Christmas Workshop on Fluid Mechanics

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Book of Abstracts

Chérif Amrouche (Université de Pau)

Stokes and Navier-Stokes equations with Navier boundary condition

We focus on the study of the incompressible fluid in a bounded domain in $\mathbb{R}^3$. We consider the stationary Stokes equation

$$-\Delta \boldsymbol{u} + \nabla \pi = \boldsymbol{f}, \quad \text{div}\ \boldsymbol{u} = 0 \ \text{ in } \Omega \tag{1}$$

and the stationary Navier-Stokes equation

$$-\Delta \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \nabla \pi = \boldsymbol{f}, \quad \text{div}\ \boldsymbol{u} = 0 \ \text{ in } \Omega \tag{2}$$

where Ω be a bounded domain in $\mathbb{R}^3$ with boundary Γ, possibly not connected, of class $\mathcal{C}^{1,1}$ and $\boldsymbol{u}$ and π are the velocity field and the pressure of the fluid respectively, $\boldsymbol{f}$ is the external force acting on the fluid. A boundary condition was suggested by Navier (in 1823) which states that on one hand, the normal component of the fluid velocity is zero at the boundary (impermeability condition) and on the other hand, the amount of slip in the tangential part of the velocity, rather than being zero, is proportional to the tangential part of the normal stress exerted by the fluid on the boundary i.e.

$$\boldsymbol{u} \cdot \boldsymbol{n} = 0, \quad 2\left[(\mathbb{D}\boldsymbol{u})\boldsymbol{n}\right]_{\boldsymbol{\tau}} + \alpha\, \boldsymbol{u}_{\boldsymbol{\tau}} = \boldsymbol{0} \ \text{ on } \Gamma \tag{3}$$

where $\boldsymbol{n}$ and $\boldsymbol{\tau}$ are the unit outward normal and tangent vectors on Γ respectively and $\mathbb{D}\boldsymbol{u} = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)$ is the rate of strain tensor. Here, α is the coefficient which measures the tendency of the fluid to slip on the boundary, called *friction coefficient*. We are interested to discuss the well-posedness of the problem (1) and (2) along with (3), in particular existence, uniqueness of weak and strong solutions in $\boldsymbol{W}^{1,p}(\Omega)$ and $\boldsymbol{W}^{2,p}(\Omega)$ for all $1 < p < \infty$ considering minimal regularity on the friction coefficient α. Moreover, we deduce estimates to analyze the behavior of the solution with respect to α which indicates in some sense, an inverse of the derivation of the Navier boundary conditions from no-slip boundary condition for rough boundaries. This is a joint work with P. Acevedo, C. Conca and A. Ghosh.

References

[1] P. Acevedo, C. Amrouche, C. Conca and A. Ghosh, Stokes and Navier-Stokes equations with Navier boundary conditions, *Journ. Diff. Equations*, **285**, (2021), 258–320

[2] C. Amrouche and A. Rejaiba, *L^p-theory for Stokes and Navier-Stokes equations with Navier boundary condition*, *Journ. Diff. Equations*, **256**, (2014), 1515–1547.

Danica Basarić (Czech Academy of Sciences)

Dynamics on open fluid systems

The main object of the present talk is the Navier-Stokes-Fourier system modeling the dynamics of a general viscous and heat-conducting fluid, confined to a bounded domain; in particular, inhomogeneous Dirichlet boundary conditions for the temperature will be imposed. The existence of global-in-time weak solutions will be discussed and a blow-up criterion in the class of strong solutions will be provided.

Hugo Beirão da Veiga (Università di Pisa)

On the Energy Equality for the Navier-Stokes Equations

We discuss on integral sufficient conditions on u, ∇u and mixed, to guarantee the equality of the energy, EE, for solutions of the Navier-Stokes equations. We turn back to a suitable interpretation of some main parameters which allows to overcome some apparent incongruence. Furthermore we refer on assumptions on the direction of vorticity which implies the energy equality.

Elvise Berchio (Politecnico di Torino)

Equilibrium configurations of a symmetric body immersed in a stationary Navier-Stokes flow in a planar channel

We consider the equilibrium configurations for several fluid-structure interaction problems. The fluid is confined in a 2D unbounded channel that contains a body, free to move inside the channel with rigid motions (transversal translations and rotations). The motion of the fluid is generated by a Poiseuille inflow/outflow at infinity and governed by the stationary Navier-Stokes equations. For a model where the fluid is the air and the body represents the cross-section of a suspension bridge, therefore also subject to restoring elastic forces, we prove that for small inflows there exists a unique equilibrium position while for large inflows we numerically show the appearance of additional equilibria. A similar uniqueness result is also obtained for a discretized 3D bridge, consisting in a finite number of cross-sections interacting with the adjacent ones. The very same model, but without restoring forces, is used to describe the mechanism of the Leonardo da Vinci ferry, which is able to cross a river without engines. We numerically determine the optimal orientation of the ferry allowing to cross the river in minimal time. Based on a joint work with D. Bonheure, P. Galdi, F. Gazzola and S. Perotto.

Denis Bonheure (Université Libre de Bruxelles)

Flow past a spring mounted body

I will give an overview of several results obtained recently concerning the motion of a rigid body subject to an undamped elastic restoring force, in the stream of a viscous liquid under the action of a prescribed flow at large distances. I will discuss several aspects like quasi-static dynamics, nonlinear dynamics in the small Reynolds regime and also global results. The framework is either an infinite 2D strip where a Poiseuille flow is imposed upstream and downstream at large distances or a 3D external domain where a uniform flow is given at spatial infinity.

Elia Bruè (Università Bocconi di Milano)

Anomalous Dissipation in Fluid Dynamics

Kolmogorov's K41 theory predicts anomalous dissipation in fully developed turbulence of incompressible fluids. Turbulent regimes transfer information to small scales, causing dissipation without viscosity, mainly due to limited solution regularity. Despite these insights, our understanding in rigorous mathematical terms remains limited. In this seminar, I will outline recent advancements, providing examples in forced three-dimensional Navier-Stokes equations and passive-scalar advection as in the Obukhov-Corrsin theory of scalar turbulence.

Ángel Castro (ICMAT Madrid)

Entropy solutions to macroscopic IPM

The IPM system models the motion of an incompressible and viscous fluid in a porous media. The case in which the density of this fluid takes just two values separated by an interface is known as the Muskat problem. When the part of the fluid with the bigger density is always above the other part, the equation of the evolution of the interface is ill-posed. In this situation some "mixing solutions" arise in which the interface disappears and is replaced by a strip where the densities mix. We will review what is known about the existence and (lack) uniqueness of this kind of solutions and introduce the concept of subsolution. Finally, we will study the existence of some mixing solutions which came from a scheme proposed by F. Otto to select a subsolution.

Michele Coti Zelati (Imperial College London)

Entropy maximization in the two-dimensional Euler equations

We investigate certain questions arising in two-dimensional statistical hydrodynamics, by relying on principles of entropy maximization for the vorticity of a two-dimensional perfect fluid in a disc. In analogy with the entropy functions used in statistical mechanics and thermodynamics, we show that similar concavity properties hold for the 2d Euler equations when maximizing entropies at fixed energy levels. The proofs rely on rearrangement inequalities, a modification of the classical min-max principle, and the properties of the Euler-Lagrange equations for the corresponding constrained optimization.

David Lannes (Université de Bordeaux)

The 2D nonlinear shallow water equations with a partially immersed obstacle

This talk (based on an joint work with T. Iguchi) is devoted to the proof of the well-posedness of a model describing waves propagating in shallow water in horizontal dimension $d = 2$ and in the presence of a fixed partially immersed object. We first show that this wave-interaction problem reduces to an initial boundary value problem for the nonlinear shallow water equations in an exterior domain, with boundary conditions that are fully nonlinear and nonlocal in space and time. This hyperbolic initial boundary value problem is characteristic, does not satisfy the constant rank assumption on the boundary matrix, and the boundary conditions do not satisfy any standard form of dissipativity. Our main result is the well-posedness of this system for irrotational data and at the quasilinear regularity threshold. In order to prove this, we introduce a new notion of weak dissipativity, that holds only after integration in time and space. This weak dissipativity allows high order energy estimates without derivative loss; the analysis is carried out for a class of linear non-characteristic hyperbolic systems, as well as for a class of characteristic systems that satisfy an algebraic structural property that allows us to define a generalized vorticity. We then show, using a change of unknowns, that it is possible to transform the linearized wave-interaction problem into a non-characteristic system, which satisfies this structural property and for which the boundary conditions are weakly dissipative. We can therefore use our general analysis to derive linear, and then nonlinear, a priori energy estimates. Existence for the linearized problem is obtained by a regularization procedure that makes the problem non-characteristic and strictly dissipative, and by the approximation of the data by more regular data satisfying higher order compatibility conditions for the regularized problem. Due to the fully nonlinear nature of the boundary conditions, it is also necessary to implement a quasilinearization procedure. Finally, we have to lower the standard requirements on the regularity of the coefficients of the operator in the linear estimates to be able to reach the quasilinear regularity threshold in the nonlinear well-posedness result.

Riccardo Montalto (Università Statale di Milano)

Vanishing viscosity quasi-periodic solutions for the 2D Navier-Stokes equations

In this talk I will discuss some recent results on Euler and Navier Stokes equations concerning the construction of quasi-periodic solutions and the problem of the invscid limit for the Navier Stokes equation. I will discuss the following two results:

1) Construction of quasi-periodic solutions for the Euler equation with a time quasi-periodic external force, bifurcating from a constant, diophantine velocity field.

2) I shall discuss the inviscid limit problem for the forced Navier Stokes equations, namely I shall prove the existence of quasi-periodic solutions of the Navier Stokes equation converging to the one of the Euler equation constructed in 1).
The main difficulty is that this is a singular limit problem. We overcome this difficulty by implementing a normal form method which allow to prove sharp estimates (global in time) w.r. to the viscosity parameter. This is a joint work with Luca Franzoi.

Konstantinas Pileckas (Vilnius University)

Non-stationary Navier-Stokes equations in 2D power-cusp domain

The initial boundary value problem for the non-stationary Navier-Stokes equations is studied in 2D bounded domain with a power-cusp singular point O on the boundary. The case of the boundary value with a nonzero flow rate is considered. In this case there is a source/sink in O and the solution necessary has infinite energy integral. The formal asymptotic expansion of the solution near the singular point is constructed. The justification of the asymptotic decomposition and the existence of a solution are proved.

Paolo Secchi (Università di Brescia)

Nonlinear stability of two-dimensional compressible current-vortex sheets

In this talk we are concerned with nonlinear stability and existence of two-dimensional current-vortex sheets in ideal compressible magnetohydrodynamics. This is a nonlinear hyperbolic initial-boundary value problem with characteristic free boundary. It is well-known that current-vortex sheets may be at most weakly (neutrally) stable due to the existence of surface waves solutions that yield a loss of derivatives in the energy estimate of the solution with respect to the source terms. We first identify a sufficient condition ensuring the weak stability of the linearized current-vortex sheets problem. Under this stability condition for the background state, we show that the linearized problem obeys an energy estimate in anisotropic weighted Sobolev spaces with a loss of derivatives. Based on the weakly linear stability results, we then establish the local-in-time existence and nonlinear stability of current-vortex sheets by a suitable Nash-Moser iteration, provided the stability condition is satisfied at each point of the initial discontinuity. This result gives a new confirmation of the stabilizing effect of sufficiently strong magnetic fields on Kelvin-Helmholtz instabilities. This is a joint work with A. Morando (Brescia), P. Trebeschi (Brescia) and D. Yuan (Beijing Normal Univ.)

References

[1] A. Morando, P. Secchi, P. Trebeschi and D. Yuan, *Nonlinear stability and existence of two-dimensional compressible current-vortex sheets*, Arch. Rational Mech. Anal. (2023) 247:50

Ana Silvestre (Universidade de Lisboa)

Steady Navier-Stokes equations with directional do-nothing boundary conditions: analysis, numerical simulation and optimal control

In this talk, we consider the steady Navier-Stokes equations (NS) with mixed boundary conditions, where, instead of the classical do-nothing (CDN) outflow condition, a directional do-nothing (DDN) or a regularized directional do-nothing (RDDN) condition are imposed. We prove the well-posedness of problems NS+DDN and NS+RDDN and present some numerical results which illustrate the relevance of the directional do-nothing boundary conditions and the application of a semi-smooth Newton method. An optimal control problem of velocity tracking type is also addressed. The regularized directional do-nothing (RDDN) condition is defined in terms of a parameter $0 < \delta \ll 1$. We discuss the limit $\delta \to 0$ in the forward and control problems. The results for problems NS+DDN and NS+RDDN are obtained under appropriate assumptions on the size of the data and the controls, which, however, are less restrictive when compared with the case of a CDN outflow condition. This is a joint work with Pedro Nogueira and Jorge Tiago (IST, ULisboa).

Stefano Spirito (Università dell'Aquila)

Finite dimensional approximation of Yudovich solutions of the 2D incompressible Euler equations

In this talk we consider the approximation of the unique Yudovich solution via a finite dimensional Galerkin method. In particular, we will focus on the convergence in strong norms of the velocity and the vorticity, and the study of the rates of convergence. We will compare the results with the analogous ones available for the vanishing viscosity approximation and comment on differences and similarities. The talk is based on joint works with L.C. Berselli (Univ. Pisa), G. Crippa (Univ. Basel), and G. Ciampa (Univ. Milano Statale).

Franck Sueur (Université de Bordeaux)

Differential transmutations

Inspired by Gromov's partial differential relations, we introduce a notion of differential transmutation, which allows to transfer some local properties of solutions of a PDE to solutions of another PDE, in particular local solvability, hypoellipticity, weak and strong unique continuation properties and the Runge approximation property. The latest refers to the possibility to approximate some given local solutions by a global solution, with point controls at preassigned positions in the holes of the initial domain. As an example we consider the steady Stokes system, which can be obtained as a differential transmutation of an appropriate tensorization of the Laplace operator.

Michael Winkler (Universität Paderborn)

Regularity in chemotaxis-fluid interaction

Evolution systems coupling chemotaxis models to the Stokes and Navier-Stokes equations from fluid mechanics are considered. The focus is on the discussion of analytical approaches toward describing how far such systems, despite their considerable complexity, may inherit essential regularity features from the correspondingly unperturbed taxis-only and fluid-only subsystems. The presentation addresses both the case in which chemotactic interaction is determined by signal production through individuals, and models which alternatively account for signal consumption.