

Novel polytopal discontinuous Galerkin methods for geophysical fluid dynamics: the rotating shallow water equations on polygonal grids

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Abstract

Modern atmospheric and oceanic dynamical cores increasingly rely on quasi-uniform, unstructured polygonal (Voronoi) grids, which avoid the polar singularity of latitude-longitude meshes and underpin models such as MPAS and the emerging Brazilian earth-system model MONAN. On these grids, the low-order finite-volume schemes in current use remain vulnerable to cell geometric irregularity: mesh distortions imprint themselves on the solution as spurious grid-scale features — the so-called grid imprinting — which degrades accuracy and can excite spurious computational modes. Raising the order of finite-volume schemes on unstructured grids typically requires wide reconstruction stencils that enlarge communication halos and erode the data locality prized at scale.

We introduce polytopal discontinuous Galerkin (PolyDG) methods for geophysical fluid dynamics, formulated directly on polygonal grids. By admitting elements of arbitrary shape and number of faces, PolyDG combines high-order accuracy with element-local data locality and reduced sensitivity to grid geometry. We formulate the scheme for the advection equation and for the nonlinear rotating shallow water equations, in both flux and vector-invariant forms — to our knowledge the first high-order DG discretization of the rotating shallow water equations on general polygonal grids — and exploit quadrature-free integration tailored to polytopes to remove the geometric overhead of high-order integration on distorted cells; explicit Runge–Kutta time stepping completes the discretization.

We rigorously benchmark the methods on bi-periodic planar centroidal Voronoi meshes that reproduce the cell distortions of quasi-uniform spherical grids, isolating the discretization from the additional complications of spherical geometry. We report hp-convergence, assess conservation of mass, energy, and potential enstrophy, and examine the response of the high-order solution to controlled grid distortion, gauging the extent to which grid imprinting is mitigated. Standard benchmarks, including geostrophically

balanced and barotropically unstable jets, are used throughout. The results establish a foundation for high-order, geometrically flexible, and scalable discretizations for weather and climate models built on polygonal spherical grids.

Keywords: discontinuous Galerkin, polytopal methods, rotating shallow water equations, grid imprinting

References

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