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A POLYDG FRAMEWORK FOR THERMO-HYDRO-MECHANICS

Stefano Bonetti[1], Michele Botti[1], Paola F. Antonietti[1]
[1] MOX-Dipartimento di Matematica, Politecnico di Milano, Milano, Italy

Corresponding author: stefano.bonetti@polimi.it

Abstract

In the framework of geosciences applications, the subsoil is often modeled as a poroelastic material. However, in several applications in the context of environmental sustainability, such as geothermal energy production procedures and greenhouse gas sequestration, temperature plays a key role in the description of the physical phenomena. Then, the differential problem should also take into account the influence of the temperature, leading to a fully coupled thermo-hydro-mechanical system of equations. In this talk, we present the discontinuous Galerkin approximation on polygonal grids for flow problems in the geophysical context, taking thermal effects into account. The models considered include a fully coupled nonlinear thermo-poroelastic model, Darcy-Forchheimer flow laws, and a first approach to treatment of viscous phenomena. Moreover, different time regimes (i.e., steady, quasi-static, and dynamical problems) are considered. For these models, we derive the corresponding mathematical models, construct the approximations—along with their respective stability and convergence analyses—and pay particular attention to an efficient strategy for solving the discretized problem. For the spatial discretization of all the presented problems, we design an arbitrary-order weighted symmetric interior penalty scheme that supports general polytopal grids and is robust with respect to heterogeneities in the model coefficients. In this class of problems, the convective transport term plays a key role, and suitable stabilization techniques must be developed to make the scheme robust in advection-dominated regimes. Finally, a complete set of numerical simulations is presented to assess the convergence and robustness of the schemes, and physically inspired test cases in both two- and three-dimensional settings demonstrate their applicability in real-life applications.

Keywords: polytopal discontinuous Galerkin, thermo-hydro-mechanics, geophysics, geothermal energy production

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