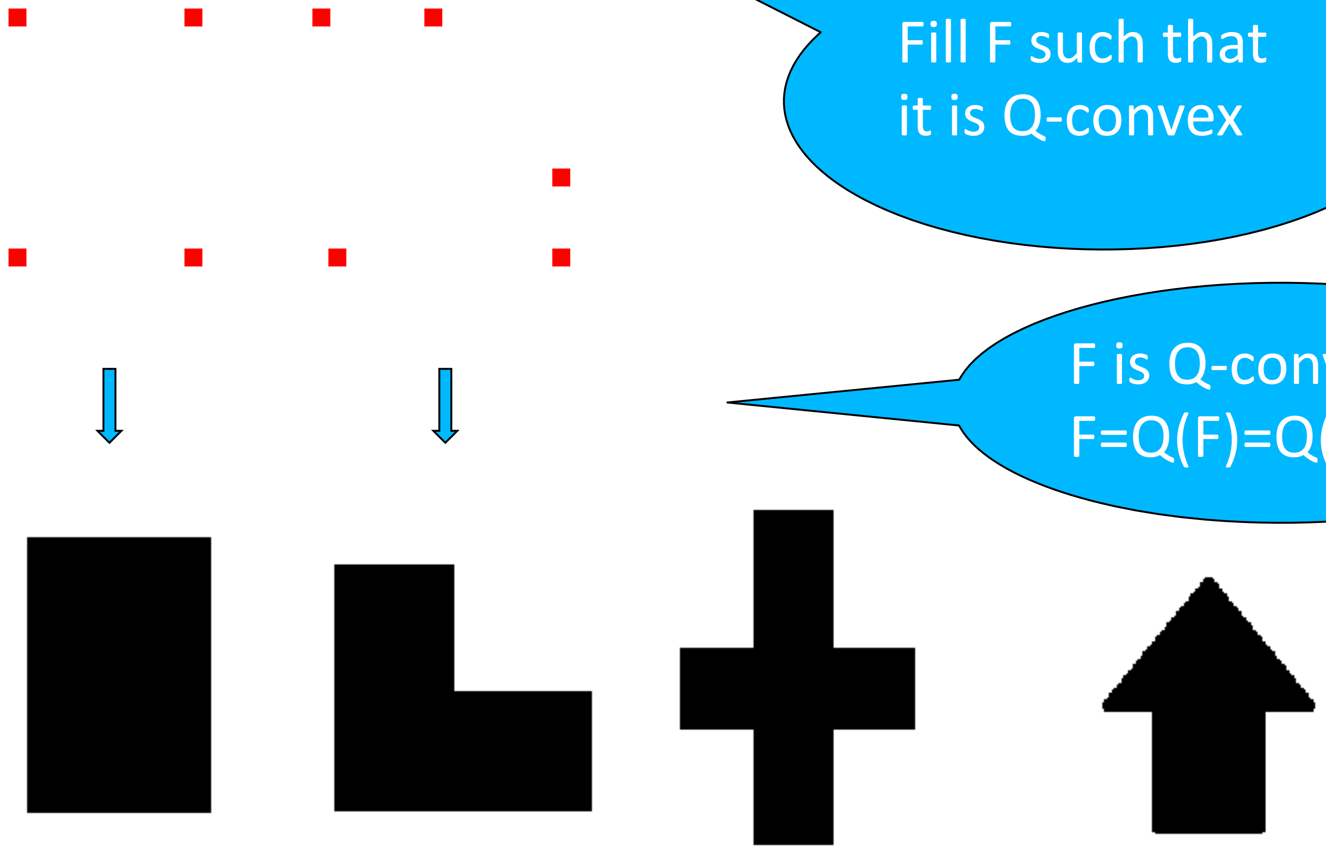
$\hat{Z}$

$$Z \cap F \setminus \{x\} = \phi$$

Q-convex hull : Q(S(F))

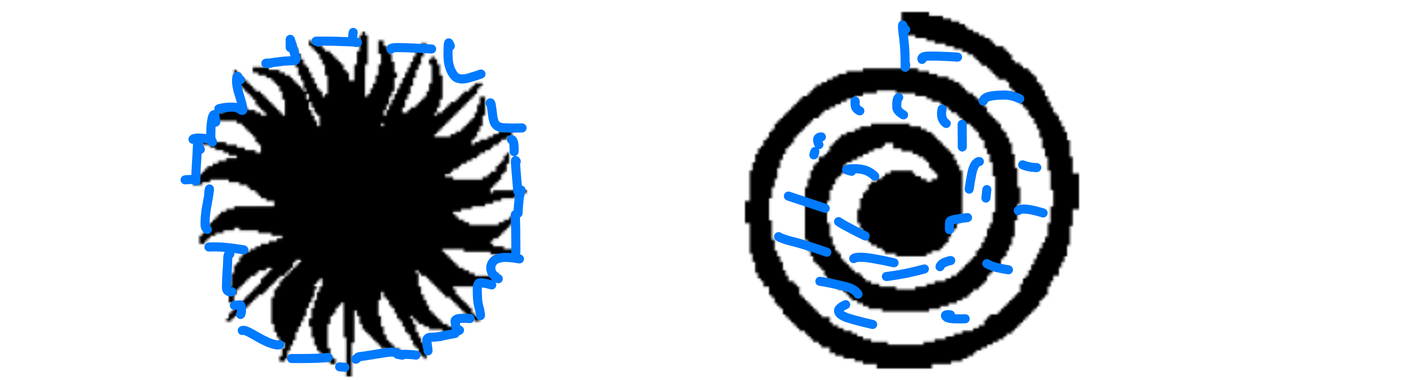
Fill F such that
it is Q-convex

F is Q-convex iff
F=Q(F)=Q(S(F))



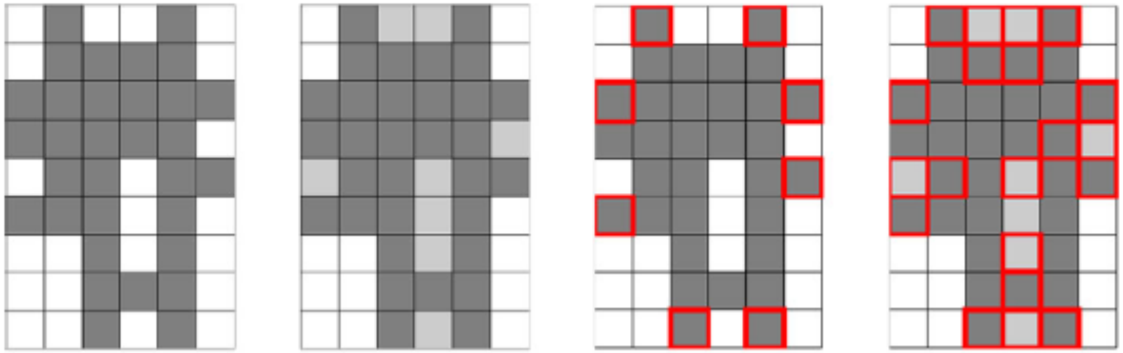
What if not Q-convex: more ‘’complex’’ binary images

What if not Q-convex: more ''complex'' binary images



Generalized salient pixels (GSp)

- If F is Q-convex, then Q(F)=F
- If F is not Q-convex, then look at Q(F)\F !
- Compute the salient pixels of Q(F)\F
- If not empty, then iterate!



The algorithm to compute the GS matrix

0	0	1	1	0	1	1	0
0	1	0	1	1	0	1	0
1	1	0	0	0	1	1	1
1	0	1	0	1	0	0	0
0	1	0	1	1	1	1	0
0	1	0	1	1	0	1	1
0	0	0	1	1	1	1	0
0	0	1	0	0	1	0	0

$k=1$

0	0	1	1	0	1	1	0
0	1	0	1	1	0	1	0
1	1	0	0	0	1	1	1
1	0	1	0	1	0	0	0
0	1	0	1	1	1	1	0
0	1	0	1	1	0	1	1
0	0	0	1	1	1	1	0
0	0	1	0	0	1	0	0

$k=2$

0	0	1	1	0	1	1	0
0	1	0	1	1	0	1	0
1	1	0	0	0	1	1	1
1	0	1	0	1	0	0	0
0	1	0	1	1	1	1	0
0	1	0	1	1	0	1	1
0	0	0	1	1	1	1	0
0	0	1	0	0	1	0	0

$k=3$

0	0	1	1	0	1	1	0
0	1	0	1	1	0	1	0
1	1	0	0	0	1	1	1
1	0	1	0	1	0	0	0
0	1	0	1	1	1	1	0
0	1	0	1	1	0	1	1
0	0	0	1	1	1	1	0
0	0	1	0	0	1	0	0

$k=4$

0	0	1	1	0	1	1	0
0	1	0	1	1	0	1	0
1	1	0	0	0	1	1	1
1	0	1	0	1	0	0	0
0	1	0	1	1	1	1	0
0	1	0	1	1	0	1	1
0	0	0	1	1	1	1	0
0	0	1	0	0	1	0	0

$k=5$

0	0	1	0	2	0	1	0
0	1	2	3	3	2	0	0
1	0	0	4	4	3	0	1
1	2	3	4	5	4	0	2
0	0	0	0	0	0	3	2
0	1	0	0	0	2	0	1
0	0	2	3	3	0	1	0
0	0	1	2	2	1	0	0

GS-matrix associated to the binary image

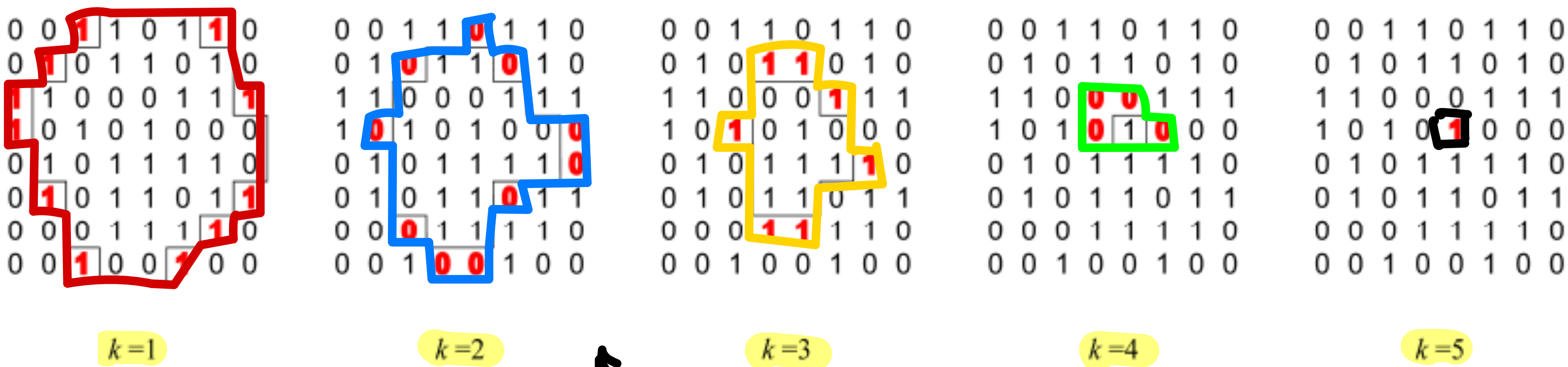
It provides a k-level description of the image

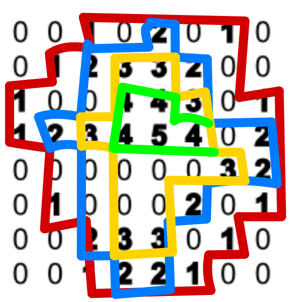
Examples

GS-matrices as grey-scale images

Every binary image can be decomposed in the k-multi level representation
k is greater for more ‘complex’ binary images
k is maximum for the chessboard (all its pixels are gsp)

Binary image and its GS matrix





-Every binary image can be decomposed in the k-multi level representation

-The image at each level is Q-convex

-By projection, images are ordered by inclusion

The algorithm to rebuild the binary image from its GS-matrix

```
0 0 1 0 2 0 1 0
0 1 2 3 3 2 0 0
1 0 0 4 4 3 0 1
1 2 3 4 5 4 0 2
0 0 0 0 0 0 3 2
0 1 0 0 0 2 0 1
0 0 2 3 3 0 1 0
0 0 1 2 2 1 0 0
```

```
0 0 . 1 . 1 . 0
0 . . . . . 1 0
. 1 2 . . . 1 .
. . . . . . 2 .
0 1 2 3 3 3 . .
0 . 2 3 3 . 1 .
0 0 . . . 1 . 0
0 0 . . . . 0 0
```

```
0 0 1 1 0 1 1 0
0 1 0 1 1 0 1 0
1 1 0 0 0 1 1 1
1 0 1 0 1 0 0 0
0 1 0 1 1 1 1 0
0 1 0 1 1 0 1 1
0 0 0 1 1 1 1 0
0 0 1 0 0 1 0 0
```

procedure (B)
  for each $b_{ij} = 0$ **do**
    for each t=0,1,2,3 **do**
      find b the maximum item of B in $Z_t(b_{ij})$
      store b in z^t_{ij} of matrix Z^t
    end for
  end for
  for each $b_{ij} = 0$ **do**
    $h \leftarrow \min(z^0_{ij}, z^1_{ij}, z^2_{ij}, z^3_{ij})$
    if h is odd **then** $f_{ij} \leftarrow 1$
    else $f_{ij} \leftarrow 0$
    end if
  end for
  for each $b_{ij} \neq 0$ **do**
    if b_{ij} is odd **then** $f_{ij} \leftarrow 1$
    else $f_{ij} \leftarrow 0$
    end if
  end for
end procedure

How does it change the image by modifying the GS-matrix?

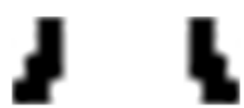
0	0	1	0	2	0	0	0	0	0	0	0	0	2	0	1	0	0
0	0	0	0	3	4	0	0	0	0	0	0	4	3	0	0	0	0
0	0	0	0	3	0	4	0	0	0	0	4	0	3	0	0	0	0
0	1	0	0	0	0	3	0	4	4	0	3	0	0	0	0	1	0
0	0	0	0	0	0	0	3	0	0	3	0	0	0	0	0	0	0
1	0	0	2	0	0	0	0	3	3	0	0	0	0	2	0	0	1
1	0	0	2	0	0	0	0	0	0	0	0	0	0	2	0	0	1

Delete 1's (and 2's)

Delete 2's

Leave 1's only

Delete 3's



Delete 4's

Erosion and delation operations

Erosion- shrink the image.

The erosion operator takes two pieces of data as inputs:
the first is the image which is to be eroded; the second is a structuring element (also known as a kernel). The kernel determines the precise effect of the erosion on the input image.

Dilation- grow the image.

It is the dual of erosion.

An iterative definition of erosion

- Repeat
  - Remove pixels of the binary image corresponding to items 1 in the GS
  - Compute the GS of the obtained binary image

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	1	1	1	0	0
0	0	1	1	1	1	0	0	0	0	1	1	1	1	0	0
0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	2	0	0	2	0	0	1	0	0
0	0	1	0	0	0	2	0	0	0	3	0	0	0	0	0
0	0	0	1	0	2	0	0	0	3	0	0	0	1	0	0
0	0	0	0	2	0	0	0	3	0	0	0	1	0	0	0
0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0
0	0	0	0	0	0	3	0	0	0	3	2	0	0	0	0
0	0	0	0	2	3	0	0	0	3	0	0	0	0	0	0
0	0	0	0	0	0	0	0	2	0	2	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

An iterative definition of erosion

- Repeat
  - Remove pixels of the binary image corresponding to items 1 in the GS
  - Compute the GS of the obtained binary image

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	0	0	0	0	1	1	1	1	0	0
0	0	0	1	1	0	0	0	0	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	2	0	0	2	0	1	0	0	0
0	0	0	0	0	0	2	0	0	0	3	0	0	1	0	0
0	0	0	1	0	2	0	0	0	3	0	0	1	0	0	0
0	0	0	0	2	0	0	0	3	0	0	0	0	0	0	0
0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0
0	0	0	0	0	0	3	0	0	0	3	2	0	0	0	0
0	0	0	0	2	3	0	0	0	3	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	2	0	2	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

An iterative definition of erosion

- Repeat
  - Remove pixels of the binary image corresponding to items 1 in the GS
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An iterative definition of erosion

- Repeat
  - Remove pixels of the binary image corresponding to items 1 in the GS
  - Compute the GS of the obtained binary image

What about dilation?

- There is not a unique way to add 1’s in the GS matrix

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	2	0	0	2	0	0	1	0	0
0	0	1	0	0	0	2	0	0	0	3	0	0	0	0	0
0	0	0	1	0	2	0	0	0	3	0	0	0	1	0	0
0	0	0	0	2	0	0	0	3	0	0	0	1	0	0	0
0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0
0	0	0	0	0	0	3	0	0	0	3	2	0	0	0	0
0	0	0	0	2	3	0	0	0	3	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	2	0	2	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Medial axis transform (MAT)

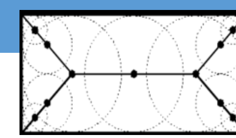
- Similarly, a grey level image

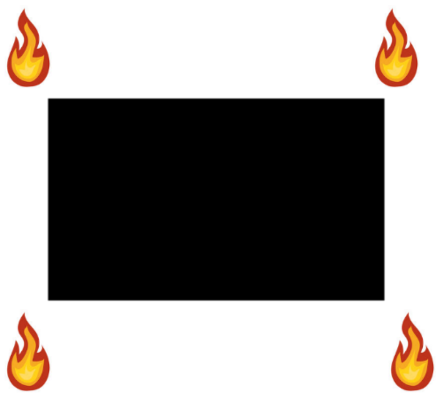
such that the intensity of its

pixels representes their ‘distance’

to the salient pixels, iteratively by erosion

The MAT is a grey level image where each pixel on the skeleton has an intensity which represents its distance to a boundary in the original object.



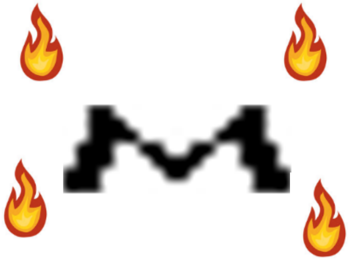


1	2	3	4	5	5	4	3	2	1
2	3	4	5	6	6	5	4	3	2
3	4	5	6	7	7	6	5	4	3
4	5	6	7	8	8	7	6	5	4
3	4	5	6	7	7	6	5	4	3
2	3	4	5	6	6	5	4	3	2
1	2	3	4	5	5	4	3	2	1

Medial axis transform (MAT)

The MAT is a grey level image where each pixel on the skeleton has an intensity which represents its distance to a boundary in the original object.

- Similarly, a grey level image such that the intensity of its pixels representes their 'distance' to the salient pixels, iteratively by erosion



0	0	1	2	0	0	0	0	0	0	0	0	0	0	2	1	0	0
0	0	2	3	4	0	0	0	0	0	0	0	0	4	3	2	0	0
0	0	3	4	5	6	0	0	0	0	0	0	6	5	4	3	0	0
0	1	4	5	0	0	7	8	0	0	8	7	0	0	5	4	1	0
0	2	5	6	0	0	0	7	8	8	7	0	0	0	6	5	2	0
1	3	4	0	0	0	0	0	5	5	0	0	0	0	0	4	3	1
1	2	3	0	0	0	0	0	0	0	0	0	0	0	0	3	2	1

Conclusions and perspectives

- Binary image ↔ GS matrix

- Binary image
  - erosion
  - dilation
  - intersection
  - union
  - Other..
- GS matrix
  - erosion
  - dilation
  - ?intersection
  - ?union
  - Other..

- IF SPARSE → COMPRESSION
- SHAPE DESCRIPTOR

References

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