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# Risk seeking, non convex remuneration and regime switching

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## **Abstract**

We investigate asset management in a regime switching framework when the fund manager aims to beat a certain target for the assets under management either in an infinite horizon or over a finite horizon. We consider both a full information and a partial information setting. In a full information setting, the asset manager tends to take more risk in the good state and less risk in the bad state with respect to the constant parameter environment. Confidence risk induces the agent to increase his risk exposure.

**Keywords:** regime switching, bonus, remuneration, investment.

# 1 Introduction

The recent financial crisis has shed some light on risk incentives and remuneration schemes. While the academic literature has investigated the issue mostly in a constant investment opportunity environment, the financial crisis has suggested the possibility that a non convex remuneration scheme may induce excess risk seeking when financial markets are bubbling. In other words, bonus compensation and irrational exuberance may create a dangerous mix. Charles Prince (the former CEO of Citi) said it explicitly: "As long as the music is playing, you have got to get up and dance".

In this paper we investigate this claim analyzing asset management in a regime switching framework when the fund manager aims to beat a certain target for the assets under management (wealth). The asset manager gets a fixed reward either the first time a target is reached or if the target is reached over a finite horizon. This type of reward fee models an absolute performance remuneration scheme in an infinite and in a finite horizon.

Regime switching is modeled assuming that the drift and the volatility of the geometric Brownian motion of the asset price evolves as a continuous time Markov chain with two states. We consider both a full information and a partial information setting. In the first case the manager observes the state of the Markov chain, in the second one he does not observe the state of the Markov chain but forms beliefs on it through the observation of the asset price. The regime switching setting is interesting because it allows us to model in a simple way a mean reverting dynamics addressing some regularities observed in the asset pricing-management literature, see [Cecchetti et al., 1990, Ang and Timmermann, 2011, Guidolin and Timmermann, 2007], and in the option pricing literature, see [Buffington and Elliott, 2002, Guo and Zhang, 2004, Elliott et al., 2005]. Our analysis shows that a non convex remuneration scheme may lead to excess risk taking when the market regime switches over time.

The main goal of the paper is to analyze how a pure target driven asset allocation is affected by the risk of switch in the state of the economy and by the estimation risk about the state. The first issue is addressed considering the full information setting, the second one considering the partial information setting.

Note that there are no transaction costs, therefore in a full information setting the manager can redefine the strategy promptly as the state of the economy changes. However, this possibility does not imply that the optimal portfolio coincides with the one obtained in a constant parameter environment. As a matter of fact, in a good (bad) state the manager may decide to overweight (underweight) the risky asset position exploiting the momentum and fearing (waiting) a switch to the bad (good) state. This consideration does not affect the optimal portfolio of an agent maximizing the expected utility from terminal wealth. In fact, [Sotomayor and Cadenillas, 2009] showed that in case of an utility function with constant relative risk aversion, the optimal portfolio is the constant weight obtained in the constant parameter environment with state dependent parameters (drift and variance matrix). Therefore, the regime switching does not affect the optimal policy obtained in case of constant parameters, simply the agent switches as the state changes always adopting the investment policy obtained in the state as if the parameters were constant.

Considering a target driven remuneration scheme we show that the risk of a regime switch affects the optimal investment policy obtained in the case of constant parameters. Differently from the constant parameter setting, see [Browne, 1995, Browne, 1997], when the agent is rewarded with a constant bonus as the assets under management touch the target, the solution is no more a constant weight. When the assets under management are low and the target is far away, the agent acts as a risk lover in a good state, i.e., he overweights the risky asset, and he takes less risk in a bad state underweighting the risky asset with respect to the constant parameter solution. The effect is reversed when the value of the assets approaches the barrier. Instead, when the horizon is fixed, the asset manager always takes more risk in the good state and less risk in the bad state with respect to the constant parameter environment.

In a partial information setting, the agent faces a confidence risk, i.e., the change of his beliefs about the state of the economy. It is worthwhile to observe that in a two regime switching model beliefs update and asset returns are positively correlated. In other words confidence risk is positively correlated with market risk. As a consequence, an agent more

risk averse than a logarithmic utility would attempt to hedge the confidence risk buying less of the risky asset with respect to the constant parameter optimal investment strategy, i.e., the hedging demand is negative, see [David, 1997, Honda, 2003]. In our setting we show that the agent's attitude towards confidence risk is similar to what we observe in a full information setting: when there is an infinite horizon, for a small wealth the agent overweights (underweights) the risky asset in the good (bad) state. The reverse occurs when the wealth approaches the target. When the horizon is fixed we observe that the agent underinvests in the bad state and overinvests in the good state and that he takes a long position also for a belief with a small probability of the favorable state.

We also extend to the analysis to the case of a manager who is remunerated when he beats a benchmark and to high water marks remuneration fees. The main results of our analysis are confirmed: in both cases, the manager takes more (less) risk in a good (bad) state.

This paper adds to the literature on incentive fees and asset management showing that a non convex remuneration scheme leads to excess risk-seeking, see [Grinblatt and Titman, 1989, Carpenter, 2000, Ross, 2004, Goetzmann et al., 2003, Panageas and Westerfield, 2009]. We show that regime switching induces excess risk taking in the favorable state when the horizon is fixed or is infinite and the target is far away.

The paper is organized as follows. In Section 2 we introduce the regime switching model in a full and in a partial information environment. In Section 3 we analyze the asset management problem when the manager goal is provided by a fixed bonus when the target for the assets under management is reached. In Section 4 we analyze the asset management problem when the manager goal is provided by maximizing the probability of reaching a given target over a finite horizon. In Section 5 we extend the analysis to the case of a manager who wants to beat a benchmark. In Section 6 we analyze portfolio choices of a manager remunerated through a high water marks contract.

## 2 The Model

Let  $(\Omega, \mathcal{F}, P)$  be a probability space on which a standard Brownian motion  $Z$  and a two state continuous Markov chain  $Y$  are defined. The process  $Y$  is right-continuous with values in  $\{0, 1\}$  and represents the regime of the economy.

In  $t = 0$ ,  $Y(0)$  has outcome 1 with probability  $p$  and 0 with probability  $1 - p$ . The process  $Y(t)$  starting in state  $i$  remains in the same state for an exponentially distributed length of time and jumps to state  $j \neq i$  with intensity  $\lambda_{ij}$ . In what follows we consider the symmetric case and we set  $\lambda_{01} = \lambda_{10} = \lambda$ .

The jump times are independent and independent of  $Z$ . The regime switching and the Brownian motion generate the information filtration  $\mathcal{F} = \{\mathcal{F}_t^{Z,Y}\}$  where  $\mathcal{F}_t^{Z,Y} = \sigma(Z(s), Y(s), s \leq t)$ , i.e.,  $\mathcal{F}_t^{Z,Y}$  is the augmented  $\sigma$ -algebra on  $\Omega$  generated by the observation of  $Z(s)$  and  $Y(s)$  up to  $t$ .

The agent can trade a riskless bond and a risky asset paying no dividend. The riskless bond price  $B(t)$  satisfies

$$dB(t) = rB(t)dt, \quad B(0) = 1$$

with a positive constant  $r$ , the risky asset price evolves as

$$dS(t) = S(t)\mu(Y(t))dt + S(t)\sigma dZ(t), \quad S(0) = S_0.$$

As far as the information set is concerned, we consider two different information environments: the full information, and the partial information one.

In the main setting, the volatility of the asset price is constant  $\sigma$ , instead the drift is a function of the state  $Y(t)$ . More precisely, we assume  $\mu(0) = \mu_0$  and  $\mu(1) = \mu_1$ . In the partial information setting we will stick to a constant volatility in the two regimes, instead under full information we will relax this assumption considering also the case of switching volatility ( $\sigma(Y(t))$ ). In the following, we denote by  $w(t)$  the wealth fraction invested in the risky asset.

In the full information setting, the investor observes  $Y(t)$ ,  $Z(t)$ , and  $S(t)$ . In this case,  $w(t)$  is adapted to  $\mathcal{F}_t^{Z,Y}$ . In the partial information setting, the agent only observes the asset price  $S(t)$ , he does not observe  $Y(t)$  and  $\mu(Y(t))$ . In this case, the investor's

information is defined by the filtration  $\mathcal{F}^S = \{\mathcal{F}_t^S\}$  where  $\mathcal{F}_t^S = \sigma(S(s), s \leq t)$ . The investment policy  $w(t)$  is adapted to  $\mathcal{F}_t^S$ . In both cases, the parameters  $\sigma$ ,  $p$ ,  $\lambda$ ,  $\mu_0$ ,  $\mu_1$  are known constants.

Let  $X(t)$  the assets under management (wealth) of the manager. In the full information setting,  $X(t)$  evolves as follows:

$$dX(t) = X(t)(w(t)(\mu(Y(t)) - r) + r)dt + w(t)\sigma X(t)dZ(t), \quad X(0) = X_0. \quad (1)$$

In the partial information case, we can identify a  $\sigma$ -algebra equivalent economy with filtered probability

$$\pi(t) = P(Y(t) = 1 | \mathcal{F}_t^S), \quad \pi(0) = p.$$

$\pi(t)$  is the probability that the current regime is state 1 given the observation  $S(s), s \leq t$ . As shown in [Honda, 2003], filtering techniques yield that  $\pi(t)$  satisfies the stochastic differential equation

$$d\pi(t) = \lambda(1 - 2\pi(t))dt + \pi(t)(1 - \pi(t))\frac{\mu_1 - \mu_0}{\sigma}d\bar{Z}(t) \quad (2)$$

where  $\bar{Z}(t)$  is the standard Brownian motion defined as

$$\bar{Z}(t) := \int_0^t \frac{dS(s) - S(s)\hat{\mu}(\pi(s))}{S(s)\sigma} ds$$

with  $\hat{\mu}(\pi(s)) = \pi(s)\mu_1 + (1 - \pi(s))\mu_0$ .

A  $\sigma$ -algebra equivalent is described by the risk-free asset, the filtered probability space and the risky price process  $S(t)$  satisfying

$$dS(t) = S(t)\hat{\mu}(\pi(t))dt + S(t)\sigma d\bar{Z}(t)$$

and the filtration  $\mathcal{F}^S$  generated by  $S$ .

A trading strategy  $w(t)$  is an adapted process and  $X(t)$  evolves as follows

$$dX(t) = X(t)(w(t)(\hat{\mu}(\pi(t)) - r) + r)dt + w(t)\sigma X(t)d\bar{Z}(t), \quad X(0) = X_0. \quad (3)$$

Note the following features of the stochastic differential equation (2) governing agent's beliefs: a) the larger the difference between the two states ( $\mu_1$  and  $\mu_0$ ), the larger the

volatility of beliefs, b) the larger the volatility of asset returns, the smaller the volatility of beliefs, c)  $\pi$  is mean reverting, the mean reversion speed is high when the switching probability  $\lambda$  is large. These features imply that the degree of confidence on a state is mean reverting and the convergence rate is proportional to the switching probability. Confidence on the state of the economy is extremely volatile when the mean returns in the two states are different and the return volatility is low.

### 3 Fixed bonus from reaching a target

Let us analyze the asset allocation problem for a manager who is rewarded with a fixed amount of money (normalized to one) when the assets under management  $X(t)$  reach a certain target  $b$  provided that bankruptcy does not occur before ( $X(t) = 0$ ). Let us denote by

$$\tau_b = \inf\{t > 0 : X(t) = b\}$$

the first hitting time of the target  $b$  of the assets under management, and by  $\delta$  the discount factor of the manager, then the asset allocation problem can be formulated defining the optimal value function as

$$V(x) := \max_w E [e^{-\delta\tau_b} | X(0) = x]. \quad (4)$$

subject to (1) under full information and (3) under partial information. In the first case  $w(t)$  is adapted to  $\mathcal{F}_t^{Z,Y}$ , in the second case to  $\mathcal{F}_t^S$ . Notice that the manager goes bankrupt when the assets under management reach the zero level, i.e., the manager cannot leverage his position.

In [Browne, 1995, Browne, 1997] the problem has been analyzed in a no switching setting (constant parameters). As a benchmark for our analysis, we report the main results. Let  $\lambda = 0$  and denote by  $\mu$  the constant drift of the asset price. In this environment the Hamilton Jacobi Bellman (HJB) equation becomes

$$\sup_w -\delta V + (w(\mu - r) + r)xV' + \frac{\sigma^2 w^2}{2}x^2V'' = 0, \quad (5)$$

with the boundary conditions

$$V(0) = 0, \quad V(b) = 1.$$

Assuming that the HJB has a classical solution, i.e.,  $V'' < 0$ , it holds that

$$w^* = \frac{r - \mu}{\sigma^2} \frac{V'(x)}{xV''(x)},$$

and thus the HJB equation becomes

$$-\delta V - \frac{1}{2} \frac{(r - \mu)^2}{\sigma^2} \frac{V'(x)^2}{V''(x)} + rxV'(x) = 0, \quad 0 < x < b. \quad (6)$$

The value function becomes

$$V(x) = \frac{x^C}{b^C}$$

with

$$C = \frac{\delta + \frac{1}{2} \frac{(r-\mu)^2}{\sigma^2} + r - \sqrt{(\delta + \frac{1}{2} \frac{(r-\mu)^2}{\sigma^2} + r)^2 - 4\delta r}}{2r} > 0 \quad (7)$$

and therefore

$$w^* = \frac{1}{C-1} \frac{r - \mu}{\sigma^2}.$$

Note that the strategy is a constant weight as the one observed maximizing a logarithmic or a power utility function from terminal wealth. So, the portfolio is the golden rule (the strategy maximizing the expected logarithmic growth of rate) multiplied by the constant  $1/(C-1)$  that depends on the parameters of the model. Note that the parameter  $C$  depends on the Sharpe ratio of the risky asset. It is easy to show that the relationship between the Sharpe ration and the fraction of wealth invested in the risky asset is not monotonic.

### 3.1 Regime Switching with full information

We denote by  $V^0, V^1$  and  $w^0, w^1$  the value function and the optimal strategy in state 0 and 1. The HJB equation for problem (4) becomes

$$\begin{aligned} \sup_{w^0} -\delta V^0 + (w^0(\mu_0 - r) + r)xV_x^0 + \frac{\sigma^2(w^0)^2}{2}x^2V_{xx}^0 - \lambda V^0 + \lambda V^1 &= 0 \\ \sup_{w^1} -\delta V^1 + (w^1(\mu_1 - r) + r)xV_x^1 + \frac{\sigma^2(w^1)^2}{2}x^2V_{xx}^1 - \lambda V^1 + \lambda V^0 &= 0, \end{aligned}$$

the boundary conditions are

$$V^i(0) = 0, \quad V^i(b) = 1.$$

If we assume that  $V_{xx}^i < 0$ , then the optimal portfolio strategy in state  $i$  becomes

$$w^i = \frac{r - \mu_i}{\sigma^2} \frac{V_x^i}{xV_{xx}^i}, \quad i = 0, 1.$$

An explicit solution for the value function is not available. We solve the above problem numerically considering a finite difference technique to discretize the partial differential equation, coupled with a fixed-point algorithm, using as guess function the solution obtained in the no switching setting, in the following denoted as  $w_0$ . More precisely, we introduce a mesh  $\{x_j\}_{j=0}^N$ , with  $x_j = j\Delta x$ ,  $\Delta x = b/N$ ; given the guess vector  $\mathbf{f}_0^i$  defined as  $\{\mathbf{f}_0^i\}_j = f_0^i(x_j) = x_j w_0^i(x_j)$ ,  $i = 0, 1$ ,  $j = 1, \dots, N-1$ , for  $k = 0, \dots$ , we solve the linear system

$$\begin{aligned} -\delta \mathbf{V}_k^0 + (\mathbf{f}^0(\mu_0 - r) + rx) \cdot * \mathcal{T} \mathbf{V}_k^0 + \frac{\sigma^2 (\mathbf{f}^0)^2}{2} \cdot * \mathcal{D} \mathbf{V}_k^0 - \lambda \mathbf{V}_k^0 + \lambda \mathbf{V}_k^1 &= 0 \\ -\delta \mathbf{V}_k^1 + (\mathbf{f}^1(\mu_1 - r) + rx) \cdot * \mathcal{T} \mathbf{V}_k^1 + \frac{\sigma^2 (\mathbf{f}^1)^2}{2} \cdot * \mathcal{D} \mathbf{V}_k^1 - \lambda \mathbf{V}_k^1 + \lambda \mathbf{V}_k^0 &= 0, \end{aligned}$$

where the finite difference operator  $\mathcal{D}$  is given by

$$\{\mathcal{D} \mathbf{V}\}_j := \frac{V(x_{j-1}) - 2V(x_j) + V(x_{j+1}))}{\Delta^2 x^2},$$

with  $V(x_0) = 0$  and  $V(x_N) = 1$ . Similarly  $\mathcal{T}$  is the upwind finite difference operator for the first order derivative. Notice that in the above formulation  $\mathbf{f} \cdot * \mathbf{g}$  and  $(\mathbf{f})^2$  represent the element-wise product and square operators. The iterative procedure is repeated till the distances  $\mathbf{V}_k^0 - \mathbf{V}_{k-1}^0$  and  $\mathbf{V}_k^1 - \mathbf{V}_{k-1}^1$ , computed according to the  $l^2$  norm, fall below a  $10^{-6}$  tolerance threshold. Here we set the number of grid points  $N + 1$  equal to 2000.

In what follows, we set  $b = 5$ ,  $\mu_0 = 0.04$ ,  $\mu_1 = 0.08$ ,  $r = 0.05$ ,  $\delta = 0.04$ ,  $\sigma = 0.3$  and  $\lambda = 0.1$ . Note that there are no transaction costs, short selling is allowed, and volatility is constant in the two states, then what is relevant in the asset allocation problem is the absolute value of the expected excess return in the two states:  $|\mu_0 - r| = 0.01$  and  $|\mu_1 - r| = 0.03$ . For this set of parameters, state 1 is the good one not because of a higher

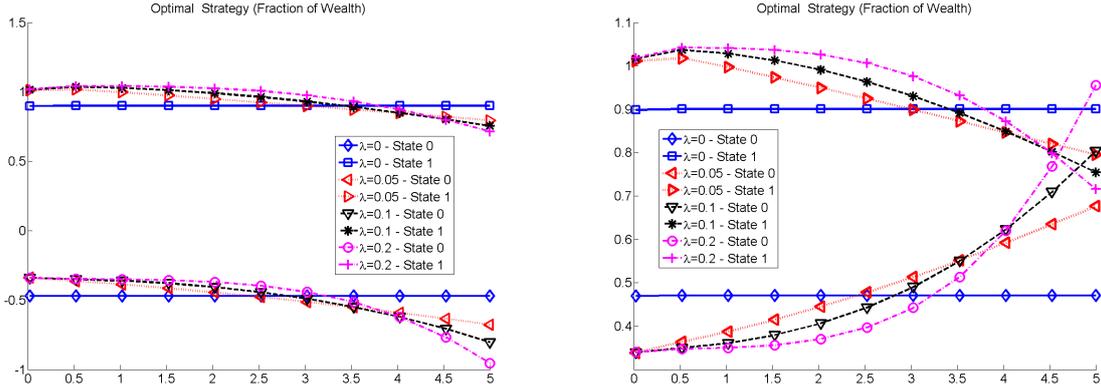


Figure 1: No Switching and Switching with Full Information:  $b = 5$ ;  $\delta = 0.04$ ;  $r = 0.05$ ;  $\mu_0 = 0.04$  (left) and  $\mu_0 = 0.06$  (right);  $\mu_1 = 0.08$ ;  $\sigma = 0.3$

expected return but because the absolute value of the expected excess return is higher than the one observed in state 0. In Figure 1 we represent the fraction of the wealth invested in the risky asset also assuming  $\mu_0 = 0.06$ . Note that in both cases the risky asset in state 1 is more favorable than in state 0, the difference is that the manager short sells the risky asset in state 0 when  $\mu_0 = 0.04$  and, instead, he invests a positive amount of wealth in both states when  $\mu_0 = 0.06$ .

These figures show that the optimal investment strategy is not a constant weight: the exposure to the risky asset ( $|w|$ ) decreases (increases) in the good (bad) state as wealth increases. If we compare the optimal portfolio obtained in the regime switching setting with full information with the one obtained in a setting with constant weights (no regime switching,  $\lambda = 0$  in the two figures), we notice that when the wealth is low the manager invests more (less or sells short less) in the risky asset in state 1 (state 0) with respect to the case without regime switching. This attitude is reversed when the reward target is approaching. The departure from the constant weight strategy increases as the switching probability increases ( $\lambda$  goes up). So the bias in the investment policy due to the switching probability is asymmetric with more risk in the good state and less risk in the bad state when the wealth is low. The opposite effect is observed when the wealth is next to the target barrier.

It seems that a reward through a fixed bonus as the target is reached induces the agent

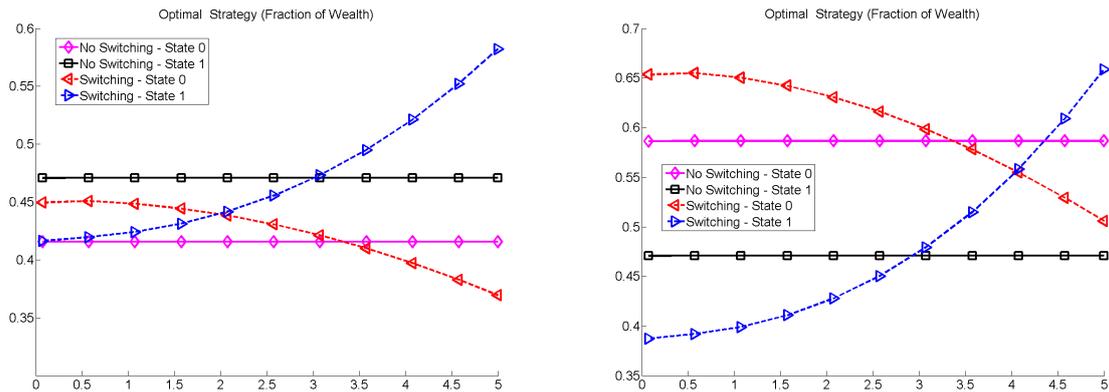


Figure 2: No Switching and Switching with Full Information ( $\lambda = 0.1$ ):  $b = 5$ ;  $\delta = 0.04$ ;  $r = 0.05$ ;  $\mu_0 = 0.08$ ;  $\mu_1 = 0.06$ ;  $\sigma_0 = 0.5$ ;  $sr_0 = 0.06$  and  $\sigma_1 = 0.3$ ;  $sr_1 = 0.0333$  (left) -  $\sigma_0 = 0.4$ ;  $sr_0 = 0.075$  and  $\sigma_1 = 0.3$ ;  $sr_1 = 0.0333$  (right)

to excess (less) risk taking in the good (bad) state when the barrier is far away. In this case, in the good state the manager exploits the momentum and takes more risk fearing a switch to the bad state. On the other hand, in case of the bad state, as the target is far away the manager expects a switch to the good state and therefore he takes less risk. Instead, when the wealth is close to the barrier, the probability that the barrier is reached before a switch increases and therefore the agent's investment attitude is reversed in both states. Notice that when  $|\mu_1 - r| = |\mu_0 - r|$  the optimal investment does not depend on  $\lambda$ , the optimal investment weights are constant and coincide with those obtained in case of constant parameters. This interpretation is confirmed by the fact that this phenomenon is magnified by an increase in the switching probability  $\lambda$ . A similar result is obtained in [Panageas and Westerfield, 2009] in case of a high water mark fee, where it is shown that the portfolio weight is increasing in the density of the jump determining the termination of the fund.

In the full information setting, we can easily extend the analysis to the case with both the drift and the volatility of the risky asset switching as the state changes ( $\mu_i, \sigma_i, i = 0, 1$ ). As both the drift and the volatility change in the two states, the risk-return profile can be evaluated according to the Sharpe ratio  $sr_i = \frac{\mu_i - r}{\sigma_i}$ ,  $i = 0, 1$ . In Figure 2 we consider two different sets of parameters, in both cases the Sharpe ratio in state 0 is

higher than the one in state 1. The figure shows that when the wealth is low the agent overinvests (underinvests) in the risky asset in the state with the higher (lower) Sharpe ratio with respect to the constant parameter setting. The reverse occurs when the wealth approaches the target. This result confirms that when the target is far away the asset manager invests more in the risky asset in the favorable state fearing the switch to the bad state. This attitude is reversed when the target is approaching.

### 3.2 Regime Switching with partial information

Considering the stochastic differential equation for beliefs (2), for a control processes  $w$  the generator of the above process becomes

$$\begin{aligned} \mathcal{A}^w g(t, x, \pi) = & g_t + (w(\hat{\mu}(\pi) - r) + r)xg_x + \lambda(1 - 2\pi)g_\pi \\ & + \frac{1}{2}\sigma^2 w^2 x^2 g_{xx} + \frac{1}{2}\pi^2(1 - \pi)^2 \frac{(\mu_1 - \mu_0)^2}{\sigma^2} g_{\pi\pi} + w\pi(1 - \pi)(\mu_1 - \mu_0)xg_{x\pi} \end{aligned} \quad (8)$$

where  $g_t, g_x, \dots$ , denote the derivatives of the function  $g$ .

Defining the optimal value function as

$$V(x, \pi) := \sup_w E [e^{-\delta\tau_b} | X(0) = x, \pi(0) = \pi],$$

the HJB equation becomes

$$\begin{aligned} \sup_w -\delta V + (w(\hat{\mu}(\pi) - r) + r)xV_x + \lambda(1 - 2\pi)V_\pi \frac{\sigma^2 w^2}{2} x^2 V_{xx} \\ + \frac{\pi^2(1 - \pi)^2}{2} \frac{(\mu_1 - \mu_0)^2}{\sigma^2} V_{\pi\pi} + w\pi(1 - \pi)(\mu_1 - \mu_0)xV_{x\pi} = 0, \quad 0 \leq x \leq b, \quad 0 \leq \pi \leq 1 \end{aligned} \quad (9)$$

where  $\hat{\mu}(\pi) = \mu_0(1 - \pi) + \mu_1\pi$ , with boundary conditions

$$V(0, \pi) = 0, \quad V(b, \pi) = 1,$$

for  $0 \leq \pi \leq 1$ . For  $\pi = 0$  the above equation becomes

$$\sup_w -\delta V + (w(\mu_0 - r) + r)xV_x + \lambda V_\pi + \frac{\sigma^2 w^2}{2} x^2 V_{xx} = 0, \quad (10)$$

and for  $\pi = 1$  it becomes

$$\sup_w -\delta V + (w(\mu_1 - r) + r)xV_x - \lambda V_\pi + \frac{\sigma^2 w^2}{2} x^2 V_{xx} = 0. \quad (11)$$

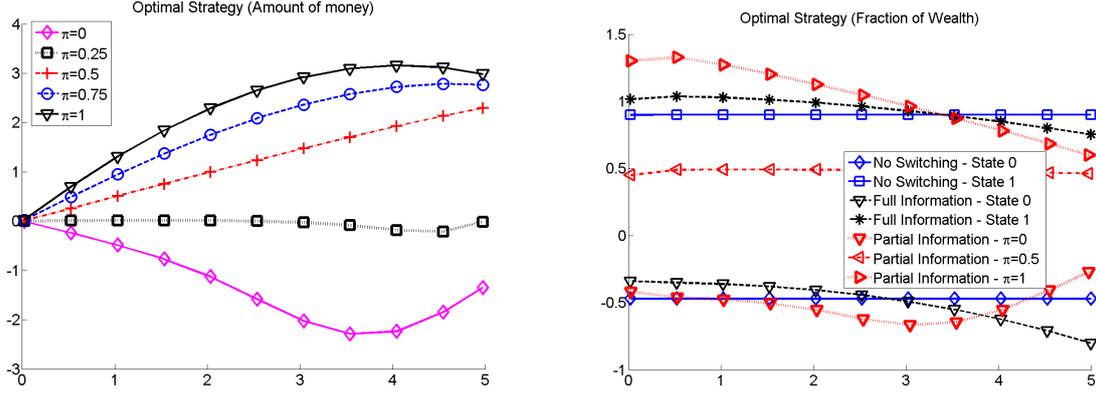


Figure 3: Switching with Partial Information, no Switching, Switching with Full Information:  $b = 5$ ;  $\delta = 0.04$ ;  $\lambda = 0.1$ ;  $r = 0.05$ ;  $\mu_0 = 0.04$ ;  $\mu_1 = 0.08$ ;  $\sigma = 0.3$

Assuming that the HJB (9) has a classical solution, i.e.,  $V_{xx} < 0$ , differentiating with respect to  $w$ , it holds

$$w^* = \frac{-(\hat{\mu}(\pi) - r)V_x - \pi(1 - \pi)(\mu_1 - \mu_0)V_{x\pi}}{\sigma^2 x V_{xx}}.$$

The partial differential equations (9)-(11) can be solved again with a finite difference scheme associated with a fixed-point algorithm. In Figure 3 we plot the optimal investment strategy for different values of  $\pi$ . First of all we notice that the optimal strategy is not a constant weight. For a given level of wealth  $x$ , the investment weight is increasing in the probability  $\pi$  that the agent assigns to the state with the higher expected return. Note that also for a low level of confidence in the favorable state (e.g.  $\pi = 0.25$ ) the agent is long in the risky asset and not short as we would expect being  $\mu_0 < r$ . It seems that the agent aiming to reach the target over a finite horizon takes a high risk investing in the risky asset also when the probability guess would suggest to sell it short. Again, the rationale is that  $|\mu_1 - r| = 0.03 > |\mu_0 - r| = 0.01$  and therefore the agent aiming at reaching the target as soon as possible "bets" on the most favorable state (1) also when the probability of being in that state is small. We can conclude that confidence risk induces the agent to take a long position hoping to be in the good state.

For  $\pi = 0$  and  $\pi = 1$  we can compare the optimal investment strategies obtained

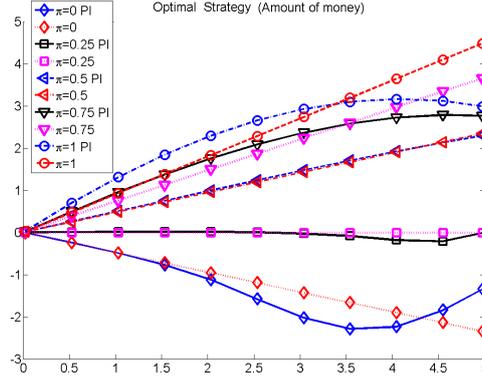


Figure 4: Switching with Partial Information comparison with a certainty equivalent drift strategy (PI):  $b = 5$ ;  $\delta = 0.04$ ;  $r = 0.05$ ;  $\mu_0 = 0.04$ ;  $\mu_1 = 0.08$ ;  $\sigma = 0.3$ ,  $\lambda = 0.01$

under partial information with those obtained under full information (see the previous subsection) and with those obtained with a constant investment opportunities set. The comparison is provided in the second picture of Figure 3. When the wealth is low, confidence risk induces the agent to invest in the risky asset more (less) than in the full information setting in the good (bad) state, when the wealth approaches the target the phenomenon is reversed.

According to these results, confidence risk leads to a higher (lower) risk exposure when the wealth is low (high). Remember that confidence and market risk are positively correlated, therefore an agent maximizing the expected utility should take less risk, instead a pure target bonus induces the agent to take excess risk when the target is far away. A reward through a fixed bonus induces the agent to act as a risk lover with respect to confidence risk. When the target approaches the target the attitude changes for arguments similar to those introduced in a full information world. The interpretation is confirmed looking at Figure 4 where, for different values of  $\pi$ , we compare the optimal investment strategy with partial information with the no switching strategy with a drift equal to the expected drift according to the agent's beliefs ( $\hat{\mu}(\pi) = \mu_0(1 - \pi) + \mu_1\pi$ ) (certainty equivalent drift strategy). The analysis confirms that the strategy obtained in the partial information setting is more risky than the no switching one if the agent's wealth is low, the reverse occurs when the wealth approaches the target.

## 4 Reaching a target by a deadline

In this section we analyze the optimal investment strategy for an agent maximizing the probability of reaching a certain target for the assets under management by the end of the horizon  $[0, T]$ . Set  $b$  the target, the optimal value function becomes

$$V(x) = E [P(X(T)) | X(0) = x].$$

where  $P(x) := \mathbf{1}_{x \geq b}$ . We assume that reaching a zero wealth corresponds to bankruptcy. Notice that for  $0 \leq t < T$  and for any wealth level  $x \geq x_{\max} = x_{\max}(t) := be^{-r(T-t)}$ , the value function  $V(x, t) = 1$ , and the optimal policy consists in investing all the wealth in the risk-free asset to reach the target level at the terminal time with probability 1.

This problem in a constant parameter setting has been addressed in [Browne, 1999]. The author shows that the problem corresponds to solve the HJB equation

$$\sup_w V_t + (w(\mu_0 - r) + r)xV_x + \frac{\sigma^2 w^2}{2}x^2V_{xx} = 0, \quad (12)$$

on  $[0, T) \times (0, x_{\max})$ , with boundary condition  $V(t, 0) = 0$  and  $V(t, x_{\max}(t)) = 1$  for any  $t \in [0, T)$  and terminal condition  $V(x, T) = P(x)$ . The author proves that the optimal solution is

$$V(x, t) = \Phi \left( \Phi^{-1} \left( \frac{x}{x_{\max}} \right) + (T - t) \left( \frac{\mu - r}{\sigma} \right)^2 \right),$$

$$w = w(x, t) = \frac{\mu - r}{\sigma |\mu - r| \sqrt{T - t}} \frac{x_{\max}}{x} \phi \left( \Phi^{-1} \left( \frac{x}{x_{\max}} \right) \right),$$

where we denote by  $\phi$  and  $\Phi$  the density and the cumulative distribution function of a standard normal random variable, respectively. Notice that the terminal condition causes a discontinuity, since  $\lim_{t \rightarrow T} V(x, t) = \frac{x}{x_{\max}} \neq P(x)$ . The solution is not a constant weight, it is time dependent and is affected through the normal density by the distance of  $x$  from  $x_{\max}$ .

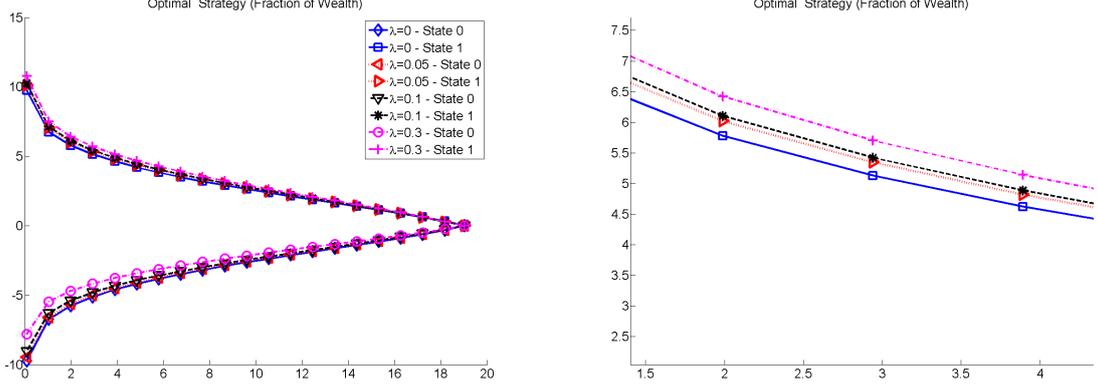


Figure 5: No Switching, Switching with Full Information:  $t = 0, b = 20, T = 1, \mu_0 = 0.04, \mu_1 = 0.08, r = 0.05, \sigma = 0.3$ . Right: zoom of the state 1 case.

#### 4.1 Regime Switching with full information

In a regime switching environment with full information, the HJB equation becomes

$$\begin{aligned} \sup_{w^0} V_t^0 + (w^0(\mu_0 - r) + r)xV_x^0 + \frac{\sigma^2(w^0)^2}{2}x^2V_{xx}^0 - \lambda V^0 + \lambda V^1 &= 0, \\ \sup_{w^1} V_t^1 + (w^1(\mu_1 - r) + r)xV_x^1 + \frac{\sigma^2(w^1)^2}{2}x^2V_{xx}^1 - \lambda V^1 + \lambda V^0 &= 0, \end{aligned}$$

for  $0 \leq t < T, 0 \leq x \leq x_{\max} = x_{\max}(t)$  with terminal condition  $V^i(T, x) = P(x), i = 0, 1$ . Due to the bankruptcy condition, it must also hold  $V^i(t, 0) = 0$  for  $0 \leq t \leq T, i = 0, 1$ . Moreover,  $V^i(t, x_{\max}) = 1$  for  $0 \leq t < T, i = 0, 1$ . If we assume that  $V_{xx}^i < 0, i = 0, 1$ , then the optimal policy is given by

$$w^i = \frac{r - \mu_i}{\sigma^2} \frac{V_x^i}{xV_{xx}^i}.$$

We solve this problem coupling a finite difference scheme with a fixed-point algorithm, using the solution of the no switching case as guess solution at each time step  $t_l = T - l\delta, l = 1, \dots, M$ , with  $\delta = T/M$ . We also assume that  $\lim_{t \rightarrow T} V^i(x, t) = \frac{x}{x_{\max}}$  for  $i = 0, 1$ , to avoid the numerical problems related to the discontinuity of the terminal function.

In Figure 5 we plot the optimal investment strategy in a full information environment in  $t = 0$  for different values of  $\lambda$  together with the optimal investment strategy obtained in case of no regime switching ( $\lambda = 0$ ). First of all, we notice that the agent takes less risk

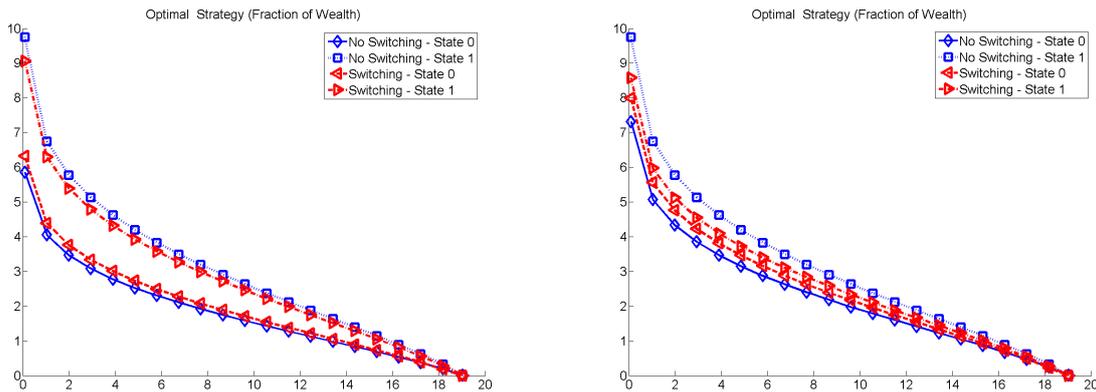


Figure 6: No Switching ( $\lambda = 0$ ) and Switching with Full Information ( $\lambda = 0.3$ ):  $t = 0$ ,  $b = 20$ ,  $T = 1$ ,  $r = 0.05$ ,  $\mu_0 = 0.08$ ,  $\mu_1 = 0.06$ ,  $\sigma_0 = 0.5$ ,  $sr_0 = 0.06$  and  $\sigma_1 = 0.3$ ,  $sr_1 = 0.0333$  (left) -  $\sigma_0 = 0.4$ ,  $sr_0 = 0.075$  and  $\sigma_1 = 0.3$ ,  $sr_1 = 0.0333$  (right)

(absolute value of the portfolio weight invested in the risky asset) as the wealth increases. As a general result, we have that the agent takes more risk in state 1 for all level of wealth (the most favorable one) and less risk in state 0 (the less favorable one) with respect to the no switching setting. Increasing  $\lambda$ , the departure is amplified.

The interpretation of these results is similar to the one provided in case of a fixed reward as the target is reached. Reaching a goal over a finite horizon induces the agent to take excess (less) risk in the good (bad) state. The manager exploits the fact of that the state is good and takes more risk fearing a switch to the bad state. On the other hand, in the bad state, the manager expects a switch to the good state and therefore he takes less risk. This interpretation is confirmed by the fact that this phenomenon is magnified by an increase of the switching probability  $\lambda$ . Differently from what has been observed in case of a reward for reaching a given target over an infinite horizon, a reward over a finite horizon induces excess (less) risk in the good (bad) state for all levels of wealth.

The analysis is confirmed considering the more general case where both the drift and the volatility of the risky asset switch in the two states. In Figure 6 we consider the case where the Sharpe ratio (volatility) in state 0 is higher (lower) than the one in state 1. From this figure it is evident that the agent performs a riskier strategy in the state characterized by the higher Sharpe ratio with respect to the strategy without switching,

while the reverse holds true in case of the state with the lower Sharpe ratio.

## 4.2 Regime Switching with partial information

In this case the HJB equation becomes

$$\begin{aligned} & \sup_w V_t + (w(\hat{\mu}(\pi) - r) + r)xV_x + \lambda(1 - 2\pi)V_\pi \\ & + \frac{\sigma^2 w^2}{2} x^2 V_{xx} + \frac{\pi^2(1 - \pi)^2 (\mu_1 - \mu_0)^2}{2\sigma^2} V_{\pi\pi} + w\pi(1 - \pi)(\mu_1 - \mu_0)xV_{x\pi} = 0, \end{aligned} \quad (13)$$

for  $0 \leq t < T$ ,  $0 \leq x \leq x_{\max}$  and  $0 \leq \pi \leq 1$ , with boundary condition  $V(t, 0, \pi) = 0$  and  $V(t, x_{\max}, \pi) = 1$  for  $0 \leq \pi \leq 1$ .

Assuming that the HJB equation (13) admits a classical solution, i.e.,  $V_{xx} < 0$ , differentiating with respect to  $w$ , we obtain

$$w^* = \frac{-(\hat{\mu}(\pi) - r)V_x - \pi(1 - \pi)(\mu_1 - \mu_0)V_{x\pi}}{\sigma^2 x V_{xx}},$$

and thus (13) becomes

$$\begin{aligned} & V_t - \frac{((\hat{\mu}(\pi) - r)V_x + \pi(1 - \pi)(\mu_1 - \mu_0)V_{x\pi})^2}{2\sigma^2 V_{xx}} + rxV_x + \lambda(1 - 2\pi)V_\pi \\ & + \frac{\pi^2(1 - \pi)^2 (\mu_1 - \mu_0)^2}{2\sigma^2} V_{\pi\pi} = 0. \end{aligned} \quad (14)$$

Again, we use a fixed-point algorithm coupled with a finite difference scheme to solve the above problem.

In Figure 7 we plot the optimal investment strategy in  $t = 0$  for  $\lambda = 0.1$  and different initial beliefs  $\pi$ . First of all we notice that the agent takes less risk (absolute value of the portfolio weight invested in the risky asset) as the wealth increases. The optimal strategies for  $\pi = 1$ ,  $\pi = 0.75$  and  $\pi = 0.5$  are very close one another with a positive investment in the risky asset. Considering the case  $\pi = 0.25$  (a small probability of being in the favorable state 1), the agent invests a positive fraction of wealth in the risky asset when the wealth is low. This behavior contrasts with the likelihood that he assigns to a favorable regime and therefore the agent assumes a very risky position with the hope that his beliefs are incorrect. On the opposite, when the wealth is high enough, the agent invests a small negative amount of wealth in the risky asset.

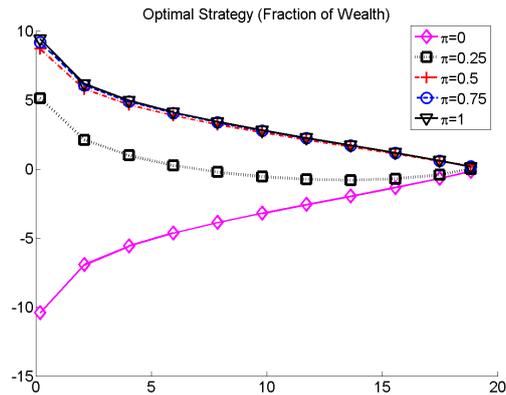


Figure 7: Switching with Partial Information:  $t = 0$ ,  $b = 20$ ,  $T = 1$ ,  $\mu_0 = 0.04$ ,  $\mu_1 = 0.08$ ,  $r = 0.05$ ,  $\sigma = 0.3$  and  $\lambda = 0.1$

To conclude, in Figure 8 we compare the no switching setting, the full information case and the partial information one. Considering  $\pi = 0$  or  $\pi = 1$ , we observe that the optimal investment strategies in the partial information setting are riskier than those obtained in the full information and in the constant parameter setting both in the bad and in the good state. Note that also the strategy for  $\pi = 0.5$  is riskier than the one obtained with constant parameter in the good state.

We can conclude that confidence risk affects the strategy of the agent inducing him to take excess risk overinvesting in the favorable state and underinvesting in the bad state. The phenomenon is observed for all level of wealth. Moreover, the agent may decide to invest a positive amount of wealth in the risky asset also when the likelihood that he assigns to a favorable state is low.

Summing up, reaching a target over a fixed horizon induces a risky strategy in a regime switching environment with full information (in the good state) and partial information reinforces the excessive risk taking attitude.

## 5 Relative performance bonus

The analysis can be extended to a remuneration scheme based on beating a benchmark over an infinite or a finite horizon. The main results on regime switching and risk seeking

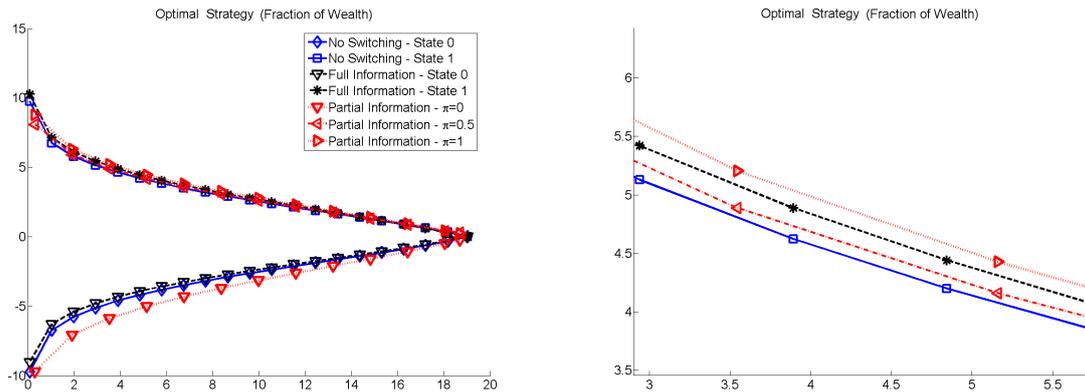


Figure 8: No Switching, Switching with Full and Partial Information:  $t = 0$ ,  $b = 20$ ,  $T = 1$ ,  $\mu_0 = 0.04$ ,  $\mu_1 = 0.08$ ,  $r = 0.05$ ,  $\sigma = 0.3$  and  $\lambda = 0.1$ . Right: zoom of the state 1 case

are confirmed. Let us assume that the benchmark is driven by a logarithmic Brownian motion with a drift switching at the same time the asset price does. The Brownian motion of the benchmark is correlated with the one of the asset price. note that we are in an incomplete market setting, i.e., the manager cannot use the stock to replicate the benchmark.

The benchmark dynamics is provided by

$$dP(t) = P(t)\alpha(Y(t))dt + P(t)\beta dZ(t) + P(t)\gamma d\widehat{Z}(t) \quad P(0) = P_0.$$

with zero correlation between the Brownian motions  $Z$  and  $\widehat{Z}$ . We follow [Browne, 1999b] assuming that the asset manager receives a fixed bonus (normalized to one) the first time the assets under management outperform the benchmark by a multiplicative constant  $\eta$ .

The analysis can be developed considering the setting of Section 3: we look for the optimal investment strategy to beat the benchmark by a fraction  $\eta$  before bankruptcy that occurs for  $X(t) = 0$ . Let

$$\tau_\eta = \inf\{t > 0 : X(t) \geq (1 + \eta)P(t)\} = \inf\{t > 0 : R(t) \geq 1 + \eta\},$$

and  $R := \frac{X}{P}$  the process of the assets under management normalized by the benchmark,

then the asset allocation problem can be formulated as follows

$$\max_w E [e^{-\delta\tau_\eta} | R(0) = X_0/P_0].$$

Assuming no switch and a drift for the risky asset  $S$  equal to  $\mu_0$  and for the benchmark  $P$  equal to  $\alpha_0$ , the problem has been solved in [Browne, 1999b]. In this setting, the evolution of the process  $R(t)$  becomes

$$dR = R (r - \alpha_0 + \beta^2 + \gamma^2 + w(t) (\mu_0 - r - \sigma\beta)) dt + R(w\sigma - \beta)dZ(t) - R\gamma d\widehat{Z}(t).$$

The optimal investment strategy is a constant weight

$$w = -\frac{\mu_0 - r - \sigma\beta}{C\sigma^2} + \frac{\beta}{\sigma},$$

where  $C$  is the unique root which belongs to the interval  $(-1, 0)$  of the cubic equation

$$\frac{\gamma^2}{2}C^3 + \left(\widehat{A} + \frac{\gamma^2}{2}\right)C^2 + \left(-\delta + \widehat{A} - \widehat{B}\right)C - \widehat{B} = 0$$

where

$$\widehat{A} = r - \alpha_0 + \gamma^2 + \frac{\mu_0 - r}{\sigma}\beta, \quad \widehat{B} = \frac{1}{2} \left( \frac{\mu_0 - r - \sigma\beta}{\sigma} \right)^2$$

see [Browne, 1999b, Section 6] for details.

Assuming a regime switching model with full information, the HJB equation becomes

$$\begin{aligned} \sup_{w^0} -(\delta + \lambda)V^0 + (r - \alpha_0 + \beta^2 + \gamma^2 + w^0(\mu_0 - r - \sigma\beta))xV_x^0 + \frac{(w^0\sigma - \beta)^2 + \gamma^2}{2}x^2V_{xx}^0 + \lambda V^1 &= 0, \\ \sup_{w^1} -(\delta + \lambda)V^1 + (r - \alpha_1 + \beta^2 + \gamma^2 + w^1(\mu_1 - r - \sigma\beta))xV_x^1 + \frac{(w^1\sigma - \beta)^2 + \gamma^2}{2}x^2V_{xx}^1 + \lambda V^0 &= 0. \end{aligned}$$

These equations can be solved concurrently with our fixed-point iterative method. In Figure 9 we show the optimal strategies considering different values of  $\lambda$ . If we consider Figure 9, it can be observed that the optimal investment strategy is similar to the one obtained in Section 3: it is not a constant weight, when the wealth is low the agent invests more (less) in the risky asset in state 1 (0) with respect to the case without regime switching. This attitude is reversed when the reward target is approaching.

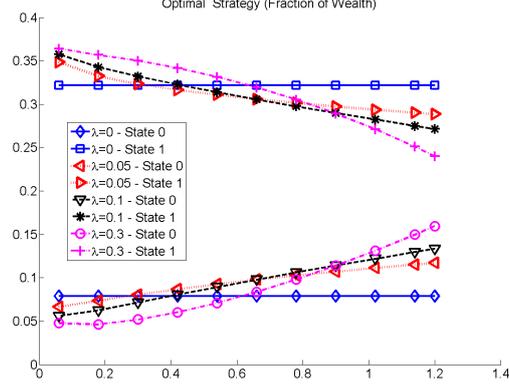


Figure 9: Switching with Full Information:  $\eta = 0.2$ ;  $\delta = 0.04$ ;  $r = 0.03$ ;  $\mu_0 = 0.06$ ;  $\mu_1 = 0.08$ ;  $\sigma = 0.3$ ;  $\alpha_0 = 0.045$ ;  $\alpha_1 = 0.085$ ;  $\beta = 0.3$ ;  $\gamma = 0.4$

Under partial information, the analysis becomes more complex. As a matter of fact, the agent observes both  $P(t)$  and  $S(t)$  and uses this information to form his beliefs  $\pi(t)$  on the probability of being in state 1.  $\pi(t)$  satisfies the stochastic differential equation

$$d\pi(t) = \lambda(1 - 2\pi(t))dt + \pi(t)(1 - \pi(t)) \left( \frac{\mu_1 - \mu_0}{\sigma} + \frac{\beta}{\gamma^2 + \beta^2}(\alpha_1 - \alpha_0) \right) d\bar{Z}(t) + \pi(t)(1 - \pi(t)) \frac{\gamma}{\gamma^2 + \beta^2}(\alpha_1 - \alpha_0) d\bar{\bar{Z}}(t),$$

where  $\bar{Z}(t)$  and  $\bar{\bar{Z}}(t)$  are independent standard Brownian motions. The HJB equation becomes

$$\begin{aligned} & \sup_w -\delta V + (r - \hat{\alpha}(\pi) + \beta^2 + \gamma^2 + w(\hat{\mu}(\pi) - r - \sigma\beta))xV_x + \lambda(1 - 2\pi)V_\pi \\ & + \frac{1}{2}\pi^2(1 - \pi)^2 \left( \left( \frac{\mu_1 - \mu_0}{\sigma} + \frac{\beta}{\beta^2 + \gamma^2}(\alpha_1 - \alpha_0) \right)^2 + \left( \frac{\gamma}{\beta^2 + \gamma^2}(\alpha_1 - \alpha_0) \right)^2 \right) V_{\pi\pi} \\ & + \pi(1 - \pi) \left( \left( \frac{\mu_1 - \mu_0}{\sigma} + \frac{\beta}{\beta^2 + \gamma^2}(\alpha_1 - \alpha_0) \right) (w\sigma - \beta) - \frac{\gamma^2}{\beta^2 + \gamma^2}(\alpha_1 - \alpha_0) \right) xV_{x\pi} \\ & + \frac{1}{2}((\sigma w - \beta)^2 + \gamma^2) x^2 V_{xx} \end{aligned}$$

with boundary conditions

$$V(0, \pi) = 0, \quad V(1 + \eta, \pi) = 1,$$

for  $0 \leq \pi \leq 1$ .

This equation can be solved with the proposed finite difference iterative method obtaining results similar to those shown in Section 3, i.e., confidence risk induces excess risk seeking with respect to the full information investment strategy. Instead, considering the case of a fixed bonus if the manager beats the benchmark by a fraction  $\eta$  by a terminal date  $T$ , we obtain results similar to those obtained in Section 4.

## 6 High water marks remuneration scheme

[Panageas and Westerfield, 2009] address the manager's optimal investment problem when he is remunerated by a high-water mark contract: the manager receives a fraction of the increase in fund value in excess of the last recorded maximum, the so-called high-water mark, if such an increase took place. Mathematically, assuming that the fund manager can invest in a risk-free and in a risky asset, and that  $k$  is the fraction of the maximum increase that the manager receives as compensation, the asset management problem becomes

$$\max_w E \left[ \int_0^{\min\{\tau, +\infty\}} e^{-(\delta+\eta)t} k dH(t) \right],$$

where  $\tau$  is the (random) bankruptcy time,  $\eta$  is the constant intensity of the Poisson process that models the termination time of the fund, and  $H(t)$  is the running maximum of the wealth  $X(t)$ , which evolves as follows

$$dX(t) = X(t)(w(t)(\mu(Y(t)) - r) + r)dt + w(t)\sigma X(t)dZ(t) - kdH_t, \quad X(0) = X_0.$$

In [Panageas and Westerfield, 2009], the HJB equation is given by

$$\sup_w -(\delta + \eta)V + (w(\mu - r) + r)xV' + \frac{\sigma^2 w^2}{2} x^2 V'' = 0,$$

where we denote with  $'$  the derivative with respect to  $x$  (here  $V = V(x, h)$  for any  $0 \leq x \leq h$  and  $h \geq 0$ ). The HJB equation is coupled with boundary conditions  $V(0, h) = 0$  for any  $h \geq 0$ , i.e., for any value of the maximum process, due to the bankruptcy condition, and  $V'(h, h) = 1 + \frac{1}{k} \frac{\partial V}{\partial h}(h, h)$ . [Panageas and Westerfield, 2009] provide a closed form solution for this problem. We can provide a formulation of the problem in the full and

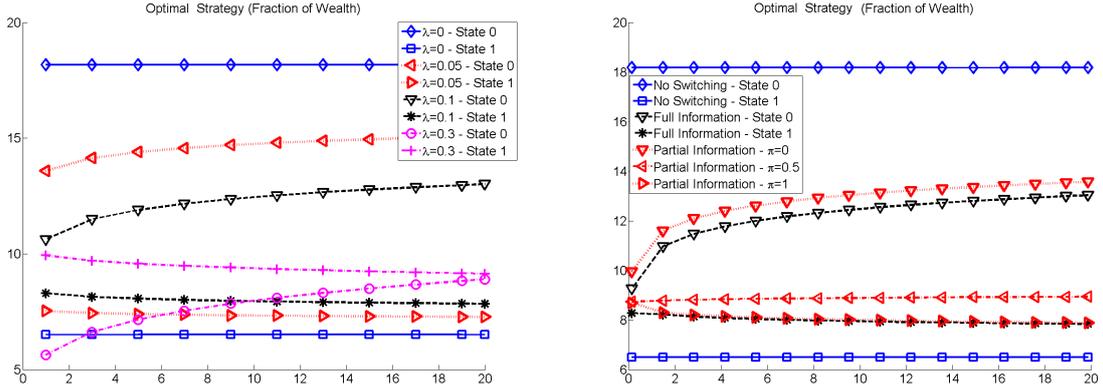


Figure 10: No Switching and Switching with Full Information and different values of  $\lambda$  (left) and Switching with both Full and Partial Information ( $\lambda = 0.1$ ) (right):  $\delta = 0.04$ ;  $\eta = 0.1$ ;  $r = 0.05$ ;  $\mu_0 = 0.06$ ;  $\mu_1 = 0.08$ ;  $\sigma = 0.3$ ;  $k = 0.2$ ;  $h = 20$ .

partial information case proceeding as above, solving it numerically. In Figure 10 we compare the no switching case with the full information one. As expected, in the good state (1), the agent invests more in the risky asset as  $\lambda$  increases, while the reverse happens in the bad state (0).

The result is similar to what we have observed in case of a bonus. It seems that a reward related to the maximum dynamics induces the agent to excess (less) risk taking in the good (bad) state for any level of wealth. In this case, in the good state the manager exploits the momentum and takes more risk fearing a switch to the bad state or the end of the fund. On the other hand, in case of the bad state, the manager expects a switch to the good state and therefore he takes less risk.

## 7 Conclusions

There are some anecdotes on how a non convex remuneration may affect management decisions in a non constant environment. The claim is that a manager remunerated through a bonus when a target is reached will take risk in excess in a bull market.

In this paper we have demonstrated this claim showing that in a two state regime switching environment the manager's risk exposure is high in a good state and is low

in a bad state. In the good state the manager exploits the momentum and takes more risk fearing a switch to the bad state. On the other hand, in case of the bad state, the manager expects a switch to the good state and therefore he takes less risk. The effect is for all level of wealth in case of a bonus over a finite horizon or a high water markets remuneration, when the horizon is infinite this effect is observed only when the target is far away. Contrary to what is observed in case of the maximization of the expected utility, confidence risk induces the agent to takes more risk in a partial information environment.

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