Fluids@PoliMi

DEPARTMENT OF MATHEMATICS, POLITECNICO DI MILANO
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Book of Abstracts

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Absence of anomalous dissipation for weak solutions of the Maxwell-Stefan system

We recall the main results about existence of weak and strong solutions for the Stefan-Maxwell system. After having introduced the notion of entropy for this problem and the related inequality satisfied by weak solutions, we show that if fact all weak solutions satisfy an entropy equality, establishing the absence of anomalous dissipation. Based on a joint work with Stefanos Georgiadis and Thanos Tzavaras.

Gennaro Ciampa
Università degli Studi dell'Aquila

Magnetic reconnection for incompressible MHD equations

The goal of this talk is to construct smooth solutions of the Magnetohydrodynamics equations (MHD) that show a change in the topological structure of magnetic field lines. This is known to be impossible in the non-resistive case by Alfvén's theorem, indeed the integral lines of a sufficiently smooth magnetic field are transported by the fluid. The reconnection of the magnetic field lines occurs instead in the resistive case, being responsible for many dynamic phenomena in astrophysics such as solar flares and the solar wind. Although numerical and experimental evidence exist, analytical examples of magnetic reconnection were not known.

Francesca Crispo
Università della Campania Vanvitelli

Estimates of a possible gap related to the energy equality
for a class of non-Newtonian fluids

We consider non-Newtonian fluids of power law type, modeled by the p-Navier-Stokes equations, and we investigate on the validity of an energy equality.

It is widely recognized that already for the Navier-Stokes equations the validity of the energy equality is not known, and a weak solution a priori enjoys an energy inequality. In this connection, in papers [1]–[2], we construct a weak solution for the Navier-Stokes equations enjoying a sort of energy equality, which adds a dissipative quantity to the classical energy equality, and we produce different expressions of a possible gap. The result is obtained under minimal assumptions on the initial datum for the existence of a weak solution. It is based on a new existence theorem, where our weak solution is the limit of a sequence of solutions to a regularized problem.

In paper [3] we extend our investigation to weak solutions of the system of p-Navier-Stokes fluids. The study involves shear-thinning fluids and includes also the case of a singular extra stress tensor. Due to the nonlinearity of the principal operator, we have to restrict our study to space-periodic domains and to use different approximations, that are now the Galerkin ones.

It is intriguing that the gap expressions for the two problems coincide, unless of course the L^2-norm of gradients of solutions to Navier-Stokes equations is substituted for that in L^p for power law fluids. Further, in both cases, the additional dissipation, that measures the possible gap with the classical energy equality, is only expressed in terms of energy quantities.

References

[1] F. Crispo, C.R. Grisanti and P. Maremonti, *Some new properties of a suitable weak solution to the Navier- Stokes equations*, in Waves in flows – the 2018 Prague-Sum Workshop lectures, 159–179, Adv. Math. Fluid Mech., Birkhäuser/Springer, Cham.; 2021.

[2] P. Maremonti, F. Crispo and C.R. Grisanti, *Navier–Stokes equations: a new estimate of a possible gap related to the energy equality of a suitable weak solution*, Meccanica **58**, 1141–1149; 2023.

[3] F. Crispo, A.P. Di Feola, C.R. Grisanti and P. Maremonti, *An estimate of a possible gap related to the energy equality for power law fluids*, forthcoming.

Raphaël Danchin

Université Paris-Est Créteil

Some asymptotics of the global solutions to the Vlasov-Navier-Stokes equations in the whole space

The incompressible Vlasov-Navier-Stokes equations are a toy model for describing the dynamics of a cloud of particles that are immersed in an incompressible viscous fluid. In the first part of the talk, we are concerned with the behavior of global strong solutions when the time goes to infinity, in the case where the equations are posed in the whole space. For small enough initial data with sufficient integrability at infinity, it is shown that the velocity decays to zero, with the same rate as the classical Navier-Stokes equations, and that the kinetic distribution of the particles is well approximated by a monokinetic distribution with velocity which is the same as that of the viscous fluid.

In the second part of the talk, we point out the connections of the Vlasov-Navier-Stokes equations with one of its formal hydrodynamic limit, which is a coupling between the incompressible Navier-Stokes equations and the pressureless compressible Euler equations. This connection is based on a multiphase formulation of Vlasov equations that has been proposed by A. Baradat, L. Ertzbischoff and D. Han-Kwan.

For simplicity, we present all the results in the frame of H^1 Navier-Stokes velocity fields, so that one can make use of a higher order energy functional that controls the regularity H^1 of the velocity and seems to have been first introduced by Li, Shou and Zhang in the context of the nonhomogeneous Vlasov-Navier-Stokes system.

Michele Dolce

École Polytechnique Fédérale de Lausanne

The long way of a viscous vortex dipole

The evolution of two counter-rotating point vortices in a 2D inviscid fluid is governed by the Helmholtz-Kirchhoff system, resulting in a translation at a constant speed. However, at large but finite Reynolds numbers, vortex core sizes increase due to diffusion, making the point vortex approximation unjustified over long times for the resulting viscous dipole. This talk aims to systematically define an asymptotic expansion accounting for streamline deformation from vortex interactions and understand the finite size effects on the dipole's translation speed. We then prove that the exact solution remains close to our approximation over a very long time interval, which extends unboundedly as the Reynolds number approaches infinity. The proof relies on energy estimates inspired from the Arnold's variational characterization of the steady states of the 2D Euler equation, as recently revised by Gallay and Šverák for viscous fluids as well. This work is a collaboration with T. Gallay.

Alessio Falocchi

Politecnico di Milano

Long-time behaviour of solutions to unforced Navier-Stokes equations under Navier boundary conditions in sector domains

We study the asymptotic behaviour of the solutions to Navier-Stokes unforced equations under Navier boundary conditions in a wide class of merely Lipschitz domains of physical interest that we call *sectors*. The main motivations come from the celebrated results by Foias-Saut related to the long time behaviour of the solutions to Navier-Stokes equations under Dirichlet conditions. Here the choice of the boundary conditions requires carefully considering the geometry of the domain Ω, due to the possible lack of the Poincaré inequality in presence of axial symmetries. In non-axially symmetric domains we show the validity of the Foias-Saut result about the limit at infinity of the Dirichlet quotient, in axially symmetric domains we provide two invariants of the flow which completely characterize the motion and we prove that the Foias-Saut result holds for initial data belonging to one of the invariants. This is a joint work with Elvise Berchio (Politecnico di Torino, Italy) and Clara Patriarca (Université libre de Bruxelles, Belgium).

Francesco Fanelli

Basque Center for Applied Mathematics

Geometric constraints in fast rotating fluids

This talk aims to give an overview of recent results about the asymptotic dynamics of geophysical flows in the regime of fast rotation (i.e. low Rossby number), in presence of non-trivial boundary effects known under the name of Ekman boundary layers. We will mainly focus in situations where either the density of the fluid experiences large variations, or the domain has a non-flat bottom (topography effects). We will show how the presence of those large heterogeneities translate into a strong geometrical constraint for the limit flow, which in turn makes the target dynamics become linear.

The talk is based on joint works with Edoardo Bocchi (Politecnico di Milano), Christophe Prange (Cergy Paris Université), Marco Bravin (Universidad de Cantabria), Jean-Yves Chemin (Université Claude Bernard Lyon 1) and Isabelle Gallagher (École Normale Supérieure).

Eduard Feireisl

Czech Academy of Sciences

On regularity and well posedness of the Euler system of gas dynamics in the class of dissipative solutions

We show that any dissipative (measure–valued) solution of the compressible Euler system that complies with Dafermos' criterion of maximal dissipation is necessarily an admissible weak solution. In addition, we propose a simple, at most two step selection procedure to identify a unique semigroup solution in the class of dissipative solutions to the Euler system.

Mikhail V. Korobkov

Fudan University and Sobolev Institute of Mathematics

On Leray problems for steady-state Navier-Stokes system in unbounded domains

In recent years, using the geometric and real analysis methods, essential progress has been achieved in some classical Leray's problems on stationary motions of viscous incompressible fluid in exterior 2d domains: the uniqueness of the solutions to the plane flow around an obstacle problem in the class of all D-solutions, the nontriviality of the Leray solutions (obtained by the "invading domains" method) and their convergence to a given limit at low Reynolds numbers; the boundedness and the uniform convergence of D-solutions in general case, etc. A review of these advances and methods will be the focus of the talk. Most of the reviewed results were obtained in our joint articles with Konstantin Pileckas, Remigio Russo, Xiao Ren, and Julien Guillod, see, e.g., the recent survey paper [1]. In the last part of the talk we consider the advances on the classical Ladyzhenskaya–Leray's problem concerning the stationary fluid motion in a system of infinite channels and pipes under nonhomogeneous Dirichlet boundary conditions. In contrast to many previous works, the domain is not assumed to be simply-connected, and the boundary fluxes are not assumed to be small. This is a joint result with Xiao Ren (Peking University), Filippo Gazzola (Politecnico di Milano), Gianmarco Sperone (Pontifical Catholic University of Chile).

References

[1] M. Korobkov and X. Ren, *Stationary Solutions to the Navier–Stokes System in an Exterior Plane Domain: 90 Years of Search, Mysteries and Insights*, J. Math. Fluid Mech. **25**, 55; 2023.

Boris Muha

University of Zagreb

Weak solutions for fluid-structure interaction problems
with three-dimensional structural displacements

This talk addresses a nonlinear fluid-structure interaction (FSI) problem involving the Navier-Stokes equations, which govern the flow of an incompressible, viscous fluid in a three-dimensional domain, coupled with the motion of a thin viscoelastic lateral wall. The wall's dynamics are modeled by a two-dimensional plate equation with fractional damping, allowing for displacements in all three spatial directions. The system is nonlinearly coupled through kinematic and dynamic conditions imposed on the moving fluid-structure interface, whose position evolves and is not known a priori.

We establish three key results for this class of FSI problems, focusing on cases with vector displacements of thin structures. First, we prove a hidden spatial regularity in the structural displacement, ensuring the absence of self-contact within a finite time interval. Second, we show temporal regularity for the fluid velocity and structural displacement, leading to a novel compactness result for three-dimensional structural dynamics. Finally, leveraging these findings, we construct a proof of local-in-time existence for weak solutions to the FSI problem using a Lie operator splitting method and time discretization.

These results contribute to the mathematical understanding of fluid-structure interaction problems with three-dimensional structural displacements, particularly in terms of solution regularity and well-posedness. This is a joint work with Sunčica Čanić and Krutika Tawri.

Šárka Nečasová

Czech Academy of Sciences

On a compressible fluid-structure interaction problem

We study a system describing the compressible barotropic fluids interacting with (visco) elastic solid shell/plate. In particular, the elastic structure is part of the moving boundary of the fluid, and the Navier-slip type boundary condition is taken into account. Depending on the reference geometry (flat or not), we show the existence of weak solutions to the coupled system provided the adiabatic exponent satisfies $\gamma > \frac{12}{7}$ without damping and $\gamma > \frac{3}{2}$ with structure damping, utilizing the domain extension and regularization approximation. Moreover, we give a rigorous justification of the incompressible inviscid limit of the compressible fluid-structure interaction problem with a flat reference geometry, in the regime of low Mach number, high Reynolds number, and well-prepared initial data. Finally, we study a nonlinear fluid-structure interaction problem between a "square-root" viscoelastic beam and a compressible viscous fluid. The beam is immersed in the fluid which fills a two-dimensional rectangular domain with periodic boundary conditions in both directions, while both the beam and the fluid are under the effect of time-periodic forces. By using a decoupling approach, at least one time-periodic weak solution to this problem is constructed which has a bounded energy and a fixed prescribed mass.

References

[1] Y. Liu, S. Mitra and Š. Nečasová: *On a compressible fluid-structure interaction problem with slip boundary conditions.*

[2] O. Kreml, V. Mácha, Š. Nečasová and S. Trifunovič, *On time-periodic solutions to an interaction problem between compressible viscous fluids and viscoelastic beams*, Nonlinearity **38**, 1; 2025

Helena Nussenzveig Lopes

Universidade Federal do Rio de Janeiro

Inviscid limit in 2D avoids inviscid dissipation and anomalous dissipation

We say inviscid dissipation occurs when a vanishing viscosity limit does not satisfy energy balance. A closely related phenomenon is anomalous dissipation, where, in the limit of vanishing viscosity, the total dissipation does not vanish. In this talk we will discuss recent results on avoiding these phenomena in 2D incompressible flows, with and without forcing.

Gianmarco Sperone

Pontificia Universidad Católica de Chile

On the steady motion of a Navier-Stokes flow across a sieve with prescribed pressure drop in a finite pipe

The steady motion of a viscous incompressible fluid through a sieve (that is, a wall perforated with a large number of small holes), in a pipe of finite length, is modeled through the Navier-Stokes equations under mixed boundary conditions involving the Bernoulli pressure and the tangential velocity on the inlet and outlet of the tube, while the pressure drop is prescribed along the pipe. Applying the classical energy method in homogenization theory, we study the asymptotic behavior of the solutions to this system, without any restriction on the magnitude of the data, as the diameters of the perforations vanish. Regardless of the initial scaling and distribution of the holes, we show that the sieve asymptotically becomes a wall, meaning that the effective equations are two, independent, stationary Navier-Stokes systems with a no-slip boundary condition on the wall. In the absence of external forces we prove, furthermore, that the fluid motion becomes quiescent in the homogenization limit.

Agnieszka Świerczewska-Gwiazda

University of Warsaw

On the relative entropy method and unconditional stability of radially symmetric steady sates of compressible viscous fluids.

I will discuss various applications of the relative enropy method in equations of fluid mechanics such as weak strong uniqueness results or asymptotic limits. I will mostly concentrate on recent result on unconditional stability of certain radially symmetric steady states of compressible viscous fluids in domains with inflow/outflow boundary conditions. This means that any not necessarily radially symmetric solution of the associated evolutionary problem converges to a single radially symmetric steady state.

Edriss S. Titi

University of Cambridge, Texas A&M University and Weizmann Institute of Science

Mathematical analysis of atmospheric and oceanic dynamics models: cloud formation and sea-ice models

In this talk we will present rigorous analytical results concerning global regularity, in the viscous case, and finite-time singularity, in the inviscid case, for oceanic and atmospheric dynamics models. Moreover, we will also provide a rigorous justification of the derivation of the Primitive Equations of planetary scale oceanic dynamics from the three-dimensional Navier-Stokes equations as the vanishing limit of the small aspect ratio of the depth to horizontal width. In addition, we will also show the global well-posedeness of the coupled three-dimensional viscous Primitive Equations with a micro-physics phase change moisture model for cloud formation. Eventually, we will also present short-time well-posedness of solutions to the Hibler's sea-ice model.

Alexis Vasseur

University of Texas at Austin

From Navier-Stokes to discontinuous solutions of compressible Euler

The compressible Euler equation can lead to the emergence of shock discontinuities in finite time, notably observed behind supersonic planes. A very natural way to justify these singularities involves studying solutions as inviscid limits of Navier-Stokes solutions with evanescent viscosities. The mathematical study of this problem is however very difficult because of the destabilization effect of the viscosities.

Bianchini and Bressan proved the inviscid limit to small BV solutions using the so-called artificial viscosities in 2004. However, until very recently, achieving this limit with physical viscosities remained an open question.

In this presentation, we will provide the basic ideas of classical mathematical theories to compressible fluid mechanics and introduce the recent method of a-contraction with shifts. This method is employed to describe the physical inviscid limit in the context of the barotropic Euler equation, and to solve the Bianchini and Bressan conjecture in this special case. This is a joint work with Geng Chen and Moon-Jin Kang.

Ewelina Zatorska

University of Warwick

One-dimensional compressible Euler equations with non-local effects

This talk will be devoted to a one-dimensional model of collective motion. The considered system consists of compressible Euler equations with nonlocal interaction term playing the same role as the pressure term in fluids' equations. I will present some of our recent results concerning existence of strong and weak solutions, long-time asymptotic of solutions, and singular limits through relative entropy method based on the two-velocity formulation.

Poster Session

Angela Bašić-Šiško
University of Rijeka

Existence results for micropolar real gas flow with spherical symmetry

Damien Galant
Université de Mons and CERAMATHS

An action approach to nodal and least energy normalized solutions for nonlinear Schrödinger equations

Chiara Lonati
Università Cattolica del Sacro Cuore

Bénard convection in a second-gradient fluid

Emanuele Pastorino
Politecnico di Milano

Unpredictable behavior of a partially damped system of PDEs modeling suspension bridges

Marko Radulović
University of Zagreb

Forchheimer extended Darcy-Brinkman flow through thin domains

Borja Rukavina
University of Zagreb

Asymptotic analysis of the nonsteady micropolar fluid flow through a system of thin pipes