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**CANDIDATE LOCAL MINIMIZER OF
BLAKE & ZISSERMAN FUNCTIONAL**

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Abstract - Almansi decomposition and explicit coefficients of asymptotic expansion around the origin for bi-harmonic functions in a disk with a crack are evaluated by symbolic computations with Mathematica 5.0 .

We deduce S.I.F. and modes coefficients of the leading term in the expansion for candidate local minimizer of Blake & Zisserman functional.

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ClearAll

ClearAll

(INSTRUCTIONS:*

*ClearAll is not enough to perform evaluations:
closing Kernel and Mathematica
program is necessary
to restart evaluation *)*

■ (10.1) *Differential Operators in polar coordinates*

$dx[u_] := \text{Cos}[t] * D[u, r] - \text{Sin}[t] * D[u, t] / r$

$dy[u_] := \text{Sin}[t] * D[u, r] + \text{Cos}[t] * D[u, t] / r$

$dxx[u_] := dx[dx[u]]$

$dyy[u_] := dy[dy[u]]$

$dxy[u_] := dx[dy[u]]$

$dyx[u_] := dy[dx[u]]$

$lap[u_] := dxx[u] + dyy[u]$

$hes2[u_] := dxx[u]^2 + dyy[u]^2 + 2 * dxy[u]^2$

■ (10.2) Candidate Modes

Notebook parameters $(a, b, \alpha, \beta, \gamma, \delta, \lambda)$ vs file.TEX parameters $(A, B, c1, c2, c3, c4)$

$$a = 1/3 A, b = 1/3 B,$$

$$a \mathcal{R} = c4, a \mathcal{G} = c2, b \mathcal{B} = c3, b \mathcal{D} = c1,$$

(* u is a generic mute variable, used in several different contexts
parameter A in the paper is 3 times parameter a in the Notebook
parameter B in the paper is 3 times parameter b in the Notebook *)

$$us := r^{3/2} * \text{Sin}[t/2]$$

$$vs := r^{3/2} * \text{Sin}[3*t/2]$$

$$uc := r^{3/2} * \text{Cos}[t/2]$$

$$vc := r^{3/2} * \text{Cos}[3*t/2]$$

$$u := a * (\mathcal{R} * us + \mathcal{G} * vs) + b * (\mathcal{B} * uc + \mathcal{D} * vc)$$

(* Mode 1 ω *)

$$\omega := (\text{Sin}[t/2] - 5/3 * \text{Sin}[3*t/2])$$

(* Mode 2 w *)

$$w := (\text{Cos}[t/2] - 7/3 * \text{Cos}[3*t/2])$$

u

$$b \left(r^{3/2} \mathcal{B} \text{Cos}\left[\frac{t}{2}\right] + r^{3/2} \mathcal{D} \text{Cos}\left[\frac{3t}{2}\right] \right) + a \left(r^{3/2} \mathcal{R} \text{Sin}\left[\frac{t}{2}\right] + r^{3/2} \mathcal{G} \text{Sin}\left[\frac{3t}{2}\right] \right)$$

■ (10.3) Euler conditions: formula (4.3) of Theorem 4.3 at $\theta = \pi$

$dy[dy[u]] /. t \rightarrow \pi$

$$-\frac{-a \left(\frac{3\sqrt{r}\mathcal{R}}{2} - \frac{3\sqrt{r}\mathcal{G}}{2} \right) - \frac{a \left(-\frac{1}{4}r^{3/2}\mathcal{R} + \frac{9}{4}r^{3/2}\mathcal{G} \right)}{r}}{r}$$

Simplify [%]

$$\frac{a(5\mathcal{R} + 3\mathcal{G})}{4\sqrt{r}}$$

■ (10.4) Euler condions: formula (4.3) of Theorem 4.3 at $\theta = -\pi$

$dy[dy[u]] /. t \rightarrow -\pi$

$$-\frac{-a \left(-\frac{3\sqrt{r}\mathcal{R}}{2} + \frac{3\sqrt{r}\mathcal{G}}{2} \right) - \frac{a \left(\frac{1}{4}r^{3/2}\mathcal{R} - \frac{9}{4}r^{3/2}\mathcal{G} \right)}{r}}{r}$$

Simplify [%]

$$-\frac{a(5A + 3G)}{4\sqrt{r}}$$

- (10.5) Euler conditions: formula (4.4) of Theorem 4.3 at $\theta = \pi$

dy[dy[dy[u]]] + 2 * dx[dx[dy[u]]] /. t -> pi

$$2 \left(-\frac{b \left(-\frac{3B}{8\sqrt{r}} + \frac{9D}{8\sqrt{r}} \right)}{r} + \frac{2b \left(-\frac{3\sqrt{r}B}{4} + \frac{9\sqrt{r}D}{4} \right)}{r^2} - \frac{2b \left(-\frac{1}{2} r^{3/2} B + \frac{3}{2} r^{3/2} D \right)}{r^3} \right) -$$

$$\frac{b \left(-\frac{3\sqrt{r}B}{4} + \frac{9\sqrt{r}D}{4} \right)}{r} - \frac{b \left(-\frac{1}{2} r^{3/2} B + \frac{3}{2} r^{3/2} D \right)}{r^2} - \frac{-2b \left(-\frac{3\sqrt{r}B}{4} + \frac{9\sqrt{r}D}{4} \right)}{r} - \frac{b \left(\frac{1}{8} r^{3/2} B - \frac{27}{8} r^{3/2} D \right)}{r} + \frac{b \left(-\frac{1}{2} r^{3/2} B + \frac{3}{2} r^{3/2} D \right)}{r}$$

Simplify [%]

$$\frac{b(7B + 3D)}{8r^{3/2}}$$

- (10.6) Euler conditions: formula (4.4) of Theorem 4.3 at $\theta = -\pi$

dy[dy[dy[u]]] + 2 * dx[dx[dy[u]]] /. t -> -pi

$$2 \left(-\frac{b \left(\frac{3B}{8\sqrt{r}} - \frac{9D}{8\sqrt{r}} \right)}{r} + \frac{2b \left(\frac{3\sqrt{r}B}{4} - \frac{9\sqrt{r}D}{4} \right)}{r^2} - \frac{2b \left(\frac{1}{2} r^{3/2} B - \frac{3}{2} r^{3/2} D \right)}{r^3} \right) -$$

$$\frac{b \left(\frac{3\sqrt{r}B}{4} - \frac{9\sqrt{r}D}{4} \right)}{r} - \frac{b \left(\frac{1}{2} r^{3/2} B - \frac{3}{2} r^{3/2} D \right)}{r^2} - \frac{-2b \left(\frac{3\sqrt{r}B}{4} - \frac{9\sqrt{r}D}{4} \right)}{r} + \frac{b \left(\frac{1}{2} r^{3/2} B - \frac{3}{2} r^{3/2} D \right)}{r} - \frac{b \left(-\frac{1}{8} r^{3/2} B + \frac{27}{8} r^{3/2} D \right)}{r}$$

Simplify [%]

$$-\frac{b(7B + 3D)}{8r^{3/2}}$$

- (10.7) Choice of parameters A, B, G, D to be compiled in order to Plot Graphics

$$A = 3; B = 3; G = -5; D = -7;$$

u

$$b \left(3 r^{3/2} \cos\left[\frac{t}{2}\right] - 7 r^{3/2} \cos\left[\frac{3t}{2}\right] \right) + a \left(3 r^{3/2} \sin\left[\frac{t}{2}\right] - 5 r^{3/2} \sin\left[\frac{3t}{2}\right] \right)$$

*Evaluation of candidate (normalized and weighted by $\text{Cos}(\theta)$) energy in a disk (in order to force equipartition of energy)
and
Evaluation of extremality conditions at crack-tip (in order to fulfil conditions of Theorem 5.5)*

- (10.8) Hessian of u (normalization of candidate W) $\text{hes2}[u] = |\nabla^2 u|^2$

Simplify[$r * \text{hes2}[u]$]

$$-\frac{9}{4} (-17 a^2 - 29 b^2 + 4 (a^2 - b^2) \text{Cos}[t] + (5 a^2 - 7 b^2) \text{Cos}[2 t] - 8 a b \text{Sin}[t] - 12 a b \text{Sin}[2 t])$$

- (10.9) Squared hessian ($|\nabla^2 u|^2$) of u integrated over the disk B_R (here we compute 9 times second equation of (7.36) in Section 7)

C1 = Simplify[**Integrate**[% , { r , 0, R }, { t , $-\text{Pi}$, Pi }]]

$$\frac{9}{2} (17 a^2 + 29 b^2) \pi R$$

- (10.10) $k_1 + k_2$ of u (here we compute 9 times equation of (7.32) in Section 7)

Integrate[$R *$

Simplify[$\text{hes2}[u] * \text{Cos}[t] + 2 * \text{dx}[u] * (D[\text{dxx}[u] * \text{Cos}[t] * \text{dx}[t] + \text{dyy}[u] * \text{Sin}[t] * \text{dy}[t] + \text{dxy}[u] * (\text{Cos}[t] * \text{dy}[t] + \text{Sin}[t] * \text{dx}[t]), t] + D[\text{lap}[u], r])]$, { t , $-\text{Pi}$, Pi }] /. $r \rightarrow R$

$$\frac{315 a^2 \pi}{4} + \frac{333 b^2 \pi}{4}$$

C2 = Simplify[%]

$$\frac{9}{4} (35 a^2 + 37 b^2) \pi$$

■ (10.11) Solution of algebraic system (equipartition of energy and $k_1+k_2=\alpha$)

$$\mathbf{sol} = \mathbf{Solve}[\{\mathbf{C1} == \alpha * \mathbf{R}, \mathbf{C2} == \alpha\}, \{\mathbf{a}, \mathbf{b}\}]$$

$$\left\{ \left\{ \mathbf{a} \rightarrow -\sqrt{\frac{7}{579\pi}} \sqrt{\alpha}, \mathbf{b} \rightarrow -\frac{\sqrt{\alpha}}{3\sqrt{193\pi}} \right\}, \left\{ \mathbf{a} \rightarrow -\sqrt{\frac{7}{579\pi}} \sqrt{\alpha}, \mathbf{b} \rightarrow \frac{\sqrt{\alpha}}{3\sqrt{193\pi}} \right\}, \right. \\ \left. \left\{ \mathbf{a} \rightarrow \sqrt{\frac{7}{579\pi}} \sqrt{\alpha}, \mathbf{b} \rightarrow -\frac{\sqrt{\alpha}}{3\sqrt{193\pi}} \right\}, \left\{ \mathbf{a} \rightarrow \sqrt{\frac{7}{579\pi}} \sqrt{\alpha}, \mathbf{b} \rightarrow \frac{\sqrt{\alpha}}{3\sqrt{193\pi}} \right\} \right\}$$

Computations related to higher order modes $uj_h, h > 0$

■ Definition of an integer variable h

$$h \in \text{Integers}$$

$$h \in \text{Integers}$$

■ (10.12) Euler conditions for uj_h (Theorem 4.3 formula (4.3)) at $\theta = \pi$

$$\begin{aligned} \mathbf{us} &= \mathbf{r}^{\mathbf{h} + 3/2} * \mathbf{Sin}[(\mathbf{h} - 1/2) * \mathbf{t}] \\ \mathbf{vs} &= \mathbf{r}^{\mathbf{h} + 3/2} * \mathbf{Sin}[(\mathbf{h} + 3/2) * \mathbf{t}] \\ \mathbf{uc} &= \mathbf{r}^{\mathbf{h} + 3/2} * \mathbf{Cos}[(\mathbf{h} - 1/2) * \mathbf{t}] \\ \mathbf{vc} &= \mathbf{r}^{\mathbf{h} + 3/2} * \mathbf{Cos}[(\mathbf{h} + 3/2) * \mathbf{t}] \\ \mathbf{u} &= \mathbf{a} * (\mathbf{fl} * \mathbf{us} + \mathbf{G} * \mathbf{vs}) + \mathbf{b} * (\mathbf{B} * \mathbf{uc} + \mathbf{D} * \mathbf{vc}) \end{aligned}$$

$$r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right) t\right]$$

$$r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]$$

$$r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right) t\right]$$

$$r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]$$

$$\begin{aligned} & b \left(3 r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right) t\right] - 7 r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) + \\ & a \left(3 r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right) t\right] - 5 r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right) \end{aligned}$$

dy[dy[u]] /. t -> pi

$$-\frac{1}{r} \left(-b \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 7 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) - \right. \\ \left. a \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] - 5 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) - \frac{1}{r} \right. \\ \left(b \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + \right. \\ \left. a \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] - 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) \right)$$

Simplify[%]

$$-\frac{1}{2} (1 + 2 h) r^{-\frac{1}{2}+h} (a (15 + 2 h) \cos [h \pi] - 2 b (9 + 2 h) \sin [h \pi])$$

■ (10.13) Euler conditions for uj_h (Theorem 4.3 formula (4.3)) at $\theta = -\pi$

dy[dy[u]] /. t -> -pi

$$-\frac{1}{r} \left(-b \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 7 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) - \right. \\ \left. a \left(-3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] + 5 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) - \frac{1}{r} \right. \\ \left(b \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + \right. \\ \left. a \left(-3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] + \right. \\ \left. \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) \right)$$

Simplify[%]

$$\frac{1}{2} (1 + 2 h) r^{-\frac{1}{2}+h} (a (15 + 2 h) \cos [h \pi] + 2 b (9 + 2 h) \sin [h \pi])$$

■ (10.14) Euler conditions for uj_h (Theorem 4.3 formula (4.4)) at $\theta = \pi$

dy[dy[dy[u]]] + 2 * dx[dx[dy[u]]] / . t → π

$$\begin{aligned}
& 2 \left(-\frac{1}{r} \left(a \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - \right. \right. \right. \\
& \quad \left. \left. \left. 5 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right)^2 r^{-\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + b \left(3 \left(\frac{1}{2} - h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \right. \right. \\
& \quad \left. \left. \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) \right) + \frac{1}{r^2} \\
& \left(2 \left(a \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 5 \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + \right. \\
& \quad \left. b \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] - \right. \right. \\
& \quad \left. \left. 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) \right) - \frac{1}{r^3} \\
& \left(2 \left(a \left(3 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 5 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + \right. \\
& \quad \left. b \left(3 \left(\frac{1}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] - 7 \left(-\frac{3}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) \right) - \\
& \frac{1}{r} \left(\frac{1}{r} \left(a \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 5 \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + \right. \\
& \quad \left. b \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) \right) - \\
& \frac{1}{r^2} \left(a \left(3 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 5 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + \right. \\
& \quad \left. b \left(3 \left(\frac{1}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] - 7 \left(-\frac{3}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) \right) - \\
& \frac{1}{r} \left(-2 \left(a \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 5 \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + \right. \\
& \quad \left. b \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] - \right. \right. \\
& \quad \left. \left. 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) \right) + \frac{1}{r} \\
& \left(a \left(3 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 5 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + \right. \\
& \quad \left. b \left(3 \left(\frac{1}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] - 7 \left(-\frac{3}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) \right) - \frac{1}{r} \\
& \left(a \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{3}{2}+h} \right. \right. \\
& \quad \left. \left. \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + b \left(3 \left(\frac{1}{2} - h \right)^2 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] - \right. \right. \\
& \quad \left. \left. 7 \left(-\frac{3}{2} - h \right)^2 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) \right) \right)
\end{aligned}$$

Simplify[%]

$$\frac{1}{4} (-1 + 4 h^2) r^{-\frac{3}{2}+h} (4 b h \cos[h \pi] + a (-3 + 2 h) \sin[h \pi])$$

■ (10.15) Euler conditions for uj_h (Theorem 4.3 formula (4.4)) at $\theta = -\pi$

dy[dy[dy[u]]] + 2 * dx[dx[dy[u]]] /. t -> -pi

$$\begin{aligned}
& 2 \left(-\frac{1}{r} \left(a \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - \right. \right. \right. \\
& \quad \left. \left. \left. 5 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right)^2 r^{-\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + b \left(-3 \left(\frac{1}{2} - h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \right. \right. \\
& \quad \left. \left. \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] + 7 \left(-\frac{3}{2} - h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) \right) + \frac{1}{r^2} \\
& \left(2 \left(a \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 5 \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + \right. \\
& \quad \left. b \left(-3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] + \right. \right. \\
& \quad \left. \left. 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) \right) - \frac{1}{r^3} \\
& \left(2 \left(a \left(3 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 5 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + \right. \\
& \quad \left. b \left(-3 \left(\frac{1}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] + 7 \left(-\frac{3}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) \right) - \\
& \frac{1}{r} \left(\frac{1}{r} \left(a \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 5 \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + b \right. \\
& \quad \left. \left(-3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] + 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) \right) - \\
& \frac{1}{r^2} \left(a \left(3 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 5 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + \right. \\
& \quad \left. b \left(-3 \left(\frac{1}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] + 7 \left(-\frac{3}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) - \right. \\
& \frac{1}{r} \left(-2 \left(a \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 5 \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + \right. \\
& \quad \left. b \left(-3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] + \right. \right. \\
& \quad \left. \left. 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) \right) + \frac{1}{r} \\
& \left(a \left(3 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 5 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + \right. \\
& \quad \left. b \left(-3 \left(\frac{1}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] + 7 \left(-\frac{3}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) \right) - \\
& \frac{1}{r} \left(a \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) \pi \right] - 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{3}{2}+h} \right. \right. \\
& \quad \left. \left. \cos \left[\left(\frac{3}{2} + h \right) \pi \right] \right) + b \left(-3 \left(\frac{1}{2} - h \right)^2 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) \pi \right] + \right. \right. \\
& \quad \left. \left. 7 \left(-\frac{3}{2} - h \right)^2 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) \pi \right] \right) \right) \right)
\end{aligned}$$

Simplify[%]

$$-\frac{1}{4} (-1 + 4 h^2) r^{-\frac{3}{2}+h} (4 b h \cos[h \pi] + a (3 - 2 h) \sin[h \pi])$$

*Integration on the boundary of a small disk :
Theorem 5.5 - Crack Tip (with $\mu=0$, e.g. without g)*

u

$$b \left(3 r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) +$$

$$a \left(3 r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 5 r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right)$$

- (10.16) Evaluation of k_1 (NEL FILE TEX k_1 e k_2 sono definiti solo per $h=0$)

`Simplify[Integrate[R * Simplify[hes2[u] * Cos[t]],
{t, -Pi, Pi}]] /. r -> R`

$$-\frac{9 (a^2 - b^2) (-1 + 2 h) (1 + 2 h)^2 R^{2 h} \sin[2 h \pi]}{2 (-1 + h) h}$$

- (10.17) Evaluation of k_2 (NEL FILE TEX k_1 e k_2 sono definiti solo per $h=0$)

`Simplify[Integrate[`

`R * Simplify[2 * dx[u] * (D[dxx[u] * Cos[t] * dx[t] +
dyy[u] * Sin[t] * dy[t] +
dxy[u] * (Cos[t] * dy[t] + Sin[t] * dx[t])), t] +
D[lap[u], r)], {t, -Pi, Pi}]] /. r -> R`

$$\frac{1}{8 h (-1 + h^2)} \left((1 + 2 h) (a^2 (189 - 72 h - 486 h^2 - 36 h^3 + 200 h^4 + 16 h^5) - \right.$$

$$\left. b^2 (297 - 378 h - 900 h^2 + 216 h^3 + 512 h^4 + 64 h^5) \right) R^{2 h} \sin[2 h \pi]$$

■ (10.18) Evaluation of k_1+k_2

Integrate[R *

Simplify[hes2[u] * Cos[t] + 2 * dx[u] * (D[dx[x[u] * Cos[t] * dx[t] + dyy[u] * Sin[t] * dy[t] + dxy[u] * (Cos[t] * dy[t] + Sin[t] * dx[t]), t] + D[lap[u], r])], {t, -Pi, Pi}] /. r -> R

$$\begin{aligned}
& -\frac{153}{2} a^2 R^{2h} \sin[2h\pi] + 99 b^2 R^{2h} \sin[2h\pi] + \frac{9 a^2 R^{2h} \sin[2h\pi]}{16(-1+h)} - \frac{9 b^2 R^{2h} \sin[2h\pi]}{16(-1+h)} - \\
& \frac{225 a^2 R^{2h} \sin[2h\pi]}{8h} + \frac{333 b^2 R^{2h} \sin[2h\pi]}{8h} - 36 a^2 h R^{2h} \sin[2h\pi] + 27 b^2 h R^{2h} \sin[2h\pi] - \\
& \frac{9 a^2 h R^{2h} \sin[2h\pi]}{-1+h} + \frac{9 b^2 h R^{2h} \sin[2h\pi]}{-1+h} - 6 a^2 h^2 R^{2h} \sin[2h\pi] + 12 b^2 h^2 R^{2h} \sin[2h\pi] - \\
& \frac{81 a^2 h^2 R^{2h} \sin[2h\pi]}{2(-1+h)} + \frac{81 b^2 h^2 R^{2h} \sin[2h\pi]}{2(-1+h)} - 30 a^2 h^3 R^{2h} \sin[2h\pi] + \\
& 42 b^2 h^3 R^{2h} \sin[2h\pi] - \frac{36 a^2 h^3 R^{2h} \sin[2h\pi]}{-1+h} + \frac{36 b^2 h^3 R^{2h} \sin[2h\pi]}{-1+h} + \\
& \frac{9 a^2 h^4 R^{2h} \sin[2h\pi]}{-1+h} - \frac{9 b^2 h^4 R^{2h} \sin[2h\pi]}{-1+h} + \frac{405 a^2 R^{2h} \sin[2h\pi]}{16(1+h)} - \\
& \frac{945 b^2 R^{2h} \sin[2h\pi]}{16(1+h)} + \frac{225 a^2 h R^{2h} \sin[2h\pi]}{2(1+h)} - \frac{252 b^2 h R^{2h} \sin[2h\pi]}{1+h} + \\
& \frac{345 a^2 h^2 R^{2h} \sin[2h\pi]}{2(1+h)} - \frac{735 b^2 h^2 R^{2h} \sin[2h\pi]}{2(1+h)} + \frac{110 a^2 h^3 R^{2h} \sin[2h\pi]}{1+h} - \\
& \frac{224 b^2 h^3 R^{2h} \sin[2h\pi]}{1+h} + \frac{25 a^2 h^4 R^{2h} \sin[2h\pi]}{1+h} - \frac{49 b^2 h^4 R^{2h} \sin[2h\pi]}{1+h}
\end{aligned}$$

C2 = Simplify[%]

$$\frac{1}{8h(-1+h^2)} \left((1+2h) (a^2 (225 - 36h - 630h^2 - 180h^3 + 200h^4 + 16h^5) - b^2 (333 - 342h - 1044h^2 + 72h^3 + 512h^4 + 64h^5)) R^{2h} \sin[2h\pi] \right)$$

■ (10.19) Deduction of parameters \mathcal{A} , \mathcal{B} , \mathcal{G} , \mathcal{D} :

Solution of algebraic system (equipartition of energy and $k_1+k_2=\alpha$)

sol = Solve[{C1 == α * R, C2 == α }, {a, b}]

{a ->

$$\begin{aligned}
& -\frac{1}{\sqrt{225 + 414h - 702h^2 - 1440h^3 - 160h^4 + 416h^5 + 32h^6}} \left(R^{-h} \sqrt{\text{Csc}[2h\pi]} \sqrt{(-8h\alpha + 8h^3\alpha + \right. \\
& (37R^{2h} (225\alpha + 414h\alpha - 702h^2\alpha - 1440h^3\alpha - 160h^4\alpha + 416h^5\alpha + 32h^6 \\
& \alpha + 612h\pi R^{-2h} \alpha \text{Csc}[2h\pi] - 612h^3\pi R^{-2h} \alpha \text{Csc}[2h\pi]) \sin[2h\pi]) /} \\
& (6093\pi + 8757h\pi - 24867h^2\pi - 38016h^3\pi + 3256h^4\pi + 15280h^5\pi + 1552h^6\pi) +} \\
& (36hR^{2h} (225\alpha + 414h\alpha - 702h^2\alpha - 1440h^3\alpha - 160h^4\alpha + 416h^5\alpha + 32h^6 \\
& \alpha + 612h\pi R^{-2h} \alpha \text{Csc}[2h\pi] - 612h^3\pi R^{-2h} \alpha \text{Csc}[2h\pi]) \sin[2h\pi]) /} \\
& (6093\pi + 8757h\pi - 24867h^2\pi - 38016h^3\pi + 3256h^4\pi + 15280h^5\pi + 1552h^6\pi) -} \\
& (192h^2R^{2h} (225\alpha + 414h\alpha - 702h^2\alpha - 1440h^3\alpha - 160h^4\alpha + 416h^5\alpha + 32h^6 \\
& \alpha + 612h\pi R^{-2h} \alpha \text{Csc}[2h\pi] - 612h^3\pi R^{-2h} \alpha \text{Csc}[2h\pi]) \sin[2h\pi]) /} \\
& (6093\pi + 8757h\pi - 24867h^2\pi - 38016h^3\pi + 3256h^4\pi + 15280h^5\pi + 1552h^6\pi) -} \\
& \left. (224h^3R^{2h} (225\alpha + 414h\alpha - 702h^2\alpha - 1440h^3\alpha - 160h^4\alpha + 416h^5\alpha + 32h^6 \right.
\end{aligned}$$

$$\begin{aligned}
& (224 h^3 R^{2h} (225 \alpha + 414 h \alpha - 702 h^2 \alpha - 1440 h^3 \alpha - 160 h^4 \alpha + 416 h^5 \alpha + 32 h^6 \alpha + \\
& \quad 612 h \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi] - 612 h^3 \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi]) \operatorname{Sin}[2 h \pi]) / \\
& \quad (6093 \pi + 8757 h \pi - 24867 h^2 \pi - 38016 h^3 \pi + 3256 h^4 \pi + 15280 h^5 \pi + 1552 h^6 \pi) + \\
& (656 h^4 R^{2h} (225 \alpha + 414 h \alpha - 702 h^2 \alpha - 1440 h^3 \alpha - 160 h^4 \alpha + 416 h^5 \alpha + 32 h^6 \alpha + \\
& \quad 612 h \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi] - 612 h^3 \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi]) \operatorname{Sin}[2 h \pi]) / \\
& \quad (9 (6093 \pi + 8757 h \pi - 24867 h^2 \pi - 38016 h^3 \pi + 3256 h^4 \pi + 15280 h^5 \pi + 1552 h^6 \pi)) + \\
& (1088 h^5 R^{2h} (225 \alpha + 414 h \alpha - 702 h^2 \alpha - 1440 h^3 \alpha - 160 h^4 \alpha + 416 h^5 \alpha + \\
& \quad 32 h^6 \alpha + 612 h \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi] - 612 h^3 \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi]) \operatorname{Sin}[2 h \pi]) / \\
& \quad (9 (6093 \pi + 8757 h \pi - 24867 h^2 \pi - 38016 h^3 \pi + 3256 h^4 \pi + 15280 h^5 \pi + 1552 h^6 \pi)) + \\
& (128 h^6 R^{2h} (225 \alpha + 414 h \alpha - 702 h^2 \alpha - 1440 h^3 \alpha - 160 h^4 \alpha + 416 h^5 \alpha + 32 h^6 \alpha + \\
& \quad 612 h \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi] - 612 h^3 \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi]) \operatorname{Sin}[2 h \pi]) / \\
& \quad (9 (6093 \pi + 8757 h \pi - 24867 h^2 \pi - 38016 h^3 \pi + 3256 h^4 \pi + 15280 h^5 \pi + 1552 h^6 \pi))), \\
b \rightarrow & - \left(\sqrt{(225 \alpha + 414 h \alpha - 702 h^2 \alpha - 1440 h^3 \alpha - 160 h^4 \alpha + 416 h^5 \alpha + 32 h^6 \alpha + \right. \\
& \quad \left. 612 h \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi] - 612 h^3 \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi])} \right) / \\
& \left(3 \sqrt{6093 \pi + 8757 h \pi - 24867 h^2 \pi - 38016 h^3 \pi + 3256 h^4 \pi + 15280 h^5 \pi + 1552 h^6 \pi} \right) \}, \\
\{a \rightarrow & \frac{1}{\sqrt{225 + 414 h - 702 h^2 - 1440 h^3 - 160 h^4 + 416 h^5 + 32 h^6}} \\
& (R^{-h} \\
& \quad \sqrt{\operatorname{Csc}[2 h \pi]} \\
& \quad \sqrt{(-8 h \alpha + 8 h^3 \alpha + (37 R^{2h} (225 \alpha + 414 h \alpha - 702 h^2 \alpha - 1440 h^3 \alpha - 160 h^4 \alpha + 416 h^5 \alpha + \\
& \quad 32 h^6 \alpha + 612 h \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi] - 612 h^3 \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi]) \operatorname{Sin}[2 h \pi]) / \\
& \quad (6093 \pi + 8757 h \pi - 24867 h^2 \pi - 38016 h^3 \pi + 3256 h^4 \pi + 15280 h^5 \pi + 1552 h^6 \pi) + \\
& (36 h R^{2h} (225 \alpha + 414 h \alpha - 702 h^2 \alpha - 1440 h^3 \alpha - 160 h^4 \alpha + 416 h^5 \alpha + 32 h^6 \alpha + \\
& \quad 612 h \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi] - 612 h^3 \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi]) \operatorname{Sin}[2 h \pi]) / \\
& \quad (6093 \pi + 8757 h \pi - 24867 h^2 \pi - 38016 h^3 \pi + 3256 h^4 \pi + 15280 h^5 \pi + 1552 h^6 \pi) - \\
& (192 h^2 R^{2h} (225 \alpha + 414 h \alpha - 702 h^2 \alpha - 1440 h^3 \alpha - 160 h^4 \alpha + 416 h^5 \alpha + 32 h^6 \alpha + \\
& \quad 612 h \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi] - 612 h^3 \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi]) \operatorname{Sin}[2 h \pi]) / \\
& \quad (6093 \pi + 8757 h \pi - 24867 h^2 \pi - 38016 h^3 \pi + 3256 h^4 \pi + 15280 h^5 \pi + 1552 h^6 \pi) - \\
& (224 h^3 R^{2h} (225 \alpha + 414 h \alpha - 702 h^2 \alpha - 1440 h^3 \alpha - 160 h^4 \alpha + 416 h^5 \alpha + 32 h^6 \alpha + \\
& \quad 612 h \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi] - 612 h^3 \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi]) \operatorname{Sin}[2 h \pi]) / \\
& \quad (6093 \pi + 8757 h \pi - 24867 h^2 \pi - 38016 h^3 \pi + 3256 h^4 \pi + 15280 h^5 \pi + 1552 h^6 \pi) + \\
& (656 h^4 R^{2h} (225 \alpha + 414 h \alpha - 702 h^2 \alpha - 1440 h^3 \alpha - 160 h^4 \alpha + 416 h^5 \alpha + 32 h^6 \alpha + \\
& \quad 612 h \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi] - 612 h^3 \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi]) \operatorname{Sin}[2 h \pi]) / \\
& \quad (9 (6093 \pi + 8757 h \pi - 24867 h^2 \pi - 38016 h^3 \pi + 3256 h^4 \pi + 15280 h^5 \pi + 1552 h^6 \pi)) + \\
& (1088 h^5 R^{2h} (225 \alpha + 414 h \alpha - 702 h^2 \alpha - 1440 h^3 \alpha - 160 h^4 \alpha + 416 h^5 \alpha + \\
& \quad 32 h^6 \alpha + 612 h \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi] - 612 h^3 \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi]) \operatorname{Sin}[2 h \pi]) / \\
& \quad (9 (6093 \pi + 8757 h \pi - 24867 h^2 \pi - 38016 h^3 \pi + 3256 h^4 \pi + 15280 h^5 \pi + 1552 h^6 \pi)) + \\
& (128 h^6 R^{2h} (225 \alpha + 414 h \alpha - 702 h^2 \alpha - 1440 h^3 \alpha - 160 h^4 \alpha + 416 h^5 \alpha + 32 h^6 \alpha + \\
& \quad 612 h \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi] - 612 h^3 \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi]) \operatorname{Sin}[2 h \pi]) / \\
& \quad (9 (6093 \pi + 8757 h \pi - 24867 h^2 \pi - 38016 h^3 \pi + 3256 h^4 \pi + 15280 h^5 \pi + 1552 h^6 \pi))}), \\
b \rightarrow & \left(\sqrt{(225 \alpha + 414 h \alpha - 702 h^2 \alpha - 1440 h^3 \alpha - 160 h^4 \alpha + 416 h^5 \alpha + 32 h^6 \alpha + \right. \\
& \quad \left. 612 h \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi] - 612 h^3 \pi R^{-2h} \alpha \operatorname{Csc}[2 h \pi])} \right) / \\
& \left(3 \sqrt{6093 \pi + 8757 h \pi - 24867 h^2 \pi - 38016 h^3 \pi + 3256 h^4 \pi + 15280 h^5 \pi + 1552 h^6 \pi} \right) \} \}
\end{aligned}$$

■ **coefficients $a=a_h$ and $b=b_h$ for uj_h**

(the above system has four solutions: $(\pm a, \pm b)$ we chose the fourth one $(+a, +b)$ related to the candidate W)

$$A_h = 3 a_h, B_h = 3 b_h$$

Simplify[sol[[4]]]

$$\left\{ \begin{aligned} a \rightarrow & \frac{\left(R^{-h} \sqrt{\text{Csc}[2 h \pi]} \sqrt{\left((225 - 36 h - 630 h^2 - 180 h^3 + 200 h^4 + 16 h^5) \alpha (1044 h (-1 + h^2) \pi + (333 + 324 h - 1728 h^2 - 2016 h^3 + 656 h^4 + 1088 h^5 + 128 h^6) R^{2h} \text{Sin}[2 h \pi]) \right) / (6093 - 3429 h - 18009 h^2 - 1998 h^3 + 7252 h^4 + 776 h^5)} \right)}{\left(3 \sqrt{225 + 414 h - 702 h^2 - 1440 h^3 - 160 h^4 + 416 h^5 + 32 h^6} \sqrt{\pi} \right)}, \\ b \rightarrow & \frac{\left(\sqrt{\left(R^{-2h} \alpha \left((225 + 414 h - 702 h^2 - 1440 h^3 - 160 h^4 + 416 h^5 + 32 h^6) R^{2h} - 612 h (-1 + h^2) \pi \text{Csc}[2 h \pi] \right) \right)} \right)}{\left(3 \sqrt{6093 + 8757 h - 24867 h^2 - 38016 h^3 + 3256 h^4 + 15280 h^5 + 1552 h^6} \sqrt{\pi} \right)} \end{aligned} \right\}$$

■ coefficients a and b for u0_h

Evaluate[%] /. h -> 0

{a -> Indeterminate, b -> Indeterminate}

{a -> Indeterminate, b -> Indeterminate}

$$\left\{ a \rightarrow \sqrt{\frac{7}{579 \pi}} \sqrt{\alpha}, b \rightarrow \frac{\sqrt{\alpha}}{3 \sqrt{193 \pi}} \right\}$$

$$\left\{ a \rightarrow \sqrt{\frac{7}{579 \pi}} \sqrt{\alpha}, b \rightarrow \frac{\sqrt{\alpha}}{3 \sqrt{193 \pi}} \right\}$$

u = u /. %

$$\frac{\sqrt{\alpha} \left(3 r^{\frac{3}{2}+h} \text{Cos}\left[\left(-\frac{1}{2}+h\right)t\right] - 7 r^{\frac{3}{2}+h} \text{Cos}\left[\left(\frac{3}{2}+h\right)t\right] \right)}{3 \sqrt{193 \pi}} +$$

$$\sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 r^{\frac{3}{2}+h} \text{Sin}\left[\left(-\frac{1}{2}+h\right)t\right] - 5 r^{\frac{3}{2}+h} \text{Sin}\left[\left(\frac{3}{2}+h\right)t\right] \right)$$

Simplify[u]

$$\frac{r^{\frac{3}{2}+h} \sqrt{\alpha} \left(3 \text{Cos}\left[\left(-\frac{1}{2}+h\right)t\right] - 7 \text{Cos}\left[\left(\frac{3}{2}+h\right)t\right] + \sqrt{21} \left(3 \text{Sin}\left[\left(-\frac{1}{2}+h\right)t\right] - 5 \text{Sin}\left[\left(\frac{3}{2}+h\right)t\right] \right) \right)}{3 \sqrt{193 \pi}}$$

$$\frac{1}{3 \sqrt{193 \pi}} \left(r^{3/2} \sqrt{\lambda} \left(3 \text{Cos}\left[\frac{t}{2}\right] - 7 \text{Cos}\left[\frac{3t}{2}\right] + \sqrt{21} \left(3 \text{Sin}\left[\frac{t}{2}\right] - 5 \text{Sin}\left[\frac{3t}{2}\right] \right) \right) \right)$$

■ (10.20) Evaluation of laplacian of the candidates W and Φ (combination of modes at index h=0): Δ_x W, Δ_x Φ

lap[u]

$$\frac{2(1+2h)r^{-\frac{1}{2}+h}\sqrt{\alpha}\left(\cos\left(-\frac{1}{2}+h\right)t+\sqrt{21}\sin\left(-\frac{1}{2}+h\right)t\right)}{\sqrt{193\pi}}$$

$$\mathbf{W[r_, t_]} := r^{(3/2)} * (1 / (\text{Sqrt}[193 * \text{Pi}])) * \\ (\text{Sqrt}[21] * (\text{Sin}[t/2] - 5/3 * \text{Sin}[3 * t/2]) + \\ (\text{Cos}[t/2] - 7/3 * \text{Cos}[3 * t/2]))$$

lap[W[r, t]]

$$\text{Sin}[t] \left(\frac{3 \text{Sin}[t] \left(\text{Cos}\left[\frac{t}{2}\right] - \frac{7}{3} \text{Cos}\left[\frac{3t}{2}\right] + \sqrt{21} \left(\text{Sin}\left[\frac{t}{2}\right] - \frac{5}{3} \text{Sin}\left[\frac{3t}{2}\right] \right) \right)}{4 \sqrt{193\pi} \sqrt{r}} + \right. \\ \left. \frac{\text{Cos}[t] \left(\sqrt{21} \left(\frac{1}{2} \text{Cos}\left[\frac{t}{2}\right] - \frac{5}{2} \text{Cos}\left[\frac{3t}{2}\right] \right) - \frac{1}{2} \text{Sin}\left[\frac{t}{2}\right] + \frac{7}{2} \text{Sin}\left[\frac{3t}{2}\right] \right)}{2 \sqrt{193\pi} \sqrt{r}} \right) + \\ \text{Cos}[t] \left(\frac{3 \text{Cos}[t] \left(\text{Cos}\left[\frac{t}{2}\right] - \frac{7}{3} \text{Cos}\left[\frac{3t}{2}\right] + \sqrt{21} \left(\text{Sin}\left[\frac{t}{2}\right] - \frac{5}{3} \text{Sin}\left[\frac{3t}{2}\right] \right) \right)}{4 \sqrt{193\pi} \sqrt{r}} - \right. \\ \left. \frac{\text{Sin}[t] \left(\sqrt{21} \left(\frac{1}{2} \text{Cos}\left[\frac{t}{2}\right] - \frac{5}{2} \text{Cos}\left[\frac{3t}{2}\right] \right) - \frac{1}{2} \text{Sin}\left[\frac{t}{2}\right] + \frac{7}{2} \text{Sin}\left[\frac{3t}{2}\right] \right)}{2 \sqrt{193\pi} \sqrt{r}} \right) + \\ \frac{1}{r} \left(\text{Cos}[t] \left(\frac{3 \sqrt{r} \text{Cos}[t] \left(\text{Cos}\left[\frac{t}{2}\right] - \frac{7}{3} \text{Cos}\left[\frac{3t}{2}\right] + \sqrt{21} \left(\text{Sin}\left[\frac{t}{2}\right] - \frac{5}{3} \text{Sin}\left[\frac{3t}{2}\right] \right) \right)}{2 \sqrt{193\pi}} + \right. \right. \\ \frac{\sqrt{r} \text{Sin}[t] \left(\sqrt{21} \left(\frac{1}{2} \text{Cos}\left[\frac{t}{2}\right] - \frac{5}{2} \text{Cos}\left[\frac{3t}{2}\right] \right) - \frac{1}{2} \text{Sin}\left[\frac{t}{2}\right] + \frac{7}{2} \text{Sin}\left[\frac{3t}{2}\right] \right)}{2 \sqrt{193\pi}} + \\ \left. \left. \frac{\sqrt{r} \text{Cos}[t] \left(-\frac{1}{4} \text{Cos}\left[\frac{t}{2}\right] + \frac{21}{4} \text{Cos}\left[\frac{3t}{2}\right] + \sqrt{21} \left(-\frac{1}{4} \text{Sin}\left[\frac{t}{2}\right] + \frac{15}{4} \text{Sin}\left[\frac{3t}{2}\right] \right) \right)}{\sqrt{193\pi}} \right) \right) - \\ \frac{1}{r} \left(\text{Sin}[t] \left(-\frac{3 \sqrt{r} \text{Sin}[t] \left(\text{Cos}\left[\frac{t}{2}\right] - \frac{7}{3} \text{Cos}\left[\frac{3t}{2}\right] + \sqrt{21} \left(\text{Sin}\left[\frac{t}{2}\right] - \frac{5}{3} \text{Sin}\left[\frac{3t}{2}\right] \right) \right)}{2 \sqrt{193\pi}} + \right. \right. \\ \frac{\sqrt{r} \text{Cos}[t] \left(\sqrt{21} \left(\frac{1}{2} \text{Cos}\left[\frac{t}{2}\right] - \frac{5}{2} \text{Cos}\left[\frac{3t}{2}\right] \right) - \frac{1}{2} \text{Sin}\left[\frac{t}{2}\right] + \frac{7}{2} \text{Sin}\left[\frac{3t}{2}\right] \right)}{2 \sqrt{193\pi}} - \\ \left. \left. \frac{\sqrt{r} \text{Sin}[t] \left(-\frac{1}{4} \text{Cos}\left[\frac{t}{2}\right] + \frac{21}{4} \text{Cos}\left[\frac{3t}{2}\right] + \sqrt{21} \left(-\frac{1}{4} \text{Sin}\left[\frac{t}{2}\right] + \frac{15}{4} \text{Sin}\left[\frac{3t}{2}\right] \right) \right)}{\sqrt{193\pi}} \right) \right)$$

Simplify[%]

$$\frac{2 \left(\text{Cos}\left[\frac{t}{2}\right] + \sqrt{21} \text{Sin}\left[\frac{t}{2}\right] \right)}{\sqrt{193\pi} \sqrt{r}}$$

■ Evaluation of laplacian of the candidate Φ (with $\alpha=1$)

$$\mathbf{\Phi[r_, t_]} := r^{(3/2)} * (1 / (\text{Sqrt}[193 * \text{Pi}])) * \\ (\text{Sqrt}[21] * (\text{Sin}[t/2] - 5/3 * \text{Sin}[3 * t/2]) - \\ (\text{Cos}[t/2] - 7/3 * \text{Cos}[3 * t/2]))$$

lap[$\Phi[r, t]$]

$$\begin{aligned} & \sin[t] \left(\frac{3 \sin[t] \left(-\cos\left[\frac{t}{2}\right] + \frac{7}{3} \cos\left[\frac{3t}{2}\right] + \sqrt{21} \left(\sin\left[\frac{t}{2}\right] - \frac{5}{3} \sin\left[\frac{3t}{2}\right] \right) \right)}{4 \sqrt{193} \pi \sqrt{r}} + \right. \\ & \quad \left. \frac{\cos[t] \left(\sqrt{21} \left(\frac{1}{2} \cos\left[\frac{t}{2}\right] - \frac{5}{2} \cos\left[\frac{3t}{2}\right] \right) + \frac{1}{2} \sin\left[\frac{t}{2}\right] - \frac{7}{2} \sin\left[\frac{3t}{2}\right] \right)}{2 \sqrt{193} \pi \sqrt{r}} \right) + \\ & \cos[t] \left(\frac{3 \cos[t] \left(-\cos\left[\frac{t}{2}\right] + \frac{7}{3} \cos\left[\frac{3t}{2}\right] + \sqrt{21} \left(\sin\left[\frac{t}{2}\right] - \frac{5}{3} \sin\left[\frac{3t}{2}\right] \right) \right)}{4 \sqrt{193} \pi \sqrt{r}} - \right. \\ & \quad \left. \frac{\sin[t] \left(\sqrt{21} \left(\frac{1}{2} \cos\left[\frac{t}{2}\right] - \frac{5}{2} \cos\left[\frac{3t}{2}\right] \right) + \frac{1}{2} \sin\left[\frac{t}{2}\right] - \frac{7}{2} \sin\left[\frac{3t}{2}\right] \right)}{2 \sqrt{193} \pi \sqrt{r}} \right) + \\ & \frac{1}{r} \left(\cos[t] \left(\frac{3 \sqrt{r} \cos[t] \left(-\cos\left[\frac{t}{2}\right] + \frac{7}{3} \cos\left[\frac{3t}{2}\right] + \sqrt{21} \left(\sin\left[\frac{t}{2}\right] - \frac{5}{3} \sin\left[\frac{3t}{2}\right] \right) \right)}{2 \sqrt{193} \pi} + \right. \right. \\ & \quad \frac{\sqrt{r} \sin[t] \left(\sqrt{21} \left(\frac{1}{2} \cos\left[\frac{t}{2}\right] - \frac{5}{2} \cos\left[\frac{3t}{2}\right] \right) + \frac{1}{2} \sin\left[\frac{t}{2}\right] - \frac{7}{2} \sin\left[\frac{3t}{2}\right] \right)}{2 \sqrt{193} \pi} + \\ & \quad \left. \left. \frac{\sqrt{r} \cos[t] \left(\frac{1}{4} \cos\left[\frac{t}{2}\right] - \frac{21}{4} \cos\left[\frac{3t}{2}\right] + \sqrt{21} \left(-\frac{1}{4} \sin\left[\frac{t}{2}\right] + \frac{15}{4} \sin\left[\frac{3t}{2}\right] \right) \right)}{\sqrt{193} \pi} \right) \right) - \\ & \frac{1}{r} \left(\sin[t] \left(-\frac{3 \sqrt{r} \sin[t] \left(-\cos\left[\frac{t}{2}\right] + \frac{7}{3} \cos\left[\frac{3t}{2}\right] + \sqrt{21} \left(\sin\left[\frac{t}{2}\right] - \frac{5}{3} \sin\left[\frac{3t}{2}\right] \right) \right)}{2 \sqrt{193} \pi} + \right. \right. \\ & \quad \frac{\sqrt{r} \cos[t] \left(\sqrt{21} \left(\frac{1}{2} \cos\left[\frac{t}{2}\right] - \frac{5}{2} \cos\left[\frac{3t}{2}\right] \right) + \frac{1}{2} \sin\left[\frac{t}{2}\right] - \frac{7}{2} \sin\left[\frac{3t}{2}\right] \right)}{2 \sqrt{193} \pi} - \\ & \quad \left. \left. \frac{\sqrt{r} \sin[t] \left(\frac{1}{4} \cos\left[\frac{t}{2}\right] - \frac{21}{4} \cos\left[\frac{3t}{2}\right] + \sqrt{21} \left(-\frac{1}{4} \sin\left[\frac{t}{2}\right] + \frac{15}{4} \sin\left[\frac{3t}{2}\right] \right) \right)}{\sqrt{193} \pi} \right) \right) \end{aligned}$$

Simplify[%]

$$\frac{2 \left(-\cos\left[\frac{t}{2}\right] + \sqrt{21} \sin\left[\frac{t}{2}\right] \right)}{\sqrt{193} \pi \sqrt{r}}$$

THE CANDIDATE IS BIHARMONIC

■ (10.21) Evaluation of bi-laplacian of the candidate W ($\Delta^2 W$)

lap[lap[u]]

$$\begin{aligned} & \sin[t] \left(\sin[t] \right. \\ & \quad \left(\sin[t] \left(\sin[t] \left(\frac{1}{3 \sqrt{193} \pi} \left(\sqrt{\alpha} \left(3 \left(-\frac{3}{2} + h \right) \left(-\frac{1}{2} + h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{5}{2}+h} \cos\left[\left(-\frac{1}{2} + h \right) t\right] - 7 \left(-\frac{3}{2} + h \right) \left(-\frac{1}{2} + h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{5}{2}+h} \cos\left[\left(\frac{3}{2} + h \right) t\right] \right) \right) + \right. \right. \\ & \quad \sqrt{\frac{7}{579} \pi} \sqrt{\alpha} \left(3 \left(-\frac{3}{2} + h \right) \left(-\frac{1}{2} + h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{5}{2}+h} \sin\left[\left(-\frac{1}{2} + h \right) t\right] - \right. \\ & \quad \left. \left. \left. 5 \left(-\frac{3}{2} + h \right) \left(-\frac{1}{2} + h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{5}{2}+h} \sin\left[\left(\frac{3}{2} + h \right) t\right] \right) \right) \right) + \end{aligned}$$

$$\begin{aligned}
& r^{\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right]\right) - \\
& \frac{1}{r} \left(\cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \\
& \quad \left. \left. \left. 5\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right] + \right. \right. \right. \\
& \quad \left. \left. \left. \frac{\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right] \right)}{3\sqrt{193\pi}} \right) \right) \right) - \\
& \frac{1}{r} \left(\sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right] \right) \right) + \right. \\
& \quad \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \\
& \quad \left. \left. \left. 5\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right] \right) \right) \right) \right) \right) + \\
& \frac{1}{r} \left(\cos[t] \left(\cos[t] \left(\sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{3}{2}+h} \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \cos\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{1}{2}+h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{3}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right] \right) \right) \right) + \right. \\
& \quad \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \\
& \quad \left. \left. \left. 5\left(-\frac{1}{2}+h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{3}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right] \right) \right) \right) \right) + \\
& \frac{1}{r} \left(\cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h} \right. \right. \right. \\
& \quad \left. \left. \cos\left[\left(-\frac{1}{2}+h\right)t\right] - 5\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)^2 r^{-\frac{1}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right] \right) \right) + \\
& \quad \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \\
& \quad \left. \left. \left. 7\left(-\frac{3}{2}-h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right] \right) \right) \right) \right) - \\
& \frac{1}{r^2} \left(2 \cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \\
& \quad \left. \left. 5\left(\frac{3}{2}+h\right)^2 r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right] \right) \right) + \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(\frac{3}{2}+h\right) \right. \right. \\
& \quad \left. \left. r^{\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right] \right) \right) \right) \right) + \\
& \frac{1}{r^3} \left(2 \cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \\
& \quad \left. \left. 5\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right] \right) \right) + \right. \\
& \quad \left. \frac{\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right] \right)}{3\sqrt{193\pi}} \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin\left[-\frac{1}{2} + h\right] t \right) - \\
& \left. 5 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right) + \\
\cos[t] & \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \cos\left[-\frac{1}{2} + h\right] t \right) - \right. \\
& \left. 5 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right)^2 r^{-\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) + \\
& \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin\left[-\frac{1}{2} + h\right] t \right) - \right. \\
& \left. 7 \left(-\frac{3}{2} - h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) - \frac{1}{r} \left(\cos[t] \right. \\
& \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos\left[-\frac{1}{2} + h\right] t \right) - 5 \left(\frac{3}{2} + h \right)^2 \right. \\
& \left. r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) + \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \right. \right. \\
& \left. \left. \sin\left[-\frac{1}{2} + h\right] t \right) - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) \right) - \\
\frac{1}{r} & \left(\sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \right. \right. \right. \right. \\
& \left. \left. \cos\left[-\frac{1}{2} + h\right] t \right) - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) + \\
& \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin\left[-\frac{1}{2} + h\right] t \right) - \\
& \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) + \frac{1}{r^2} \left(\cos[t] \left(\sqrt{\frac{7}{579\pi}} \right. \right. \\
& \left. \left. \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos\left[-\frac{1}{2} + h\right] t \right) - 5 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) + \right. \\
& \left. \frac{\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) r^{\frac{3}{2}+h} \sin\left[-\frac{1}{2} + h\right] t \right) - 7 \left(-\frac{3}{2} - h \right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]}{3\sqrt{193\pi}} \right) \right) \right) \\
& + \frac{1}{r^2} \left(\sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \right. \right. \right. \right. \right. \\
& \left. \left. r^{\frac{3}{2}+h} \cos\left[-\frac{1}{2} + h\right] t \right) - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) + \\
& \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin\left[-\frac{1}{2} + h\right] t \right) - \\
& \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) \right) \right) + \frac{1}{r^2} \left(\cos[t] \right. \\
& \left. \left(-\sin[t] \left(\frac{\sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos\left[-\frac{1}{2} + h\right] t \right) - 7 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]}{3\sqrt{193\pi}} \right) + \right. \right. \\
& \left. \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin\left[-\frac{1}{2} + h\right] t \right) - \right. \right. \\
& \left. \left. 5 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& 7 \left(-\frac{3}{2} - h \right)^2 \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \Bigg) - \\
& \frac{1}{r^2} \left(\sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \quad \left. \left. \left. 5 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \right. \right. \\
& \quad \left. \left. \left. \frac{\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right)}{3 \sqrt{193\pi}} \right) \right) \right) \Bigg) \\
& + \frac{1}{r^2} \left(2 \cos[t] \left(\frac{1}{3 \sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) + \right. \\
& \quad \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \quad \left. \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \Bigg) + \\
& \frac{1}{r^2} \left(\sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \quad \left. \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \right. \\
& \quad \left. \frac{1}{3 \sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right)^2 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \quad \left. \left. \left. 7 \left(-\frac{3}{2} - h \right)^2 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) \Bigg) \right) + \frac{1}{r^2} \left(\sin[t] \right. \\
& \left. \left(-\cos[t] \left(\frac{\sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right)}{3 \sqrt{193\pi}} \right. \right. \right. + \\
& \quad \left. \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \right. \\
& \quad \left. \left. \left. 5 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) - \right. \\
& \quad \left. 2 \sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \right. \\
& \quad \left. \left. 5 \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \frac{1}{3 \sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) \right. \right. \right. \\
& \quad \left. \left. \left. r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) \Bigg) + \\
& \cos[t] \left(\frac{1}{3 \sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\right. \right. \right. \right. \\
& \quad \left. \left. \left. \left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) + \\
& \quad \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{r^2} \left(\cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right)t\right] - \right. \right. \right. \\
& \quad \left. \left. \left. 5 \left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right)t\right] \right) + \right. \right. \\
& \quad \left. \left. \frac{\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right)t\right] - 7 \left(-\frac{3}{2} - h\right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right)t\right] \right)}{3\sqrt{193\pi}} \right) \right) \\
& + \frac{1}{r^2} \left(2 \sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(-\frac{1}{2} + h\right) \right. \right. \right. \right. \\
& \quad \left. \left. \left. r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right)t\right] - 7 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right)t\right] \right) \right) + \right. \\
& \quad \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right)t\right] - \right. \right. \\
& \quad \left. \left. 5 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right)t\right] \right) \right) \right) - \\
& \frac{1}{r^2} \left(\cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(-\frac{1}{2} + h\right)^2 r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right)t\right] - \right. \right. \right. \\
& \quad \left. \left. 5 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right)^2 r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right)t\right] \right) + \right. \\
& \quad \left. \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right)^2 \left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right)t\right] - \right. \right. \right. \\
& \quad \left. \left. \left. 7 \left(-\frac{3}{2} - h\right)^2 \left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right)t\right] \right) \right) \right) \right) - \frac{1}{r^2} \left(\cos[t] \right. \\
& \left. \left(-\sin[t] \left(\frac{\sqrt{\alpha} \left(3 \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right)t\right] - 7 \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right)t\right] \right)}{3\sqrt{193\pi}} + \right. \right. \right. \\
& \quad \left. \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right)t\right] - \right. \right. \right. \\
& \quad \left. \left. \left. 5 \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right)t\right] \right) \right) \right) + \right. \\
& 2 \cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right)t\right] - \right. \right. \\
& \quad \left. \left. 5 \left(\frac{3}{2} + h\right)^2 r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right)t\right] \right) + \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(\frac{3}{2} + h\right) \right. \right. \right. \\
& \quad \left. \left. \left. r^{\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right)t\right] - 7 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right)t\right] \right) \right) \right) + \\
& \sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(-\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right)t\right] - \right. \right. \right. \\
& \quad \left. \left. \left. 7 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right)^2 r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right)t\right] \right) \right) + \right. \\
& \quad \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(-\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right)t\right] - \right. \right. \\
& \quad \left. \left. 5 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right)^2 r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right)t\right] \right) \right) - \frac{1}{r} \left(\cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \right. \right. \\
& \quad \left. \left. \left(3 \left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right)t\right] - 5 \left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right)t\right] \right) + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. r^{\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right]\right) \Bigg) - \\
& \frac{1}{r} \left(\sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \\
& \quad \left. \left. \left. 5\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \right. \right. \\
& \quad \left. \left. \frac{\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right]\right)}{3\sqrt{193\pi}} \right) \right) \right) \Bigg) \\
& + \frac{1}{r} \left(\cos[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right) \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right]\right) \right) \right) \right) + \\
& \quad \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \\
& \quad \left. \left. 5\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right]\right) \right) \right) \Bigg) \Bigg) + \\
& \cos[t] \left(-\sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. 7\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right]\right) \right) \right) + \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+ \right. \right. \\
& \quad \left. \left. h\right)r^{-\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - 5\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right]\right) \right) \Bigg) + \\
& \cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \\
& \quad \left. \left. 5\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)^2 r^{-\frac{1}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \frac{1}{3\sqrt{193\pi}} \right. \\
& \quad \left. \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \\
& \quad \left. \left. \left. 7\left(-\frac{3}{2}-h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right]\right) \right) \right) \Bigg) - \\
& \frac{1}{r} \left(\cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \right. \\
& \quad \left. \left. \left. 5\left(\frac{3}{2}+h\right)^2 r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(\frac{3}{2}+h\right) \right. \right. \right. \right. \\
& \quad \left. \left. \left. r^{\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right]\right) \right) \right) \Bigg) \Bigg) - \\
& \frac{1}{r} \left(\sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)^2 r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right]\right) \right) \right) + \\
& \quad \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \\
& \quad \left. \left. 5\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)^2 r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right]\right) \right) \Bigg) \Bigg) +
\end{aligned}$$

$$\begin{aligned}
& 5 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \Bigg) + \\
& 2 \cos[t] \left[\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \quad \left. \left. 5 \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) \right. \right. \right. \\
& \quad \left. \left. r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \Bigg] + \\
& \sin[t] \left[\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\right. \right. \right. \right. \\
& \quad \left. \left. \left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) + \\
& \quad \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \quad \left. \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) - \frac{1}{r} \left(\cos[t] \left[\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \right. \right. \\
& \quad \left. \left. \left(3 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \right. \right. \\
& \quad \left. \left. \frac{\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right)}{3\sqrt{193\pi}} \right) \right) \Bigg] \\
& - \frac{1}{r} \left(2 \sin[t] \left[\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) + \\
& \quad \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \quad \left. \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \Bigg] \Bigg) + \\
& \frac{1}{r} \left(\cos[t] \left[\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \quad \left. \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) + \\
& \quad \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right)^2 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \quad \left. \left. 7 \left(-\frac{3}{2} - h \right)^2 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \Bigg] \Bigg) - \frac{1}{r} \left(\sin[t] \right. \\
& \quad \left. \left(-\cos[t] \left[\frac{\sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right)}{3\sqrt{193\pi}} \right. \right. \right. \\
& \quad \left. \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \quad \left. \left. \left. 5 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(\cos[t] \left[\frac{\sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos\left[-\frac{1}{2} + h\right] t\right) - 7 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]}{3 \sqrt{193 \pi}} \right. \right. \\
& \quad \left. \left. \sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin\left[-\frac{1}{2} + h\right] t\right) - \right. \right. \\
& \quad \left. \left. 5 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]\right] \right) + \\
& \sin[t] \left[\sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \cos\left[-\frac{1}{2} + h\right] t\right) - \right. \\
& \quad \left. 5 \left(\frac{3}{2} + h\right)^2 r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]\right] + \frac{1}{3 \sqrt{193 \pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(\frac{3}{2} + h\right) \right. \right. \\
& \quad \left. \left. r^{\frac{1}{2}+h} \sin\left[-\frac{1}{2} + h\right] t\right) - 7 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]\right) \right] - \\
& \frac{1}{r} \left(\sin[t] \left[\sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[-\frac{1}{2} + h\right] t\right) - \right. \right. \\
& \quad \left. \left. 5 \left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]\right] + \right. \\
& \quad \left. \frac{\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) r^{\frac{3}{2}+h} \sin\left[-\frac{1}{2} + h\right] t\right) - 7 \left(-\frac{3}{2} - h\right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]}{3 \sqrt{193 \pi}} \right) \right] \\
& + \frac{1}{r} \left(\cos[t] \left[\frac{1}{3 \sqrt{193 \pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(-\frac{1}{2} + h\right) \right. \right. \right. \right. \\
& \quad \left. \left. \left. r^{\frac{3}{2}+h} \cos\left[-\frac{1}{2} + h\right] t\right) - 7 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]\right) \right) \right] + \\
& \quad \left. \sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \sin\left[-\frac{1}{2} + h\right] t\right) - \right. \\
& \quad \left. \left. 5 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]\right) \right] \right) \\
& \frac{1}{r} \left(\sin[t] \left[-\sin[t] \left[\frac{1}{3 \sqrt{193 \pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \cos\left[\right. \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left(-\frac{1}{2} + h\right) t\right) - 7 \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]\right) \right) \right] + \right. \\
& \quad \left. \sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \sin\left[-\frac{1}{2} + h\right] t\right) - \right. \\
& \quad \left. \left. 5 \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]\right) \right] \right) + \\
& \cos[t] \left[\sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h\right) \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \cos\left[-\frac{1}{2} + h\right] t\right) - \right. \\
& \quad \left. 5 \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right)^2 r^{-\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]\right] + \\
& \quad \frac{1}{3 \sqrt{193 \pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \sin\left[-\frac{1}{2} + h\right] t\right) - \right. \\
& \quad \left. \left. 7 \left(-\frac{3}{2} - h\right) \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]\right) \right) \right] - \\
& \frac{1}{r} \left(\cos[t] \left[\sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \cos\left[-\frac{1}{2} + h\right] t\right) - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 5 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \Bigg) + \\
& 2 \cos [t] \left[\sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \left. \left. 5 \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \frac{1}{3 \sqrt{193 \pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) \right. \right. \right. \\
& \left. \left. \left. r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right] + \\
& \sin [t] \left[\frac{1}{3 \sqrt{193 \pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\right. \right. \right. \right. \\
& \left. \left. \left. \left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right] + \\
& \sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \\
& \left. \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) - \frac{1}{r} \left(\cos [t] \left[\sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \right. \right. \\
& \left. \left. \left(3 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \right. \right. \\
& \left. \left. \frac{\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right)}{3 \sqrt{193 \pi}} \right) \right) \Bigg) \\
& - \frac{1}{r} \left(2 \sin [t] \left[\frac{1}{3 \sqrt{193 \pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \right. \right. \right. \right. \right. \\
& \left. \left. \left. r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right] + \right. \\
& \left. \sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \left. \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \Bigg) \Bigg) + \\
& \frac{1}{r} \left(\cos [t] \left[\sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \left. \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right] + \right. \\
& \left. \frac{1}{3 \sqrt{193 \pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right)^2 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \left. \left. \left. 7 \left(-\frac{3}{2} - h \right)^2 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) \Bigg) \Bigg) - \\
& \frac{1}{r} \left(\sin [t] \left[-\cos [t] \left[\frac{1}{3 \sqrt{193 \pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \cos \left[\right. \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left(-\frac{1}{2} + h \right) t \right] - 7 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right] + \right. \\
& \left. \sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \left. \left. 5 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \Bigg) -
\end{aligned}$$

$$\begin{aligned}
& r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \\
& \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \\
& \left. 5\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right]\right) \Bigg) - \\
& \frac{1}{r} \left(\sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)^2 r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \\
& \left. \left. 5\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)^2 r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \right. \\
& \left. \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \\
& \left. \left. 7\left(-\frac{3}{2}-h\right)^2\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right]\right) \right) \right) \Bigg) - \\
& \frac{1}{r} \left(\sin[t] \left(\cos[t] \left(\sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h} \right. \right. \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \cos\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right]\right) \right) + \right. \right. \\
& \left. \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \\
& \left. \left. 5\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right]\right) \right) + \frac{1}{r} \left(\cos[t] \right. \\
& \left. \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2}+h\right)t\right] - 5\left(\frac{3}{2}+h\right)^2 \right. \right. \right. \\
& \left. \left. r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h} \right. \right. \right. \\
& \left. \left. \sin\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right]\right) \right) \right) \Bigg) - \\
& \frac{1}{r^2} \left(\cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \\
& \left. \left. 5\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \right. \\
& \left. \frac{\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right]\right)}{3\sqrt{193\pi}} \right) \Bigg) \Bigg) \\
& - \sin[t] \left(\cos[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right) \right. \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. r^{-\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right]\right) \right) + \right. \\
& \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \\
& \left. \left. 5\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h} \sin\left[\left(\frac{3}{2}+h\right)t\right]\right) \right) - \frac{1}{r} \left(\sin[t] \right. \\
& \left. \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2}+h\right)t\right] - 5\left(\frac{3}{2}+h\right)^2 \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right] + \frac{1}{3\sqrt{193}\pi} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\right.\right. \\
& \left.\left.\sin\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)\right) \Bigg) + \\
& \frac{1}{r^2} \left(\sin[t] \left(\sqrt{\frac{7}{579}\pi} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right.\right. \right. \\
& \left. \left. 5\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \right. \\
& \left. \left. \frac{\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)r^{\frac{3}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)r^{\frac{3}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)}{3\sqrt{193}\pi} \right) \right) \Bigg) \\
& + \sin[t] \left(\cos[t] \left(\frac{1}{3\sqrt{193}\pi} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)\right.\right.\right. \right. \\
& \left.\left.\left. r^{-\frac{1}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right)\right) + \right. \\
& \left. \sqrt{\frac{7}{579}\pi} \sqrt{\alpha} \left(3\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \\
& \left. \left. 5\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right) \right) + \\
& \sin[t] \left(\sqrt{\frac{7}{579}\pi} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \\
& \left. \left. 5\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)^2 r^{-\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \right. \\
& \left. \frac{1}{3\sqrt{193}\pi} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \\
& \left. \left. \left. 7\left(-\frac{3}{2}-h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)\right) \right) - \\
& \frac{1}{r} \left(\sin[t] \left(\sqrt{\frac{7}{579}\pi} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \\
& \left. \left. 5\left(\frac{3}{2}+h\right)^2 r^{\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \right. \\
& \left. \frac{1}{3\sqrt{193}\pi} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \\
& \left. \left. \left. 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)\right) \right) \Bigg) + \\
& \frac{1}{r} \left(\cos[t] \left(\frac{1}{3\sqrt{193}\pi} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\right.\right.\right. \right. \\
& \left.\left.\left. \cos\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)^2 r^{\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right)\right) + \right. \\
& \left. \sqrt{\frac{7}{579}\pi} \sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \\
& \left. \left. 5\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)^2 r^{\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right) \right) \Bigg) + \\
& \frac{1}{r^2} \left(\sin[t] \left(\sqrt{\frac{7}{579}\pi} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{3 \left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[-\frac{1}{2} + h\right] t - 5 \left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] + \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) r^{\frac{3}{2}+h} \sin\left[-\frac{1}{2} + h\right] t - 7 \left(-\frac{3}{2} - h\right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]\right)}{3 \sqrt{193} \pi} \right) \\
& \left. - \frac{1}{r} \left(2 \sin[t] \left(\frac{1}{3 \sqrt{193} \pi} \right. \right. \right. \\
& \quad \left. \left. \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[-\frac{1}{2} + h\right] t - 7 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]\right) \right. \right. \right. \\
& \quad \left. \left. \left. + \sqrt{\frac{7}{579} \pi} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \sin\left[-\frac{1}{2} + h\right] t - 5 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]\right) \right) \right) \right) + \\
& \frac{1}{r} \left(\cos[t] \left(\sqrt{\frac{7}{579} \pi} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(-\frac{1}{2} + h\right)^2 r^{\frac{3}{2}+h} \cos\left[-\frac{1}{2} + h\right] t - \right. \right. \right. \\
& \quad \left. \left. \left. 5 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right)^2 r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]\right) + \frac{1}{3 \sqrt{193} \pi} \right. \right. \\
& \quad \left. \left. \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right)^2 \left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \sin\left[-\frac{1}{2} + h\right] t - 7 \left(-\frac{3}{2} - h\right)^2 \right. \right. \right. \\
& \quad \left. \left. \left. \left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]\right) \right) \right) \right) \right) - \frac{1}{r} \left(\sin[t] \left(-\cos[t] \right. \right. \\
& \left. \left. \left(\frac{\sqrt{\alpha} \left(3 \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \cos\left[-\frac{1}{2} + h\right] t - 7 \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]\right)}{3 \sqrt{193} \pi} + \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{\frac{7}{579} \pi} \sqrt{\alpha} \left(3 \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \sin\left[-\frac{1}{2} + h\right] t - \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. 5 \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]\right) \right) \right) \right) - 2 \sin[t] \right) \\
& \left(\sqrt{\frac{7}{579} \pi} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \cos\left[-\frac{1}{2} + h\right] t - 5 \left(\frac{3}{2} + h\right)^2 \right. \right. \\
& \quad \left. \left. r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]\right) + \frac{1}{3 \sqrt{193} \pi} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \right. \right. \right. \\
& \quad \left. \left. \left. \sin\left[-\frac{1}{2} + h\right] t - 7 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]\right) \right) \right) + \\
& \cos[t] \left(\frac{1}{3 \sqrt{193} \pi} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(-\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \cos\left[\right. \right. \right. \right. \\
& \quad \left. \left. \left. \left(-\frac{1}{2} + h\right) t - 7 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right)^2 r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]\right) \right) \right) + \\
& \quad \left. \sqrt{\frac{7}{579} \pi} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(-\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \sin\left[-\frac{1}{2} + h\right] t - \right. \right. \\
& \quad \left. \left. 5 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right)^2 r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]\right) \right) + \\
& \frac{1}{r} \left(\sin[t] \left(\sqrt{\frac{7}{579} \pi} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[-\frac{1}{2} + h\right] t - \right. \right. \right. \\
& \quad \left. \left. \left. 5 \left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]\right) + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \\
& \quad \left. 5 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) - \frac{1}{r} \left(\sin[t] \right. \\
& \quad \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(\frac{3}{2} + h \right)^2 \right. \right. \\
& \quad \left. \left. r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \right. \right. \right. \\
& \quad \left. \left. \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) + \\
& \frac{1}{r^2} \left(\sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \quad \left. \left. 5 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \right. \\
& \quad \left. \frac{\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right)}{3\sqrt{193\pi}} \right) \right) \right) \\
& + 2 \cos[t] \left(\cos[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. r^{-\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) + \\
& \quad \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \\
& \quad \left. 5 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) + \\
& \sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \quad \left. \left. 5 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right)^2 r^{-\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \right. \\
& \quad \left. \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \quad \left. \left. \left. 7 \left(-\frac{3}{2} - h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) - \\
& \frac{1}{r} \left(\sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \quad \left. \left. 5 \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \right. \\
& \quad \left. \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \quad \left. \left. \left. 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) \right) + \\
& \frac{1}{r} \left(\cos[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \Bigg) + \frac{1}{r} \left(\sin[t] \left(-\sin[t] \right. \right. \\
& \left. \left. \left(\frac{\sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right)}{3 \sqrt{193} \pi} + \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{7}{579} \pi} \sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \right. \\
& \left. \left. \left. \left. 5 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) + \cos[t] \right. \\
& \left. \left(\sqrt{\frac{7}{579} \pi} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(\frac{3}{2} + h \right)^2 \right. \right. \right. \right. \\
& \left. \left. \left. \left. r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \frac{1}{3 \sqrt{193} \pi} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \right. \right. \right. \right. \\
& \left. \left. \left. \left. \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) \right) - \\
& \frac{1}{r} \left(\cos[t] \left(\sqrt{\frac{7}{579} \pi} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \right. \\
& \left. \left. \left. \left. 5 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right)}{3 \sqrt{193} \pi} \right) \right) \right) \right) \\
& \left. - \frac{1}{r} \left(\sin[t] \left(\frac{1}{3 \sqrt{193} \pi} \right. \right. \right. \\
& \left. \left. \left. \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) + \sqrt{\frac{7}{579} \pi} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \right. \right. \right. \\
& \left. \left. \left. \left. \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) \right) \right) \Bigg) + \\
& \sin[t] \left(-\sin[t] \left(\frac{1}{3 \sqrt{193} \pi} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \cos \left[\right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left(-\frac{1}{2} + h \right) t \right] - 7 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) + \right. \\
& \left. \sqrt{\frac{7}{579} \pi} \sqrt{\alpha} \left(3 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \left. \left. \left. \left. 5 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) \right) + \\
& 2 \cos[t] \left(\sqrt{\frac{7}{579} \pi} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \right. \right. \\
& \left. \left. \left. \left. \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right)^2 r^{-\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) + \right. \\
& \left. \frac{1}{3 \sqrt{193} \pi} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \left. \left. \left. \left. 7 \left(-\frac{3}{2} - h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \Bigg) + \\
& \frac{1}{3 \sqrt{193 \pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right)^2 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \left. \left. 7 \left(-\frac{3}{2} - h \right)^2 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \Bigg) \Bigg) + \\
\cos[t] & \left(-\cos[t] \left(\frac{1}{3 \sqrt{193 \pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \cos \left[\right. \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left(-\frac{1}{2} + h \right) t \right] - 7 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) + \\
& \sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \\
& \left. 5 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \Bigg) - \\
2 \sin[t] & \left(\sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \right. \right. \\
& \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right)^2 r^{-\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \\
& \frac{1}{3 \sqrt{193 \pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \left. \left. 7 \left(-\frac{3}{2} - h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \Bigg) + \\
\cos[t] & \left(\frac{1}{3 \sqrt{193 \pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \cos \left[\right. \right. \right. \right. \right. \right. \\
& \left. \left. \left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right)^2 r^{-\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) + \\
& \sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin \left[\right. \right. \\
& \left. \left. \left(-\frac{1}{2} + h \right) t \right] - 5 \left(-\frac{3}{2} - h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right)^2 r^{-\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \Bigg) + \\
\frac{1}{r} & \left(\sin[t] \left(\sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \right. \\
& \left. \left. 5 \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \right. \\
& \left. \frac{1}{3 \sqrt{193 \pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \right. \\
& \left. \left. 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \Bigg) - \\
\frac{1}{r} & \left(2 \cos[t] \left(\frac{1}{3 \sqrt{193 \pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \right. \right. \right. \right. \right. \right. \\
& \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) + \\
& \sqrt{\frac{7}{579 \pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right.
\end{aligned}$$

$$\begin{aligned}
& 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \Bigg) - \\
& \frac{1}{r} \left(\sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right)^2 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \right. \right. \right. \\
& \quad \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^3 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \\
& \quad \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right)^2 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \quad \left. \left. 7 \left(-\frac{3}{2} - h \right)^2 \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \Bigg) - \\
& \frac{1}{r^2} \left(\sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \quad 5 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \\
& \quad \left. \frac{\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right)}{3\sqrt{193\pi}} \right) \Bigg) \\
& + \frac{1}{r^2} \left(2 \cos[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) + \\
& \quad \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \\
& \quad \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \Bigg) + \\
& \frac{1}{r^2} \left(\sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \quad 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \frac{1}{3\sqrt{193\pi}} \\
& \quad \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right)^2 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right)^2 \right. \right. \\
& \quad \left. \left. \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \Bigg) - \frac{1}{r} \left(2 \cos[t] \left(-\cos[t] \right. \right. \\
& \quad \left. \left. \frac{\sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right)}{3\sqrt{193\pi}} \right) + \right. \\
& \quad \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \quad \left. \left. 5 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) - 2 \sin[t] \\
& \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(\frac{3}{2} + h \right)^2 \right. \right. \\
& \quad \left. \left. r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \right. \right. \right. \\
& \quad \left. \left. \left. \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \cos[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) + \right. \\
& \quad \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \quad \left. \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) + \frac{1}{r} \left(\sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \right. \right. \\
& \quad \left. \left. \left(3 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \right. \right. \\
& \quad \left. \left. \frac{\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right)}{3\sqrt{193\pi}} \right) \right) \\
& \left. \right) - \frac{1}{r} \left(2 \cos[t] \left(\frac{1}{3\sqrt{193\pi}} \right. \right. \\
& \quad \left. \left. \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) + \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \right. \right. \\
& \quad \left. \left. \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) - \\
& \quad \frac{1}{r} \left(\sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \quad \left. \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \frac{1}{3\sqrt{193\pi}} \right. \\
& \quad \left. \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right)^2 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right)^2 \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) \right) \right) + \frac{1}{r} \left(\cos[t] \left(-\cos[t] \right. \right. \\
& \quad \left. \left. \left(\frac{\sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right)}{3\sqrt{193\pi}} + \right. \right. \\
& \quad \left. \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \quad \left. \left. \left. 5 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) - 3 \sin[t] \right) \right) \\
& \quad \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(\frac{3}{2} + h \right)^2 \right. \right. \\
& \quad \left. \left. r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \right. \right. \right. \\
& \quad \left. \left. \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) + \\
& \quad 3 \cos[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \right. \right. \right. \\
& \quad \left. \left. \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \begin{aligned}
& \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \\
& \left. 5 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) - \\
& 3 \cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \left. \left. 5 \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \right. \\
& \left. \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \left. \left. \left. 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) - \\
& 3 \sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \right. \right. \right. \\
& \left. \left. \left. \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) + \\
& \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \left. \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) + \cos[t] \\
& \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right)^2 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \\
& \left. \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^3 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \frac{1}{3\sqrt{193\pi}} \right. \\
& \left. \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right)^2 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \left. \left. \left. 7 \left(-\frac{3}{2} - h \right)^2 \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) + \\
& \frac{1}{r} \left(\cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
& \left. \left. 5 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \right. \\
& \left. \frac{\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right)}{3\sqrt{193\pi}} \right) \right) \\
& \left. + \frac{1}{r} \left(3 \sin[t] \left(\frac{1}{3\sqrt{193\pi}} \right. \right. \right. \\
& \left. \left. \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) \right. \right. \right. \right. \\
& \left. \left. \left. r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) + \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \right. \right. \\
& \left. \left. \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) - \\
& \frac{1}{r} \left(3 \cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \cos\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{1}{2}+h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{3}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \\
& \sqrt{\frac{7}{579\pi}}\sqrt{\alpha}\left(3\left(-\frac{1}{2}+h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{3}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \\
& \left. 5\left(-\frac{1}{2}+h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{3}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right) - \\
& \frac{1}{r}\left(\sin[t]\left(\sqrt{\frac{7}{579\pi}}\sqrt{\alpha}\left(3\left(-\frac{1}{2}+h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\right.\right.\right. \\
& \left.\left.\cos\left[\left(-\frac{1}{2}+h\right)t\right] - 5\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)^2r^{-\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \right. \\
& \left.\frac{1}{3\sqrt{193\pi}}\left(\sqrt{\alpha}\left(3\left(\frac{1}{2}-h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right.\right.\right. \\
& \left.\left.\left.7\left(-\frac{3}{2}-h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)\right)\right) + \\
& \frac{1}{r^2}\left(2\sin[t]\left(\sqrt{\frac{7}{579\pi}}\sqrt{\alpha}\left(3\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right.\right.\right. \\
& \left.\left.5\left(\frac{3}{2}+h\right)^2r^{\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \frac{1}{3\sqrt{193\pi}}\left(\sqrt{\alpha}\left(3\left(\frac{1}{2}-h\right)\left(\frac{3}{2}+h\right)\right.\right.\right. \\
& \left.\left.\left.r^{\frac{1}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)\right)\right) - \\
& \frac{1}{r^3}\left(2\sin[t]\left(\sqrt{\frac{7}{579\pi}}\sqrt{\alpha}\left(3\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right.\right.\right. \\
& \left.\left.5\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \right. \\
& \left.\left.\frac{\sqrt{\alpha}\left(3\left(\frac{1}{2}-h\right)r^{\frac{3}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)r^{\frac{3}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)}{3\sqrt{193\pi}}\right)\right) + \\
& \frac{1}{r}\left(\cos[t]\left(\cos[t]\left(\frac{1}{3\sqrt{193\pi}}\left(\sqrt{\alpha}\left(3\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right.\right.\right.\right.\right. \\
& \left.\left.\left.7\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right)\right) + \right. \\
& \left.\sqrt{\frac{7}{579\pi}}\sqrt{\alpha}\left(3\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right.\right. \\
& \left.\left.5\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right) + \right. \\
& \left.\sin[t]\left(\sqrt{\frac{7}{579\pi}}\sqrt{\alpha}\left(3\left(-\frac{1}{2}+h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right.\right.\right. \\
& \left.\left.5\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)^2r^{-\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \right. \\
& \left.\frac{1}{3\sqrt{193\pi}}\left(\sqrt{\alpha}\left(3\left(\frac{1}{2}-h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right.\right.\right. \\
& \left.\left.\left.7\left(-\frac{3}{2}-h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)\right)\right) - \frac{1}{r}\left(\sin[t]\right. \\
& \left.\left(\sqrt{\frac{7}{579\pi}}\sqrt{\alpha}\left(3\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right] - 5\left(\frac{3}{2}+h\right)^2\right.\right.\right.
\end{aligned}$$

$$\begin{aligned}
 & r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right] + \frac{1}{3\sqrt{193}\pi} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\right.\right. \\
 & \left.\left.\sin\left[-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)\right) + \\
 & \frac{1}{r} \left(\cos[t] \left(\frac{1}{3\sqrt{193}\pi} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\right.\right.\right.\right. \\
 & \left.\left.\left.\cos\left[-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)^2 r^{\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right)\right) + \right. \\
 & \left.\sqrt{\frac{7}{579}\pi} \sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\sin\left[-\frac{1}{2}+h\right)t\right] - \right. \\
 & \left.5\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)^2 r^{\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)\right) + \frac{1}{r^2} \left(\sin[t] \left(\sqrt{\frac{7}{579}\pi} \right.\right. \\
 & \left.\left.\sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h}\cos\left[-\frac{1}{2}+h\right)t\right] - 5\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right] + \right.\right. \\
 & \left.\left.\frac{\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)r^{\frac{3}{2}+h}\sin\left[-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)r^{\frac{3}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)}{3\sqrt{193}\pi}\right)\right) \\
 & - \frac{1}{r^2} \left(\cos[t] \left(\frac{1}{3\sqrt{193}\pi} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)\right.\right.\right.\right. \\
 & \left.\left.\left.\left.r^{\frac{3}{2}+h}\cos\left[-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right)\right)\right) + \right. \\
 & \left.\sqrt{\frac{7}{579}\pi} \sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h}\sin\left[-\frac{1}{2}+h\right)t\right] - \right. \\
 & \left.5\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)\right)\right) - \frac{1}{r^2} \left(\cos[t] \right. \\
 & \left.\cos[t] \left(\frac{\sqrt{\alpha} \left(3\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\cos\left[-\frac{1}{2}+h\right)t\right] - 7\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right)}{3\sqrt{193}\pi} + \right. \\
 & \left.\sqrt{\frac{7}{579}\pi} \sqrt{\alpha} \left(3\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\sin\left[-\frac{1}{2}+h\right)t\right] - \right. \\
 & \left.5\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)\right) + \\
 & \sin[t] \left(\sqrt{\frac{7}{579}\pi} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\cos\left[-\frac{1}{2}+h\right)t\right] - \right. \\
 & \left.5\left(\frac{3}{2}+h\right)^2 r^{\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \frac{1}{3\sqrt{193}\pi} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(\frac{3}{2}+h\right)\right.\right. \\
 & \left.\left.r^{\frac{1}{2}+h}\sin\left[-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)\right) - \\
 & \frac{1}{r} \left(\sin[t] \left(\sqrt{\frac{7}{579}\pi} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h}\cos\left[-\frac{1}{2}+h\right)t\right] - \right. \right. \\
 & \left.5\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \right. \\
 & \left.\frac{\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)r^{\frac{3}{2}+h}\sin\left[-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)r^{\frac{3}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)}{3\sqrt{193}\pi}\right)\right) \\
 & + \frac{1}{r} \left(\cos[t] \left(\frac{1}{3\sqrt{193}\pi} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)\right.\right.\right.\right.
 \end{aligned}$$

$$\begin{aligned}
& r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2}+h\right)t\right] + \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\right.\right. \\
& \left.\left.\sin\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)\right) - \\
& \frac{1}{r} \left(\sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right.\right. \right. \\
& \left. \left. 5\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \right. \\
& \left. \frac{\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)r^{\frac{3}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)r^{\frac{3}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)}{3\sqrt{193\pi}} \right) \\
& \left. \right) + \frac{1}{r} \left(\cos[t] \left(\frac{1}{3\sqrt{193\pi}} \right. \right. \\
& \left. \left. \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right] - 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)\right.\right.\right.\right. \\
& \left. \left. \left. r^{\frac{3}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h}\right.\right.\right. \\
& \left. \left. \left. \sin\left[\left(-\frac{1}{2}+h\right)t\right] - 5\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)\right)\right) \right) \right) + \\
& \cos[t] \left(-\sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right.\right.\right.\right. \right. \\
& \left. \left. \left. \left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right)\right) + \right. \\
& \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \\
& \left. \left. 5\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right) \right) + \\
& \cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \\
& \left. \left. 5\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)^2 r^{-\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \right. \\
& \left. \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \\
& \left. \left. \left. 7\left(-\frac{3}{2}-h\right)\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)\right) \right) - \\
& \frac{1}{r} \left(\cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \\
& \left. \left. 5\left(\frac{3}{2}+h\right)^2 r^{\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right) + \right. \\
& \left. \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right] - \right. \right. \right. \\
& \left. \left. \left. 7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)\right) \right) \right) - \\
& \frac{1}{r} \left(\sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2}-h\right)\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\right.\right.\right.\right. \right.
\end{aligned}$$

$$\begin{aligned}
 & \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \right. \\
 & \quad \left. \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \Bigg) + \frac{1}{r} \left(\cos[t] \left(-\sin[t] \right. \right. \\
 & \quad \left. \left. \left(\frac{\sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right)}{3\sqrt{193\pi}} + \right. \right. \right. \\
 & \quad \left. \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \\
 & \quad \left. \left. \left. 5 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) + 2 \cos[t] \right) \right. \\
 & \quad \left. \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(\frac{3}{2} + h \right)^2 \right. \right. \right. \right. \\
 & \quad \left. \left. \left. r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) \Bigg) + \\
 & \sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \Bigg) + \\
 & \quad \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \\
 & \quad \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \Bigg) - \frac{1}{r} \left(\cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \right. \right. \\
 & \quad \left. \left(3 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \right. \\
 & \quad \left. \frac{\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right)}{3\sqrt{193\pi}} \right) \Bigg) \\
 & \quad \left. \right) - \frac{1}{r} \left(2 \sin[t] \left(\frac{1}{3\sqrt{193\pi}} \right. \right. \\
 & \quad \left. \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) \right. \right. \right. \right. \\
 & \quad \left. \left. \left. r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) + \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \right. \right. \\
 & \quad \left. \left. \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \Bigg) \Bigg) + \\
 & \frac{1}{r} \left(\cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - \right. \right. \right. \right. \\
 & \quad \left. \left. \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) + \frac{1}{3\sqrt{193\pi}} \right. \\
 & \quad \left. \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right)^2 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right)^2 \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) \Bigg) \Bigg) - \frac{1}{r} \left(\sin[t] \left(-\cos[t] \right. \right.
 \end{aligned}$$

$$\begin{aligned}
& \cos\left[\left(-\frac{1}{2}+h\right)t\right]-7\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right)+ \\
& \sqrt{\frac{7}{579\pi}}\sqrt{\alpha}\left(3\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right]-\right. \\
& \left.5\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)+ \\
& \frac{1}{r}\left(\cos[t]\left(\sqrt{\frac{7}{579\pi}}\sqrt{\alpha}\left(3\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right]-\right.\right.\right. \\
& \left.\left.5\left(\frac{3}{2}+h\right)^2r^{\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right)+\right. \\
& \left.\frac{1}{3\sqrt{193\pi}}\left(\sqrt{\alpha}\left(3\left(\frac{1}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right]-\right.\right.\right. \\
& \left.\left.7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)\right)\right)- \\
& \frac{1}{r^2}\left(\cos[t]\left(\sqrt{\frac{7}{579\pi}}\sqrt{\alpha}\left(3\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right]-\right.\right.\right. \\
& \left.\left.5\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right)+\right. \\
& \left.\left.\frac{\sqrt{\alpha}\left(3\left(\frac{1}{2}-h\right)r^{\frac{3}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right]-7\left(-\frac{3}{2}-h\right)r^{\frac{3}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)}{3\sqrt{193\pi}}\right)\right)\right) \\
& -\cos[t]\left(\cos[t]\left(\frac{1}{3\sqrt{193\pi}}\left(\sqrt{\alpha}\left(3\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)\right.\right.\right.\right. \\
& \left.\left.\left.r^{-\frac{1}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right]-7\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right)\right)\right)+ \\
& \sqrt{\frac{7}{579\pi}}\sqrt{\alpha}\left(3\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right]-\right. \\
& \left.5\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)-\frac{1}{r}\left(\sin[t]\right. \\
& \left.\left(\sqrt{\frac{7}{579\pi}}\sqrt{\alpha}\left(3\left(-\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right]-5\left(\frac{3}{2}+h\right)^2\right.\right.\right. \\
& \left.\left.\left.r^{\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right)+\frac{1}{3\sqrt{193\pi}}\left(\sqrt{\alpha}\left(3\left(\frac{1}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\right.\right.\right. \\
& \left.\left.\left.\sin\left[\left(-\frac{1}{2}+h\right)t\right]-7\left(-\frac{3}{2}-h\right)\left(\frac{3}{2}+h\right)r^{\frac{1}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)\right)\right)\right)+ \\
& \frac{1}{r^2}\left(\sin[t]\left(\sqrt{\frac{7}{579\pi}}\sqrt{\alpha}\left(3\left(-\frac{1}{2}+h\right)r^{\frac{3}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right]-\right.\right.\right. \\
& \left.\left.5\left(\frac{3}{2}+h\right)r^{\frac{3}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right)+\right. \\
& \left.\left.\frac{\sqrt{\alpha}\left(3\left(\frac{1}{2}-h\right)r^{\frac{3}{2}+h}\sin\left[\left(-\frac{1}{2}+h\right)t\right]-7\left(-\frac{3}{2}-h\right)r^{\frac{3}{2}+h}\sin\left[\left(\frac{3}{2}+h\right)t\right]\right)}{3\sqrt{193\pi}}\right)\right)\right) \\
& +2\cos[t]\left(\cos[t]\left(\frac{1}{3\sqrt{193\pi}}\left(\sqrt{\alpha}\left(3\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)\right.\right.\right.\right. \\
& \left.\left.\left.r^{-\frac{1}{2}+h}\cos\left[\left(-\frac{1}{2}+h\right)t\right]-7\left(\frac{1}{2}+h\right)\left(\frac{3}{2}+h\right)r^{-\frac{1}{2}+h}\cos\left[\left(\frac{3}{2}+h\right)t\right]\right)\right)\right)+
\end{aligned}$$

$$\begin{aligned}
 & \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right) t\right] - \right. \right. \\
 & \quad \left. \left. 5 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) + \sin[t] \\
 & \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right) t\right] - 5 \left(\frac{3}{2} + h\right)^2 \right. \right. \\
 & \quad \left. \left. r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) + \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \right. \right. \right. \\
 & \quad \left. \left. \sin\left[\left(-\frac{1}{2} + h\right) t\right] - 7 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) \right) - \\
 & \frac{1}{r} \left(\sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right) t\right] - \right. \right. \right. \\
 & \quad \left. \left. 5 \left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) + \right. \\
 & \quad \left. \frac{\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right) t\right] - 7 \left(-\frac{3}{2} - h\right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right)}{3\sqrt{193\pi}} \right) \right) \\
 & \left. \right) + \frac{1}{r} \left(\cos[t] \left(\frac{1}{3\sqrt{193\pi}} \right. \right. \\
 & \quad \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right) t\right] - 7 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right) \right. \right. \\
 & \quad \left. \left. r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) + \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \right. \\
 & \quad \left. \left. \sin\left[\left(-\frac{1}{2} + h\right) t\right] - 5 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) \right) \right) - \\
 & 2 \sin[t] \left(-\sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right) t\right] - \right. \right. \right. \right. \right. \\
 & \quad \left. \left. \left(-\frac{1}{2} + h\right) t\right] - 7 \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) + \right. \\
 & \quad \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right) t\right] - \right. \right. \\
 & \quad \left. \left. 5 \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) \right) + \\
 & \cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h\right) \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right) t\right] - \right. \right. \\
 & \quad \left. \left. 5 \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right)^2 r^{-\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) + \right. \\
 & \quad \left. \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h\right) \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right) t\right] - \right. \right. \right. \\
 & \quad \left. \left. 7 \left(-\frac{3}{2} - h\right) \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) \right) - \\
 & \frac{1}{r} \left(\cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right) t\right] - \right. \right. \right. \\
 & \quad \left. \left. 5 \left(\frac{3}{2} + h\right)^2 r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) + \right.
 \end{aligned}$$

$$\begin{aligned}
& \left. \left. \left. \left. \left. -\frac{1}{r} \left(\sin[t] \left(\frac{1}{3\sqrt{193\pi}} \right. \right. \right. \right. \right. \right. \right. \right. \right. \\
& \quad \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right) t\right] - 7 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right) \right. \right. \\
& \quad \quad \left. \left. r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) + \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \right. \\
& \quad \quad \left. \sin\left[\left(-\frac{1}{2} + h\right) t\right] - 5 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) \right) \right) \right) + \\
& \sin[t] \left(-\sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right) t\right] - 7 \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) + \right. \right. \\
& \quad \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right) t\right] - 5 \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) \right) + \\
& 2 \cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \right. \right. \\
& \quad \left. \left. \cos\left[\left(-\frac{1}{2} + h\right) t\right] - 5 \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right)^2 r^{-\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) + \right. \\
& \quad \left. \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right) t\right] - 7 \left(-\frac{3}{2} - h\right) \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) \right) + \\
& \sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right) t\right] - 7 \left(-\frac{3}{2} - h\right) \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right)^2 r^{-\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) + \right. \\
& \quad \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{-\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right) t\right] - 5 \left(-\frac{3}{2} - h\right) \left(\frac{1}{2} + h\right) \left(\frac{3}{2} + h\right)^2 r^{-\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) - \\
& \frac{1}{r} \left(\cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right) t\right] - 5 \left(\frac{3}{2} + h\right)^2 r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) + \right. \right. \\
& \quad \left. \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right) t\right] - 7 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) \right) - \\
& \frac{1}{r} \left(2 \sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \cos\left[\left(-\frac{1}{2} + h\right) t\right] - 7 \left(-\frac{3}{2} - h\right) \left(\frac{3}{2} + h\right)^2 r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) \right) \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \sin\left[\left(-\frac{1}{2} + h\right) t\right] - 7\left(-\frac{3}{2} - h\right)\left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]\right) + \\
& \sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2} - h\right)\left(-\frac{1}{2} + h\right)\left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right) t\right] - \right. \right. \right. \\
& \quad \left. \left. \left(-\frac{1}{2} + h\right) t\right] - 7\left(-\frac{3}{2} - h\right)\left(\frac{3}{2} + h\right)^2 r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]\right) \right) + \\
& \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(\frac{1}{2} - h\right)\left(-\frac{1}{2} + h\right)\left(\frac{3}{2} + h\right) r^{\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right) t\right] - \right. \\
& \quad \left. 5\left(-\frac{3}{2} - h\right)\left(\frac{3}{2} + h\right)^2 r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]\right) - \frac{1}{r} \left(\cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \right. \right. \\
& \quad \left. \left. \left(3\left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right) t\right] - 5\left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right] \right) + \right. \right. \\
& \quad \left. \left. \frac{\sqrt{\alpha} \left(3\left(\frac{1}{2} - h\right) r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right) t\right] - 7\left(-\frac{3}{2} - h\right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right] \right)}{3\sqrt{193\pi}} \right) \right) \\
& \left. \right) - \frac{1}{r} \left(2 \sin[t] \left(\frac{1}{3\sqrt{193\pi}} \right. \right. \\
& \quad \left. \left. \left(\sqrt{\alpha} \left(3\left(\frac{1}{2} - h\right)\left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right) t\right] - 7\left(-\frac{3}{2} - h\right)\left(\frac{3}{2} + h\right) \right. \right. \right. \right. \\
& \quad \left. \left. \left. r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]\right) \right) + \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(\frac{1}{2} - h\right)\left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \right. \right. \\
& \quad \left. \left. \sin\left[\left(-\frac{1}{2} + h\right) t\right] - 5\left(-\frac{3}{2} - h\right)\left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]\right) \right) \right) + \\
& \frac{1}{r} \left(\cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(\frac{1}{2} - h\right)\left(-\frac{1}{2} + h\right)^2 r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right) t\right] - \right. \right. \right. \\
& \quad \left. \left. 5\left(-\frac{3}{2} - h\right)\left(\frac{3}{2} + h\right)^2 r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]\right) \right) + \\
& \quad \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2} - h\right)^2\left(-\frac{1}{2} + h\right) r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right) t\right] - \right. \right. \\
& \quad \left. \left. 7\left(-\frac{3}{2} - h\right)^2\left(\frac{3}{2} + h\right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]\right) \right) \right) \right) + \\
& \cos[t] \left(-\cos[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2} + h\right)\left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right) t\right] - \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left(-\frac{1}{2} + h\right) t\right] - 7\left(\frac{1}{2} + h\right)\left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]\right) \right) + \\
& \quad \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(\frac{1}{2} + h\right)\left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right) t\right] - \right. \\
& \quad \left. 5\left(\frac{1}{2} + h\right)\left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right) t\right]\right) - \\
& 2 \sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3\left(-\frac{1}{2} + h\right)\left(\frac{1}{2} + h\right)\left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \right. \right. \\
& \quad \left. \left. \cos\left[\left(-\frac{1}{2} + h\right) t\right] - 5\left(\frac{1}{2} + h\right)\left(\frac{3}{2} + h\right)^2 r^{-\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right) t\right]\right) \right) + \\
& \quad \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3\left(\frac{1}{2} - h\right)\left(\frac{1}{2} + h\right)\left(\frac{3}{2} + h\right) r^{-\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right) t\right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
 & \frac{1}{r} \left(\sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right)t\right] - \right. \right. \right. \\
 & \quad \left. \left. \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right)t\right] \right) + \frac{1}{3\sqrt{193\pi}} \right. \right. \\
 & \quad \left. \left. \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right)^2 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right)t\right] - 7 \left(-\frac{3}{2} - h \right)^2 \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right)t\right] \right) \right) \right) \right) + \frac{1}{r} \left(\cos[t] \left(-\cos[t] \right. \right. \\
 & \quad \left. \left. \left(\frac{\sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right)t\right] - 7 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right)t\right] \right)}{3\sqrt{193\pi}} \right) + \right. \right. \\
 & \quad \left. \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right)t\right] - \right. \right. \right. \\
 & \quad \left. \left. \left. 5 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right)t\right] \right) \right) - 3 \sin[t] \right) \right) \\
 & \quad \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right)t\right] - 5 \left(\frac{3}{2} + h \right)^2 \right. \right. \\
 & \quad \left. \left. r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right)t\right] \right) + \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \right. \right. \right. \\
 & \quad \left. \left. \left. \sin\left[\left(-\frac{1}{2} + h\right)t\right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right)t\right] \right) \right) \right) + \\
 & \quad 3 \cos[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \cos\left[\left(-\frac{1}{2} + h\right)t\right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right)t\right] \right) \right) \right) + \\
 & \quad \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right)t\right] - \right. \right. \\
 & \quad \left. \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right)t\right] \right) \right) + \sin[t] \right) \\
 & \quad \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right)^2 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right)t\right] - \right. \right. \\
 & \quad \left. \left. 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^3 r^{\frac{1}{2}+h} \cos\left[\left(\frac{3}{2} + h\right)t\right] \right) + \frac{1}{3\sqrt{193\pi}} \right. \\
 & \quad \left. \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right)^2 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right)t\right] - \right. \right. \right. \\
 & \quad \left. \left. \left. 7 \left(-\frac{3}{2} - h \right)^2 \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \sin\left[\left(\frac{3}{2} + h\right)t\right] \right) \right) \right) + \\
 & \quad \frac{1}{r} \left(\sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos\left[\left(-\frac{1}{2} + h\right)t\right] - \right. \right. \right. \\
 & \quad \left. \left. \left. 5 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \cos\left[\left(\frac{3}{2} + h\right)t\right] \right) + \right. \right. \\
 & \quad \left. \left. \frac{\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) r^{\frac{3}{2}+h} \sin\left[\left(-\frac{1}{2} + h\right)t\right] - 7 \left(-\frac{3}{2} - h \right) r^{\frac{3}{2}+h} \sin\left[\left(\frac{3}{2} + h\right)t\right] \right)}{3\sqrt{193\pi}} \right) \right) \\
 & \quad \left. - \frac{1}{r} \left(3 \cos[t] \left(\frac{1}{3\sqrt{193\pi}} \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) \right. \right. \\
 & \quad \left. \left. r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \right. \right. \\
 & \quad \left. \left. \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) - \\
 & \frac{1}{r} \left(3 \sin[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \right. \\
 & \quad \left. \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right)^2 \left(-\frac{1}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right)^2 \left(\frac{3}{2} + h \right) r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) + \\
 & \frac{1}{r} \left(\cos[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right)^2 \left(-\frac{1}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right)^2 \left(\frac{3}{2} + h \right)^2 r^{\frac{3}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \right. \right. \\
 & \quad \left. \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right)^2 \left(-\frac{1}{2} + h \right)^2 r^{\frac{3}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(-\frac{3}{2} - h \right)^2 \left(\frac{3}{2} + h \right)^2 r^{\frac{3}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) - \frac{1}{r} \left(\sin[t] \left(\sin \left[\left(-\frac{1}{2} + h \right) t \right] \right. \right. \\
 & \quad \left. \left. \left(\frac{\sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right)}{3\sqrt{193\pi}} + \right. \right. \\
 & \quad \left. \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) - \\
 & 3 \cos[t] \left(\sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \right. \\
 & \quad \left. \frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) \right) - \\
 & 3 \sin[t] \left(\frac{1}{3\sqrt{193\pi}} \left(\sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \cos \left[\left(-\frac{1}{2} + h \right) t \right] - 7 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \cos \left[\left(\frac{3}{2} + h \right) t \right] \right) + \right. \\
 & \quad \left. \sqrt{\frac{7}{579\pi}} \sqrt{\alpha} \left(3 \left(\frac{1}{2} - h \right) \left(-\frac{1}{2} + h \right) \left(\frac{3}{2} + h \right) r^{\frac{1}{2}+h} \sin \left[\left(-\frac{1}{2} + h \right) t \right] - 5 \left(-\frac{3}{2} - h \right) \left(\frac{3}{2} + h \right)^2 r^{\frac{1}{2}+h} \sin \left[\left(\frac{3}{2} + h \right) t \right] \right) \right) + \cos[t]
 \end{aligned}$$

- (10.22) Candidate W in polar coordinates ($\text{sign}(A)=\text{sign}(B)$)

$$W = \text{Sqrt}[\alpha / (193 * \text{Pi})] * r^{(3/2)} * (\text{Cos}[t/2] - (7/3) * \text{Cos}[3*t/2] + \text{Sqrt}[21] * (\text{Sin}[t/2] - (5/3) * \text{Sin}[3*t/2]))$$

$$\frac{r^{3/2} \sqrt{\alpha} (\text{Cos}[\frac{t}{2}] - \frac{7}{3} \text{Cos}[\frac{3t}{2}] + \sqrt{21} (\text{Sin}[\frac{t}{2}] - \frac{5}{3} \text{Sin}[\frac{3t}{2}]))}{\sqrt{193} \pi}$$

- (10.23) Re-normalization u of leading coefficients in W (sum of the two modes)

$$u = W /. \alpha \rightarrow 1$$

$$\frac{r^{3/2} (\text{Cos}[\frac{t}{2}] - \frac{7}{3} \text{Cos}[\frac{3t}{2}] + \sqrt{21} (\text{Sin}[\frac{t}{2}] - \frac{5}{3} \text{Sin}[\frac{3t}{2}]))}{\sqrt{193} \pi}$$

- (10.24) Re-normalization of α in Φ (difference of the two modes)

- (10.25) Graphs of candidates W and Φ in a disk (polar coordinates): Figg. 4 e 7

(* Graph in polar coordinates (true coefficients, normalized $\alpha=1$) *)

(* Graph of W in polar coordinates (true coefficients, normalized $\alpha=1$) *)

ClearAll

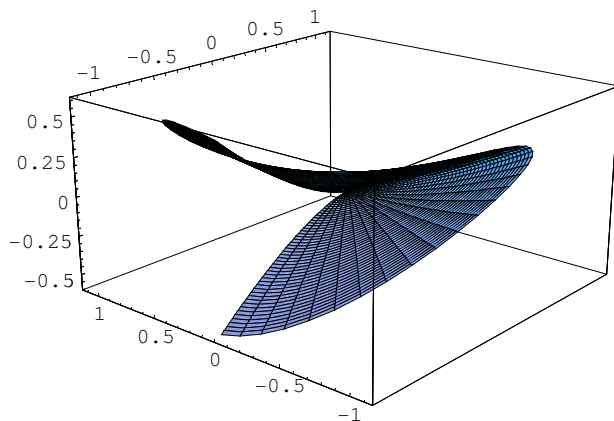
ClearAll

```

W[r_, t_] := r^(3/2) * 1 / (Sqrt[193 * Pi]) *
            ( Sqrt[21] * ( Sin[t/2] - 5/3 * Sin[3 * t/2] ) +
              ( Cos[t/2] ) - 7/3 * Cos[3 * t/2] )

ParametricPlot3D[{r * Cos[t], r * Sin[t], W[r, t], Hue[W[r, t]] },
                 {r, 0, 1.1}, {t, -Pi, Pi},
                 PlotPoints -> 50, ViewPoint -> {-2.651, -2.271, 1.2}]

```



- Graphics3D -

(* **Graph of Φ in polar coordinates**
(true coefficients, normalized $\alpha=1$) *)

```
ClearAll
```

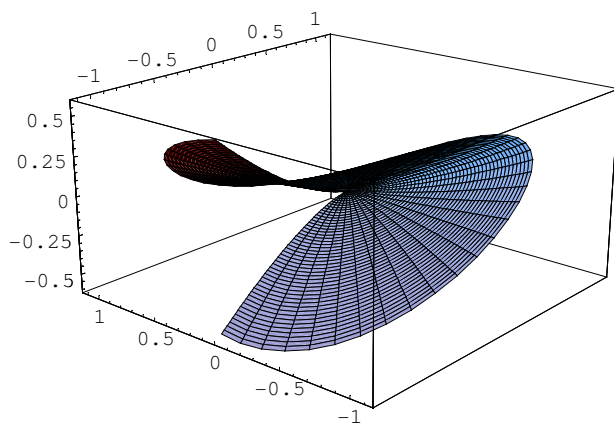
```
ClearAll
```

```

Φ[r_, t_] := r^(3/2) * 1 / (Sqrt[193 * Pi]) *
            ( Sqrt[21] * ( Sin[t/2] - 5/3 * Sin[3 * t/2] ) -
              ( Cos[t/2] ) - 7/3 * Cos[3 * t/2] )

ParametricPlot3D[{r * Cos[t], r * Sin[t], Φ[r, t], Hue[Φ[r, t]] },
                 {r, 0, 1.1}, {t, -Pi, Pi}, PlotPoints -> 50, ViewPoint -> {-2.651, -2.271, 1.2}]

```



- Graphics3D -

■ (10.26) Candidate W in cartesian coordinates

candW[x_, y_] =

$$\begin{aligned} & 1 / \text{Sqrt}[193 * 2 * \text{Pi}] (\text{Sqrt}[21] * \\ & (\text{Sign}[y] * (x^2 + y^2)^{(1/2)} * \\ & \text{Sqrt}[(x^2 + y^2)^{(1/2)} - x] - \\ & 5 / 3 * (y * \text{Sqrt}[(x^2 + y^2)^{(1/2)} + x] + \\ & x * \text{Sign}[y] * \text{Sqrt}[(x^2 + y^2)^{(1/2)} - x])) + \\ & (x^2 + y^2) * \text{Sqrt}[(x^2 + y^2)^{(1/2)} + x] - \\ & 7 / 3 (x * \text{Sqrt}[(x^2 + y^2)^{(1/2)} + x] - \\ & \text{Abs}[y] * \text{Sqrt}[(x^2 + y^2)^{(1/2)} - x])) \end{aligned}$$

$$\frac{1}{\sqrt{386} \pi} \left((x^2 + y^2) \sqrt{x + \sqrt{x^2 + y^2}} - \frac{7}{3} \left(x \sqrt{x + \sqrt{x^2 + y^2}} - \sqrt{-x + \sqrt{x^2 + y^2}} \text{Abs}[y] \right) + \right. \\ \left. \sqrt{21} \left(\sqrt{x^2 + y^2} \sqrt{-x + \sqrt{x^2 + y^2}} \text{Sign}[y] - \frac{5}{3} \left(y \sqrt{x + \sqrt{x^2 + y^2}} + x \sqrt{-x + \sqrt{x^2 + y^2}} \text{Sign}[y] \right) \right) \right)$$

■ (10.27) Candidate Φ in cartesian coordinates

candΦ[x_, y_] =

$$\begin{aligned} & 1 / \text{Sqrt}[193 * 2 * \text{Pi}] (\text{Sqrt}[21] * \\ & (\text{Sign}[y] * (x^2 + y^2)^{(1/2)} * \\ & \text{Sqrt}[(x^2 + y^2)^{(1/2)} - x] - \\ & 5 / 3 * (y * \text{Sqrt}[(x^2 + y^2)^{(1/2)} + x] + \\ & x * \text{Sign}[y] * \text{Sqrt}[(x^2 + y^2)^{(1/2)} - x])) - \\ & ((x^2 + y^2)^{(1/2)} * \text{Sqrt}[(x^2 + y^2)^{(1/2)} + x] - \\ & 7 / 3 (x * \text{Sqrt}[(x^2 + y^2)^{(1/2)} + x] - \\ & \text{Abs}[y] * \text{Sqrt}[(x^2 + y^2)^{(1/2)} - x])) \end{aligned}$$

$$\frac{1}{\sqrt{386} \pi} \left(-\sqrt{x^2 + y^2} \sqrt{x + \sqrt{x^2 + y^2}} + \frac{7}{3} \left(x \sqrt{x + \sqrt{x^2 + y^2}} - \sqrt{-x + \sqrt{x^2 + y^2}} \text{Abs}[y] \right) + \right. \\ \left. \sqrt{21} \left(\sqrt{x^2 + y^2} \sqrt{-x + \sqrt{x^2 + y^2}} \text{Sign}[y] - \frac{5}{3} \left(y \sqrt{x + \sqrt{x^2 + y^2}} + x \sqrt{-x + \sqrt{x^2 + y^2}} \text{Sign}[y] \right) \right) \right)$$

■ (10.28) Graphs of candidate W in a square domain : Fig.6

```

candW[x_, y_] =
  1 / Sqrt[193 * 2 * Pi] (Sqrt[21] *
    (Sign[y] * (x^2 + y^2)^(1/2) *
      Sqrt[(x^2 + y^2)^(1/2) - x] -
      5 / 3 * (y * Sqrt[(x^2 + y^2)^(1/2) + x] +
        x * Sign[y] * Sqrt[(x^2 + y^2)^(1/2) - x])) +
    (x^2 + y^2) * Sqrt[(x^2 + y^2)^(1/2) + x] -
    7 / 3 (x * Sqrt[(x^2 + y^2)^(1/2) + x] -
      Abs[y] * Sqrt[(x^2 + y^2)^(1/2) - x]))

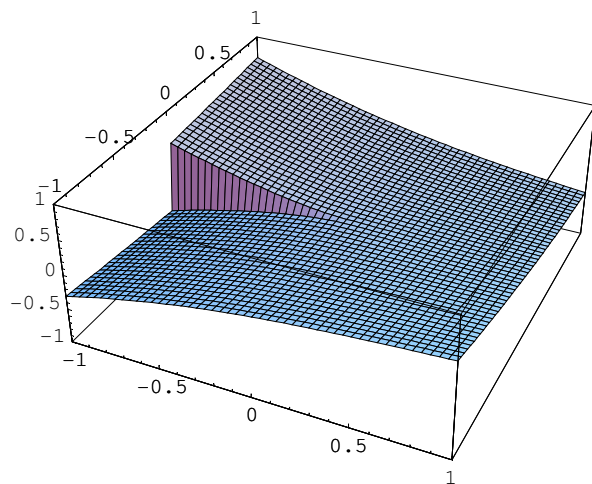
```

$$\frac{1}{\sqrt{386 \pi}} \left((x^2 + y^2) \sqrt{x + \sqrt{x^2 + y^2}} - \frac{7}{3} \left(x \sqrt{x + \sqrt{x^2 + y^2}} - \sqrt{-x + \sqrt{x^2 + y^2}} \text{Abs}[y] \right) + \sqrt{21} \left(\sqrt{x^2 + y^2} \sqrt{-x + \sqrt{x^2 + y^2}} \text{Sign}[y] - \frac{5}{3} \left(y \sqrt{x + \sqrt{x^2 + y^2}} + x \sqrt{-x + \sqrt{x^2 + y^2}} \text{Sign}[y] \right) \right) \right)$$

```

Plot3D[candW[x, y], {x, -1, 1}, {y, -1, 1},
  PlotPoints -> 50, PlotRange -> {-1, 1},
  ViewPoint -> {1.196, -2.581, 1.833}]

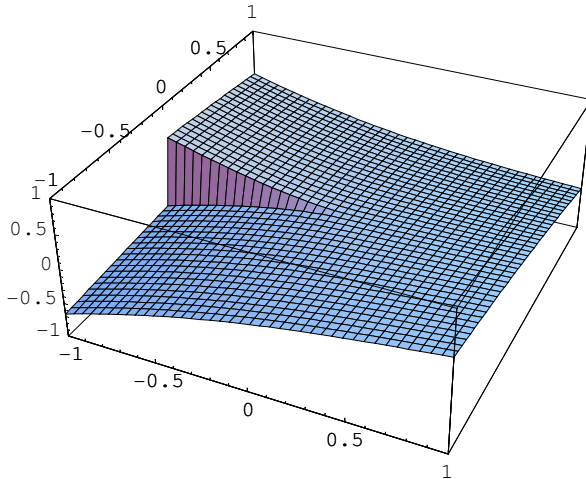
```



- SurfaceGraphics -

■ (10.29) Graphs of candidates Φ in a square domain : Fig.9

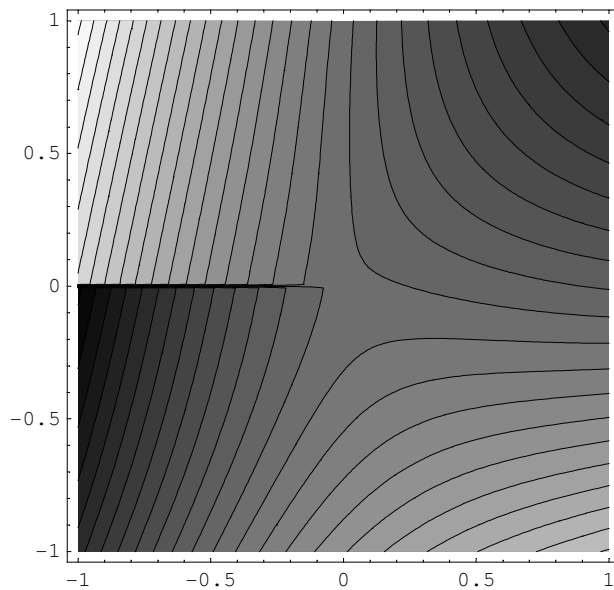
```
(* Generazione della Fig. 5 di BZEE, caption: Level lines of W *)
Plot3D[cand@[x, y], {x, -1, 1}, {y, -1, 1},
  PlotPoints -> 40, PlotRange -> {-1, 1}, ViewPoint -> {1.196, -2.581, 1.833}]
```



- SurfaceGraphics -

■ (10.30) Level lines of candidate W (cartesian coordinates): Fig. 5

```
ContourPlot[candW[x, y], {x, -1, 1},
  {y, -1, 1}, PlotPoints -> 140, Contours -> 30]
```



- ContourGraphics -

■ (10.31) Level lines of candidate Φ (cartesian coordinates): Fig. 8

```

cand̄[x_, y_] =
  1 / Sqrt[193 * 2 * Pi] (Sqrt[21] *
    (Sign[y] * (x^2 + y^2)^(1/2) *
      Sqrt[(x^2 + y^2)^(1/2) - x] -
      5 / 3 * (y * Sqrt[(x^2 + y^2)^(1/2) + x] +
        x * Sign[y] * Sqrt[(x^2 + y^2)^(1/2) - x])) -
    ((x^2 + y^2)^(1/2) * Sqrt[(x^2 + y^2)^(1/2) + x] -
      7 / 3 (x * Sqrt[(x^2 + y^2)^(1/2) + x] -
        Abs[y] * Sqrt[(x^2 + y^2)^(1/2) - x])))

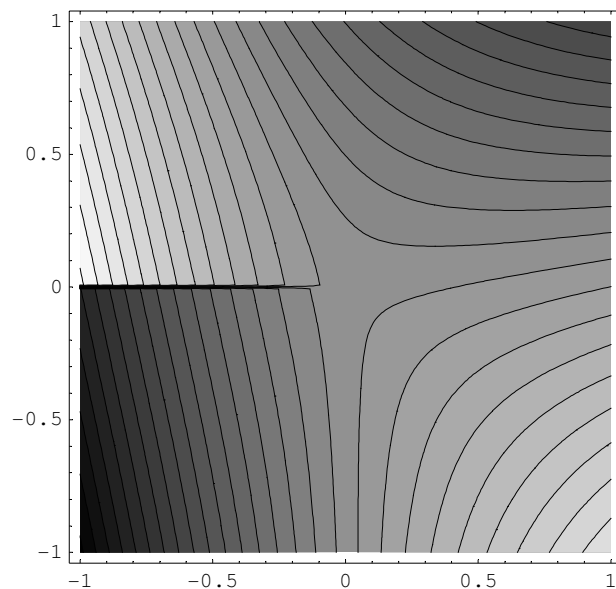
```

$$\frac{1}{\sqrt{386} \pi} \left(-\sqrt{x^2 + y^2} \sqrt{x + \sqrt{x^2 + y^2}} + \frac{7}{3} \left(x \sqrt{x + \sqrt{x^2 + y^2}} - \sqrt{-x + \sqrt{x^2 + y^2}} \text{Abs}[y] \right) + \sqrt{21} \left(\sqrt{x^2 + y^2} \sqrt{-x + \sqrt{x^2 + y^2}} \text{Sign}[y] - \frac{5}{3} \left(y \sqrt{x + \sqrt{x^2 + y^2}} + x \sqrt{-x + \sqrt{x^2 + y^2}} \text{Sign}[y] \right) \right) \right)$$

```

ContourPlot[cand̄[x, y], {x, -1, 1},
  {y, -1, 1}, PlotPoints → 140, Contours → 30]

```



• ContourGraphics •

ClearAll

ClearAll

COMPLEX BASIS : H^2 complete set for V (redundant in L^2) : $\{\{v_k\}, \{z_k\}\}$ $k \in \mathbb{Z}$ (indexed by k here, parametrized by h in the paper)

and scalar products $(\cdot, \cdot)_\rho$ in $H^2(B_\rho \setminus \Gamma)$

■ Differential operators

```

dx[u_] := Cos[ϑ] * D[u, r] - Sin[ϑ] * D[u, ϑ] / r
dy[u_] := Sin[ϑ] * D[u, r] + Cos[ϑ] * D[u, ϑ] / r
dxx[u_] := dx[dx[u]]
dyy[u_] := dy[dy[u]]
dxy[u_] := dx[dy[u]]
dyx[u_] := dy[dx[u]]
lap[u_] := dxx[u] + dyy[u]
hes2[u_] := dxx[u]^2 + dyy[u]^2 + 2 * dxy[u]^2
prodscalH2[u_, v_] := Integrate[
  (dxx[u] * dxx[v] + dyy[u] * dyy[v] + 2 dxy[u] * dxy[v]) * r, {r, 0, R}, {ϑ, -Pi, Pi}]

prodscalHERMITIANOH2[u_, v_] := Integrate[
  (Simplify[dxx[u] * Conjugate[dxx[v]]) + Simplify[dyy[u] * Conjugate[dyy[v]]) +
  Simplify[2 dxy[u] * Conjugate[dxy[v]]) * r, {r, 0, R},
  {ϑ, -Pi, Pi}, Assumptions -> {k ∈ Integers, ϑ ∈ Reals, r ∈ Reals}]

lap[lap[r^p * ψ[ϑ]]]

Simplify[%]

r^-4+p ((-2 + p)^2 p^2 ψ[ϑ] + 2 (2 - 2 p + p^2) ψ''[ϑ] + ψ^(4)[ϑ])

```

■ v_k, z_k (L^2) Redundant complete sets (parameter h in the paper)

■ v_k, z_k (H^2) Minimal complete sets (parameter h in the paper)

```

v[k_] := r^(Abs[k] - 1/2) * E^(I * (k - 1/2) * ϑ)
z[k_] := r^(Abs[k] + 3/2) * E^(I * (k - 1/2) * ϑ)
cv[k_] := r^(Abs[k] - 1/2) * E^(-I * (k - 1/2) * ϑ)
cz[k_] := r^(Abs[k] + 3/2) * E^(-I * (k - 1/2) * ϑ)

hes11[u_] := dxx[u]
hes22[u_] := dyy[u]
hes12[u_] := dxy[u]
hes21[u_] := dyx[u]

```

(10.32) HESSIAN OF FUNCTIONS OF FIRST KIND (V_k)

hes11[v[k]]

$$\begin{aligned} & \text{Cos}[\vartheta] \left(e^{i(-\frac{1}{2}+k)\vartheta} r^{-\frac{5}{2}+\text{Abs}[k]} \left(-\frac{3}{2} + \text{Abs}[k] \right) \left(-\frac{1}{2} + \text{Abs}[k] \right) \text{Cos}[\vartheta] - \right. \\ & \quad \left. i e^{i(-\frac{1}{2}+k)\vartheta} \left(-\frac{1}{2} + k \right) r^{-\frac{5}{2}+\text{Abs}[k]} \left(-\frac{3}{2} + \text{Abs}[k] \right) \text{Sin}[\vartheta] \right) - \\ & \frac{1}{r} \left(\text{Sin}[\vartheta] \left(-i e^{i(-\frac{1}{2}+k)\vartheta} \left(-\frac{1}{2} + k \right) r^{-\frac{3}{2}+\text{Abs}[k]} \text{Cos}[\vartheta] + \right. \right. \\ & \quad \left. \left. i e^{i(-\frac{1}{2}+k)\vartheta} \left(-\frac{1}{2} + k \right) r^{-\frac{3}{2}+\text{Abs}[k]} \left(-\frac{1}{2} + \text{Abs}[k] \right) \text{Cos}[\vartheta] + \right. \right. \\ & \quad \left. \left. e^{i(-\frac{1}{2}+k)\vartheta} \left(-\frac{1}{2} + k \right)^2 r^{-\frac{3}{2}+\text{Abs}[k]} \text{Sin}[\vartheta] - e^{i(-\frac{1}{2}+k)\vartheta} r^{-\frac{3}{2}+\text{Abs}[k]} \left(-\frac{1}{2} + \text{Abs}[k] \right) \text{Sin}[\vartheta] \right) \right) \end{aligned}$$

Simplify[%, {k} ∈ Integers]

$$\begin{aligned} & \frac{1}{4} e^{i(-\frac{1}{2}+k)\vartheta} r^{-\frac{5}{2}+\text{Abs}[k]} (2k + (3 - 2k + 4k^2) \text{Cos}[2\vartheta] - 6i \text{Cos}[\vartheta] \text{Sin}[\vartheta] + \\ & \quad 6ik \text{Sin}[2\vartheta] - 2\text{Abs}[k] (1 + 3 \text{Cos}[2\vartheta] + i(-1 + 2k) \text{Sin}[2\vartheta])) \end{aligned}$$

TeXForm[%]

$$\begin{aligned} & \frac{1}{4} e^{i(-\frac{1}{2}+k)\vartheta} r^{-\frac{5}{2}+\text{Abs}[k]} (2k + (3 - 2k + 4k^2) \text{Cos}[2\vartheta] - 6i \text{Cos}[\vartheta] \text{Sin}[\vartheta] + \\ & \quad 6ik \text{Sin}[2\vartheta] - 2\text{Abs}[k] (1 + 3 \text{Cos}[2\vartheta] + i(-1 + 2k) \text{Sin}[2\vartheta])) \end{aligned}$$

hes22[v[k]]

$$\begin{aligned} & \frac{1}{r} \left(\text{Cos}[\vartheta] \left(-e^{i(-\frac{1}{2}+k)\vartheta} \left(-\frac{1}{2} + k \right)^2 r^{-\frac{3}{2}+\text{Abs}[k]} \text{Cos}[\vartheta] + \right. \right. \\ & \quad \left. \left. e^{i(-\frac{1}{2}+k)\vartheta} r^{-\frac{3}{2}+\text{Abs}[k]} \left(-\frac{1}{2} + \text{Abs}[k] \right) \text{Cos}[\vartheta] - i e^{i(-\frac{1}{2}+k)\vartheta} \left(-\frac{1}{2} + k \right) r^{-\frac{3}{2}+\text{Abs}[k]} \text{Sin}[\vartheta] + \right. \right. \\ & \quad \left. \left. i e^{i(-\frac{1}{2}+k)\vartheta} \left(-\frac{1}{2} + k \right) r^{-\frac{3}{2}+\text{Abs}[k]} \left(-\frac{1}{2} + \text{Abs}[k] \right) \text{Sin}[\vartheta] \right) \right) + \\ & \text{Sin}[\vartheta] \left(i e^{i(-\frac{1}{2}+k)\vartheta} \left(-\frac{1}{2} + k \right) r^{-\frac{5}{2}+\text{Abs}[k]} \left(-\frac{3}{2} + \text{Abs}[k] \right) \text{Cos}[\vartheta] + \right. \\ & \quad \left. e^{i(-\frac{1}{2}+k)\vartheta} r^{-\frac{5}{2}+\text{Abs}[k]} \left(-\frac{3}{2} + \text{Abs}[k] \right) \left(-\frac{1}{2} + \text{Abs}[k] \right) \text{Sin}[\vartheta] \right) \end{aligned}$$

Simplify[%, {k} ∈ Integers]

$$\begin{aligned} & -\frac{1}{4} e^{i(-\frac{1}{2}+k)\vartheta} r^{-\frac{5}{2}+\text{Abs}[k]} (-2k + (3 - 2k + 4k^2) \text{Cos}[2\vartheta] - \\ & \quad 6i \text{Cos}[\vartheta] \text{Sin}[\vartheta] + 6ik \text{Sin}[2\vartheta] + \text{Abs}[k] (2 - 6 \text{Cos}[2\vartheta] - 2i(-1 + 2k) \text{Sin}[2\vartheta])) \end{aligned}$$

TeXForm[%]

```

\frac{e^{\i \left( -\left( \frac{1}{2} \right) + k \right) \vartheta},
r^{-\left( \frac{5}{2} \right) + \text{Abs}(k)},
\left( \left( -3 \i \right) \left( -1 + 2k \right) \cos(2\vartheta) + \left( 2 \i \right) \text{Abs}(k) \left( -1 + 2k \right) \cos(2\vartheta) + \left( 3 \i \right) \sin(2\vartheta) \right) + \left( 3 - 2k + 4k^2 \right) \sin(2\vartheta)}{4}

```

(10.33) HESSIAN OF FUNCTIONS OF SECOND KIND (Z_k)
hes11[z[k]]

$$\begin{aligned} & \text{Cos}[\vartheta] \left(e^{\i \left(-\frac{1}{2}+k\right)\vartheta} r^{-\frac{1}{2}+\text{Abs}[k]} \left(\frac{1}{2}+\text{Abs}[k]\right) \left(\frac{3}{2}+\text{Abs}[k]\right) \text{Cos}[\vartheta] - \right. \\ & \quad \left. \i e^{\i \left(-\frac{1}{2}+k\right)\vartheta} \left(-\frac{1}{2}+k\right) r^{-\frac{1}{2}+\text{Abs}[k]} \left(\frac{1}{2}+\text{Abs}[k]\right) \text{Sin}[\vartheta] \right) - \\ & \frac{1}{r} \left(\text{Sin}[\vartheta] \left(-\i e^{\i \left(-\frac{1}{2}+k\right)\vartheta} \left(-\frac{1}{2}+k\right) r^{\frac{1}{2}+\text{Abs}[k]} \text{Cos}[\vartheta] + \right. \right. \\ & \quad \left. \i e^{\i \left(-\frac{1}{2}+k\right)\vartheta} \left(-\frac{1}{2}+k\right) r^{\frac{1}{2}+\text{Abs}[k]} \left(\frac{3}{2}+\text{Abs}[k]\right) \text{Cos}[\vartheta] + \right. \\ & \quad \left. \left. e^{\i \left(-\frac{1}{2}+k\right)\vartheta} \left(-\frac{1}{2}+k\right)^2 r^{\frac{1}{2}+\text{Abs}[k]} \text{Sin}[\vartheta] - e^{\i \left(-\frac{1}{2}+k\right)\vartheta} r^{\frac{1}{2}+\text{Abs}[k]} \left(\frac{3}{2}+\text{Abs}[k]\right) \text{Sin}[\vartheta] \right) \right) \end{aligned}$$
Simplify[%, {k} ∈ Integers]

$$\frac{1}{4} e^{\i \left(-\frac{1}{2}+k\right)\vartheta} r^{-\frac{1}{2}+\text{Abs}[k]} (4 + 2k + (-1 - 2k + 4k^2) \text{Cos}[2\vartheta] + 2\i \text{Cos}[\vartheta] \text{Sin}[\vartheta] - 4\i k \text{Cos}[\vartheta] \text{Sin}[\vartheta] + 2\text{Abs}[k] (3 + \text{Cos}[2\vartheta] - \i (-1 + 2k) \text{Sin}[2\vartheta]))$$
TeXForm[%]

```

\frac{e^{\i \left( -\left( \frac{1}{2} \right) + k \right) \vartheta},
r^{-\left( \frac{1}{2} \right) + \text{Abs}(k)},
\left( 4 + 2k + \left( -1 - 2k + 4k^2 \right) \cos(2\vartheta) + \left( 2 \i \right) \cos(\vartheta) \sin(\vartheta) + \left( 2 \i \right) k \cos(\vartheta) \sin(\vartheta) - \left( 4 \i \right) k \cos(\vartheta) \sin(\vartheta) + 2 \text{Abs}(k) \left( 3 + \cos(2\vartheta) - \i \left( -1 + 2k \right) \sin(2\vartheta) \right) \right)}{4}

```

hes22[z[k]]

$$\begin{aligned} & \frac{1}{r} \left(\text{Cos}[\vartheta] \left(-e^{\i \left(-\frac{1}{2}+k\right)\vartheta} \left(-\frac{1}{2}+k\right)^2 r^{\frac{1}{2}+\text{Abs}[k]} \text{Cos}[\vartheta] + \right. \right. \\ & \quad \left. e^{\i \left(-\frac{1}{2}+k\right)\vartheta} r^{\frac{1}{2}+\text{Abs}[k]} \left(\frac{3}{2}+\text{Abs}[k]\right) \text{Cos}[\vartheta] - \i e^{\i \left(-\frac{1}{2}+k\right)\vartheta} \left(-\frac{1}{2}+k\right) r^{\frac{1}{2}+\text{Abs}[k]} \text{Sin}[\vartheta] + \right. \\ & \quad \left. \left. \i e^{\i \left(-\frac{1}{2}+k\right)\vartheta} \left(-\frac{1}{2}+k\right) r^{\frac{1}{2}+\text{Abs}[k]} \left(\frac{3}{2}+\text{Abs}[k]\right) \text{Sin}[\vartheta] \right) \right) + \\ & \text{Sin}[\vartheta] \left(\i e^{\i \left(-\frac{1}{2}+k\right)\vartheta} \left(-\frac{1}{2}+k\right) r^{-\frac{1}{2}+\text{Abs}[k]} \left(\frac{1}{2}+\text{Abs}[k]\right) \text{Cos}[\vartheta] + \right. \\ & \quad \left. e^{\i \left(-\frac{1}{2}+k\right)\vartheta} r^{-\frac{1}{2}+\text{Abs}[k]} \left(\frac{1}{2}+\text{Abs}[k]\right) \left(\frac{3}{2}+\text{Abs}[k]\right) \text{Sin}[\vartheta] \right) \end{aligned}$$
Simplify[%, {k} ∈ Integers]

$$\frac{1}{4} e^{\i \left(-\frac{1}{2}+k\right)\vartheta} r^{-\frac{1}{2}+\text{Abs}[k]} (4 + 2k + (1 + 2k - 4k^2) \text{Cos}[2\vartheta] - \i \text{Sin}[2\vartheta] + 2\i k \text{Sin}[2\vartheta] - 2\text{Abs}[k] (-3 + \text{Cos}[2\vartheta] - \i (-1 + 2k) \text{Sin}[2\vartheta]))$$

TeXForm[%]

```
\frac{e^{\i \operatorname{Imag} \left( -\left( \frac{1}{2} \right) + k \right) \vartheta}, \operatorname{Abs}(k)}{r^{-\left( \frac{1}{2} \right) + \operatorname{Abs}(k)} \left( 4 + 2k + \left( 1 + 2k - 4k^2 \right) \cos(2\vartheta) - \i \sin(2\vartheta) + \left( 2\i \operatorname{Imag} \right) k \sin(2\vartheta) - 2\operatorname{Abs}(k) \left( -3 + \cos(2\vartheta) - \i \operatorname{Imag} \left( -1 + 2k \right) \sin(2\vartheta) \right) \right)}
```

hes12[z[k]]

$$-\frac{1}{r} \left(\sin[\vartheta] \left(-e^{\i \left(-\frac{1}{2}+k\right)\vartheta} \left(-\frac{1}{2}+k\right)^2 r^{\frac{1}{2}+\operatorname{Abs}[k]} \cos[\vartheta] + e^{\i \left(-\frac{1}{2}+k\right)\vartheta} r^{\frac{1}{2}+\operatorname{Abs}[k]} \left(\frac{3}{2}+\operatorname{Abs}[k]\right) \cos[\vartheta] - \i e^{\i \left(-\frac{1}{2}+k\right)\vartheta} \left(-\frac{1}{2}+k\right) r^{\frac{1}{2}+\operatorname{Abs}[k]} \sin[\vartheta] + \i e^{\i \left(-\frac{1}{2}+k\right)\vartheta} \left(-\frac{1}{2}+k\right) r^{\frac{1}{2}+\operatorname{Abs}[k]} \left(\frac{3}{2}+\operatorname{Abs}[k]\right) \sin[\vartheta] \right) + \cos[\vartheta] \left(\i e^{\i \left(-\frac{1}{2}+k\right)\vartheta} \left(-\frac{1}{2}+k\right) r^{-\frac{1}{2}+\operatorname{Abs}[k]} \left(\frac{1}{2}+\operatorname{Abs}[k]\right) \cos[\vartheta] + e^{\i \left(-\frac{1}{2}+k\right)\vartheta} r^{-\frac{1}{2}+\operatorname{Abs}[k]} \left(\frac{1}{2}+\operatorname{Abs}[k]\right) \left(\frac{3}{2}+\operatorname{Abs}[k]\right) \sin[\vartheta] \right) \right)$$

Simplify[%, {k} ∈ Integers]

$$\frac{1}{4} e^{\i \left(-\frac{1}{2}+k\right)\vartheta} r^{-\frac{1}{2}+\operatorname{Abs}[k]} \left(\i (-1+2k) \cos[2\vartheta] + (-1-2k+4k^2) \sin[2\vartheta] + 2\operatorname{Abs}[k] \left(\i (-1+2k) \cos[2\vartheta] + \sin[2\vartheta] \right) \right)$$

TeXForm[%]

```
\frac{e^{\i \operatorname{Imag} \left( -\left( \frac{1}{2} \right) + k \right) \vartheta}, \operatorname{Abs}(k)}{r^{-\left( \frac{1}{2} \right) + \operatorname{Abs}(k)} \left( \i \operatorname{Imag} \left( -1 + 2k \right) \cos(2\vartheta) + \left( -1 - 2k + 4k^2 \right) \sin(2\vartheta) + 2\operatorname{Abs}(k) \left( \i \operatorname{Imag} \left( -1 + 2k \right) \cos(2\vartheta) + \sin(2\vartheta) \right) \right)}
```

hes21[z[k]]

$$\sin[\vartheta] \left(e^{\i \left(-\frac{1}{2}+k\right)\vartheta} r^{-\frac{1}{2}+\operatorname{Abs}[k]} \left(\frac{1}{2}+\operatorname{Abs}[k]\right) \left(\frac{3}{2}+\operatorname{Abs}[k]\right) \cos[\vartheta] - \i e^{\i \left(-\frac{1}{2}+k\right)\vartheta} \left(-\frac{1}{2}+k\right) r^{-\frac{1}{2}+\operatorname{Abs}[k]} \left(\frac{1}{2}+\operatorname{Abs}[k]\right) \sin[\vartheta] + \frac{1}{r} \left(\cos[\vartheta] \left(-\i e^{\i \left(-\frac{1}{2}+k\right)\vartheta} \left(-\frac{1}{2}+k\right) r^{\frac{1}{2}+\operatorname{Abs}[k]} \cos[\vartheta] + \i e^{\i \left(-\frac{1}{2}+k\right)\vartheta} \left(-\frac{1}{2}+k\right) r^{\frac{1}{2}+\operatorname{Abs}[k]} \left(\frac{3}{2}+\operatorname{Abs}[k]\right) \cos[\vartheta] + e^{\i \left(-\frac{1}{2}+k\right)\vartheta} \left(-\frac{1}{2}+k\right)^2 r^{\frac{1}{2}+\operatorname{Abs}[k]} \sin[\vartheta] - e^{\i \left(-\frac{1}{2}+k\right)\vartheta} r^{\frac{1}{2}+\operatorname{Abs}[k]} \left(\frac{3}{2}+\operatorname{Abs}[k]\right) \sin[\vartheta] \right) \right)$$

Simplify[%, {k} ∈ Integers]

$$\frac{1}{4} e^{\i \left(-\frac{1}{2}+k\right)\vartheta} r^{-\frac{1}{2}+\operatorname{Abs}[k]} \left(\i (-1+2k) \cos[2\vartheta] + (-1-2k+4k^2) \sin[2\vartheta] + 2\operatorname{Abs}[k] \left(\i (-1+2k) \cos[2\vartheta] + \sin[2\vartheta] \right) \right)$$

TeXForm[%]

```
\frac{e^{\mathrm{i} \operatorname{Im} \left( -\left( \frac{1}{2} \right) + k \right) \vartheta }, r^{-\left( \frac{1}{2} \right) + \operatorname{Abs}(k)} \left( \operatorname{Im} \left( -1 + 2k \right) \cos (2\vartheta ) + \left( -1 - 2k + 4k^2 \right) \sin (2\vartheta ) + 2 \operatorname{Abs}(k) \left( \operatorname{Im} \left( -1 + 2k \right) \cos (2\vartheta ) + \sin (2\vartheta ) \right) \right) \right) \right) \{4}
```

(10.34) COMPUTATION OF SCALAR PRODUCTS IN H^2 FOR $\{v_k\}$, $\{z_k\}$ $k \in \mathbb{Z}$

prodscalH2[v[k], cv[h]]

$$\frac{1}{2(h-k)} \left((9 - 12h + 3h^2 - 12k + 22hk - 4h^2k + 3k^2 - 4hk^2 + 4h^2k^2 + 3h \operatorname{Conjugate}[h] - 2 \operatorname{Abs}[k] (6 - 5h + 2h^2 - 3k + 6hk + 4h \operatorname{Conjugate}[h]) + 3k \operatorname{Conjugate}[k] + 4hk \operatorname{Conjugate}[h] \operatorname{Conjugate}[k] + 2 \operatorname{Abs}[h] (-6 + 3h + 5k - 6hk - 2k^2 + (11 - 2h - 2k + 4hk) \operatorname{Abs}[k] - 4k \operatorname{Conjugate}[k])) \right)$$

If $\left[\operatorname{Re}[h+k] > 3, \frac{R^{-3+\operatorname{Abs}[h]+\operatorname{Abs}[k]}}{-3+\operatorname{Abs}[h]+\operatorname{Abs}[k]}, \operatorname{Integrate}[r^{-4+\operatorname{Abs}[h]+\operatorname{Abs}[k]}, \{r, 0, R\}, \operatorname{Assumptions} \rightarrow \operatorname{Re}[h+k] \leq 3] \right] \operatorname{Sin}[(h-k)\pi]$

Simplify[%, {h, k} ∈ Integers]

0

prodscalH2[v[k], cz[h]]

$$\frac{1}{2(h-k)} \left((-3 + 3h^2 + 8k - 2hk - 4h^2k - 5k^2 - 4hk^2 + 4h^2k^2 + 3h \operatorname{Conjugate}[h] - 2 \operatorname{Abs}[k] ((-1 + 2h)(h-k) + 4h \operatorname{Conjugate}[h]) + 3k \operatorname{Conjugate}[k] + 4hk \operatorname{Conjugate}[h] \operatorname{Conjugate}[k] + 2 \operatorname{Abs}[h] (3h + 5k - 6hk - 2k^2 + (-5 - 2h - 2k + 4hk) \operatorname{Abs}[k] + 4k \operatorname{Conjugate}[k])) \right)$$

If $\left[\operatorname{Re}[h+k] > 1, \frac{R^{-1+\operatorname{Abs}[h]+\operatorname{Abs}[k]}}{-1+\operatorname{Abs}[h]+\operatorname{Abs}[k]}, \operatorname{Integrate}[r^{-2+\operatorname{Abs}[h]+\operatorname{Abs}[k]}, \{r, 0, R\}, \operatorname{Assumptions} \rightarrow \operatorname{Re}[h+k] \leq 1] \right] \operatorname{Sin}[(h-k)\pi]$

Simplify[%, {h, k} ∈ Integers]

0

prodscalH2[z[k], cz[h]]

$$\frac{1}{2(h-k)} \left((9 + 4h - 5h^2 + 4k + 6hk - 4h^2k - 5k^2 - 4hk^2 + 4h^2k^2 + 3h \operatorname{Conjugate}[h] + 2 \operatorname{Abs}[k] (6 + h - 2h^2 - k + 2hk + 4h \operatorname{Conjugate}[h]) + 3k \operatorname{Conjugate}[k] + 4hk \operatorname{Conjugate}[h] \operatorname{Conjugate}[k] + 2 \operatorname{Abs}[h] (6 - h + k + 2hk - 2k^2 + (11 - 2h - 2k + 4hk) \operatorname{Abs}[k] + 4k \operatorname{Conjugate}[k])) \right)$$

If $\left[\operatorname{Re}[h+k] > -1, \frac{R^{1+\operatorname{Abs}[h]+\operatorname{Abs}[k]}}{1+\operatorname{Abs}[h]+\operatorname{Abs}[k]}, \operatorname{Integrate}[r^{\operatorname{Abs}[h]+\operatorname{Abs}[k]}, \{r, 0, R\}, \operatorname{Assumptions} \rightarrow \operatorname{Re}[h+k] \leq -1] \right] \operatorname{Sin}[(h-k)\pi]$

```
Simplify[%, {h, k} ∈ Integers]
```

```
0
```

REAL BASIS : H^2 complete set for V (redundant in L^2) : $\{ \{f1_k\}, \{f2_k\}, \{f3_k\}, \{f4_k\} \}$, $k \in \mathbb{Z}$
(indexed by k here, parametrized by h in the paper)

and scalar products $(., .)_\rho$ in $H^2(B_\rho \setminus \Gamma)$

```
ClearAll
```

```
ClearAll
```

```
dx[u_] := Cos[t] * D[u, r] - Sin[t] * D[u, t] / r
dy[u_] := Sin[t] * D[u, r] + Cos[t] * D[u, t] / r
dxx[u_] := dx[dx[u]]
dyy[u_] := dy[dy[u]]
dxy[u_] := dx[dy[u]]
dyx[u_] := dx[dy[u]]
lap[u_] := dxx[u] + dyy[u]
hes2[u_] := dxx[u]^2 + dyy[u]^2 + 2 * dxy[u]^2
prodscalH2[u_, v_] := Integrate[(Simplify[dxx[u]] * Simplify[dxx[v]] +
  Simplify[dyy[u]] * Simplify[dyy[v]] + 2 Simplify[dxy[u]] * Simplify[dxy[v]]) * r,
  {r, 0, R}, {t, -Pi, Pi}]
```

- $f1_k, f2_k, f3_k, f4_k$ (L^2) Redundant complete set (parameter h in the paper)
- $f1_k, f2_k, f3_k, f4_k$ (H^2) Minimal complete set (parameter h in the paper)

```
f1[k_] := r^(k + 3/2) Cos[(k + 3/2) t]
```

```
f2[k_] := r^(k + 3/2) Sin[(k + 3/2) t]
```

```
f3[k_] := r^(k + 3/2) Cos[(k - 1/2) t]
```

```
f4[k_] := r^(k + 3/2) Sin[(k - 1/2) t]
```

```
hes11[u_] := dxx[u]
```

```
hes22[u_] := dyy[u]
```

```
hes12[u_] := dxy[u]
```

```
General::spell11 : Possible spelling error: new symbol name "hes12" is similar to existing symbol "hes2".
```

```
hes21[u_] := dyx[u]
```

```
General::spell11 : Possible spelling error: new symbol name "hes21" is similar to existing symbol "hes12".
```

(10.35) HESSIAN OF FUNCTIONS OF TYPE f OF FIRST KIND $(f1_k)$

hes11[f1[k]]

$$\begin{aligned}
 & -\frac{1}{r} \left(\sin[t] \left(-\left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right) t\right] \sin[t] - \right. \right. \\
 & \quad \left. \left(-\frac{3}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right) t\right] \sin[t] - \left(-\frac{3}{2} - k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] + \right. \\
 & \quad \left. \left. \left(-\frac{3}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] \right) \right) + \\
 & \cos[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right) t\right] - \right. \\
 & \quad \left. \left(-\frac{3}{2} - k\right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] \right)
 \end{aligned}$$

Simplify[%]

$$\frac{1}{4} (3 + 8k + 4k^2) r^{-\frac{1}{2}+k} \cos\left[\frac{1}{2} (t - 2kt)\right]$$

hes22[f1[k]]

$$\begin{aligned}
 & \sin[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right) t\right] \sin[t] + \right. \\
 & \quad \left. \left(-\frac{3}{2} - k\right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] + \frac{1}{r} \left(\cos[t] \right. \right. \\
 & \quad \left. \left. \left(\left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right) t\right] + \left(-\frac{3}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right) t\right] - \right. \right. \right. \\
 & \quad \left. \left. \left. \left(-\frac{3}{2} - k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] + \left(-\frac{3}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] \right) \right) \right)
 \end{aligned}$$

Simplify[%]

$$-\frac{1}{4} (3 + 8k + 4k^2) r^{-\frac{1}{2}+k} \cos\left[\frac{1}{2} (t - 2kt)\right]$$

hes12[f1[k]]

$$\begin{aligned}
 & \cos[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right) t\right] \sin[t] + \right. \\
 & \quad \left. \left(-\frac{3}{2} - k\right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] - \frac{1}{r} \left(\sin[t] \right. \right. \\
 & \quad \left. \left. \left(\left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right) t\right] + \left(-\frac{3}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right) t\right] - \right. \right. \right. \\
 & \quad \left. \left. \left. \left(-\frac{3}{2} - k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] + \left(-\frac{3}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] \right) \right) \right)
 \end{aligned}$$

Simplify[%]

$$\frac{1}{4} (3 + 8k + 4k^2) r^{-\frac{1}{2}+k} \sin\left[\frac{1}{2} (t - 2kt)\right]$$

hes21[f1[k]]

$$\begin{aligned}
 & \cos[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right) t\right] \sin[t] + \right. \\
 & \quad \left. \left(-\frac{3}{2} - k\right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] - \frac{1}{r} \left(\sin[t] \right. \right. \\
 & \quad \left. \left. \left(\left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right) t\right] + \left(-\frac{3}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right) t\right] - \right. \right. \right. \\
 & \quad \left. \left. \left. \left(-\frac{3}{2} - k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] + \left(-\frac{3}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] \right) \right) \right)
 \end{aligned}$$

Simplify[%]

$$\frac{1}{4} (3 + 8k + 4k^2) r^{-\frac{1}{2}+k} \sin\left[\frac{1}{2} (t - 2kt)\right]$$

**(10.36) HESSIAN OF FUNCTIONS OF TYPE f OF SECOND KIND
(f_{2k})**

hes11[f2[k]]

$$\begin{aligned} & \cos[t] \left(-\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right)t\right] \sin[t] + \right. \\ & \quad \left. \left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right)t\right] \right) - \\ & \frac{1}{r} \left(\sin[t] \left(-\left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right)t\right] + \left(\frac{3}{2} + k\right)^2 r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right)t\right] - \right. \right. \\ & \quad \left. \left. \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right)t\right] - \left(-\frac{3}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right)t\right] \right) \right) \end{aligned}$$

Simplify[%]

$$-\frac{1}{4} (3 + 8k + 4k^2) r^{-\frac{1}{2}+k} \sin\left[\frac{1}{2} (t - 2kt)\right]$$

hes22[f2[k]]

$$\begin{aligned} & \frac{1}{r} \left(\cos[t] \left(-\left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right)t\right] \sin[t] + \left(\frac{3}{2} + k\right)^2 r^{\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right)t\right] \sin[t] + \right. \right. \\ & \quad \left. \left. \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right)t\right] + \left(-\frac{3}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right)t\right] \right) \right) + \\ & \sin[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right)t\right] + \right. \\ & \quad \left. \left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right)t\right] \right) \end{aligned}$$

Simplify[%]

$$\frac{1}{4} (3 + 8k + 4k^2) r^{-\frac{1}{2}+k} \sin\left[\frac{1}{2} (t - 2kt)\right]$$

hes12[f2[k]]

$$\begin{aligned} & -\frac{1}{r} \left(\sin[t] \left(-\left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right)t\right] \sin[t] + \left(\frac{3}{2} + k\right)^2 r^{\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right)t\right] \sin[t] + \right. \right. \\ & \quad \left. \left. \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right)t\right] + \left(-\frac{3}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right)t\right] \right) \right) + \\ & \cos[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right)t\right] + \right. \\ & \quad \left. \left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right)t\right] \right) \end{aligned}$$

Simplify[%]

$$\frac{1}{4} (3 + 8k + 4k^2) r^{-\frac{1}{2}+k} \cos\left[\frac{1}{2} (t - 2kt)\right]$$

hes21[f2[k]]

$$-\frac{1}{r} \left(\sin[t] \left(-\left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right) t\right] \sin[t] + \left(\frac{3}{2} + k\right)^2 r^{\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right) t\right] \sin[t] + \right. \right. \\ \left. \left. \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] + \left(-\frac{3}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] \right) \right) + \\ \cos[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right) t\right] + \right. \\ \left. \left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] \right)$$

Simplify[%]

$$\frac{1}{4} (3 + 8k + 4k^2) r^{-\frac{1}{2}+k} \cos\left[\frac{1}{2} (t - 2kt)\right]$$

(10.37) HESSIAN OF FUNCTIONS OF TYPE f OF THIRD KIND (f_{3k})
hes11[f3[k]]

$$-\frac{1}{r} \left(\sin[t] \left(-\left(\frac{1}{2} - k\right) \left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(-\frac{1}{2} + k\right) t\right] \sin[t] - \right. \right. \\ \left. \left. \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(-\frac{1}{2} + k\right) t\right] \sin[t] - \left(\frac{1}{2} - k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] + \right. \right. \\ \left. \left. \left(\frac{1}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right) \right) + \\ \cos[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \cos\left[\left(-\frac{1}{2} + k\right) t\right] - \right. \\ \left. \left(\frac{1}{2} - k\right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \sin[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right)$$

Simplify[%]

$$\frac{1}{4} (1 + 2k) r^{-\frac{1}{2}+k} \left((-1 + 2k) \cos\left[\left(\frac{5}{2} - k\right) t\right] + 4 \cos\left[\left(-\frac{1}{2} + k\right) t\right] \right)$$

hes22[f3[k]]

$$\sin[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos\left[\left(-\frac{1}{2} + k\right) t\right] \sin[t] + \right. \\ \left. \left(\frac{1}{2} - k\right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right) + \frac{1}{r} \left(\cos[t] \right. \\ \left. \left(\left(\frac{1}{2} - k\right) \left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(-\frac{1}{2} + k\right) t\right] + \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(-\frac{1}{2} + k\right) t\right] - \right. \right. \\ \left. \left. \left(\frac{1}{2} - k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] + \left(\frac{1}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right) \right)$$

Simplify[%]

$$-\frac{1}{4} (1 + 2k) r^{-\frac{1}{2}+k} \left((-1 + 2k) \cos\left[\left(\frac{5}{2} - k\right) t\right] - 4 \cos\left[\left(-\frac{1}{2} + k\right) t\right] \right)$$

hes12[f3[k]]

$$\cos[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos\left[\left(-\frac{1}{2} + k\right) t\right] \sin[t] + \right. \\ \left. \left(\frac{1}{2} - k\right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right) - \frac{1}{r} \left(\sin[t] \right. \\ \left. \left(\left(\frac{1}{2} - k\right) \left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(-\frac{1}{2} + k\right) t\right] + \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(-\frac{1}{2} + k\right) t\right] - \right. \right. \\ \left. \left. \left(\frac{1}{2} - k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] + \left(\frac{1}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right) \right)$$

Simplify[%]

$$\frac{1}{4} (-1 + 2k) (1 + 2k) r^{-\frac{1}{2}+k} \sin\left[\left(\frac{5}{2} - k\right) t\right]$$

hes21[f3[k]]

$$\begin{aligned} & \cos[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos\left[\left(-\frac{1}{2} + k\right) t\right] \sin[t] + \right. \\ & \quad \left. \left(\frac{1}{2} - k\right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right) - \frac{1}{r} \left(\sin[t] \right. \\ & \quad \left. \left(\left(\frac{1}{2} - k\right) \left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(-\frac{1}{2} + k\right) t\right] + \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(-\frac{1}{2} + k\right) t\right] - \right. \right. \\ & \quad \left. \left. \left(\frac{1}{2} - k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] + \left(\frac{1}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right) \right) \end{aligned}$$

Simplify[%]

$$\frac{1}{4} (-1 + 2k) (1 + 2k) r^{-\frac{1}{2}+k} \sin\left[\left(\frac{5}{2} - k\right) t\right]$$

(10.38) HESSIAN OF FUNCTIONS OF TYPE f OF FOURTH KIND (f_{4k})
hes11[f4[k]]

$$\begin{aligned} & \cos[t] \left(-\left(-\frac{1}{2} + k\right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \cos\left[\left(-\frac{1}{2} + k\right) t\right] \sin[t] + \right. \\ & \quad \left. \left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right) - \frac{1}{r} \left(\sin[t] \right. \\ & \quad \left. \left(-\left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(-\frac{1}{2} + k\right) t\right] + \left(-\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(-\frac{1}{2} + k\right) t\right] - \right. \right. \\ & \quad \left. \left. \left(\frac{1}{2} - k\right) \left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] - \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right) \right) \end{aligned}$$

Simplify[%]

$$\frac{1}{4} (1 + 2k) r^{-\frac{1}{2}+k} \left((1 - 2k) \sin\left[\left(\frac{5}{2} - k\right) t\right] + 4 \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right)$$

hes22[f4[k]]

$$\begin{aligned} & \frac{1}{r} \left(\cos[t] \right. \\ & \quad \left. \left(-\left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(-\frac{1}{2} + k\right) t\right] \sin[t] + \left(-\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(-\frac{1}{2} + k\right) t\right] \sin[t] + \right. \right. \\ & \quad \left. \left. \left(\frac{1}{2} - k\right) \left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] + \right. \right. \\ & \quad \left. \left. \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right) \right) + \\ & \quad \sin[t] \left(\left(-\frac{1}{2} + k\right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \cos\left[\left(-\frac{1}{2} + k\right) t\right] + \right. \\ & \quad \left. \left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \sin[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right) \end{aligned}$$

Simplify[%]

$$\frac{1}{4} (1 + 2k) r^{-\frac{1}{2}+k} \left((-1 + 2k) \sin\left[\left(\frac{5}{2} - k\right) t\right] + 4 \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right)$$

hes12[f4[k]]

$$-\frac{1}{r} \left(\sin[t] \left(-\left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(-\frac{1}{2} + k\right) t\right] \sin[t] + \left(-\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(-\frac{1}{2} + k\right) t\right] \sin[t] + \left(\frac{1}{2} - k\right) \left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] + \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right) + \cos[t] \left(\left(-\frac{1}{2} + k\right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \cos\left[\left(-\frac{1}{2} + k\right) t\right] + \left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \sin[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right)$$

Simplify[%]

$$\frac{1}{4} (-1 + 2k) (1 + 2k) r^{-\frac{1}{2}+k} \cos\left[\left(\frac{5}{2} - k\right) t\right]$$

hes21[f4[k]]

$$-\frac{1}{r} \left(\sin[t] \left(-\left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(-\frac{1}{2} + k\right) t\right] \sin[t] + \left(-\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(-\frac{1}{2} + k\right) t\right] \sin[t] + \left(\frac{1}{2} - k\right) \left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] + \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right) + \cos[t] \left(\left(-\frac{1}{2} + k\right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \cos\left[\left(-\frac{1}{2} + k\right) t\right] + \left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \sin[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right)$$

Simplify[%]

$$\frac{1}{4} (-1 + 2k) (1 + 2k) r^{-\frac{1}{2}+k} \cos\left[\left(\frac{5}{2} - k\right) t\right]$$

lap[lap[f1]]

0

lap[lap[f2]]

0

lap[lap[f3]]

0

lap[lap[f4]]

0

(10.39) COMPUTATION OF SCALAR PRODUCTS IN H^2 FOR $\{f1_k\}$, $\{f2_k\}$, $\{f3_k\}$, $\{f4_k\}$, $k \in \mathbb{Z}$

prodscaleH2[f1[k], f1[k]]

$$\frac{1}{4} (1 + 2k) (3 + 2k)^2 \pi R^{1+2k}$$

TeXForm[%]

$\frac{\left(1 + 2k\right) \sqrt{\left(3 + 2k\right)^2 \pi R^{1 + 2k}}}{4}$

prodscalH2[f1[k], f2[k]]

0

prodscalH2[f1[k], f4[k]]

0

prodscalH2[f1[k], f3[k]]

0

prodscalH2[f2[k], f4[k]]

0

prodscalH2[f2[k], f3[k]]

0

prodscalH2[f2[k], f2[k]]

$\frac{1}{4} (1 + 2k) (3 + 2k)^2 \pi R^{1+2k}$

TeXForm $\left[\frac{1}{4} (1 + 2k) (3 + 2k)^2 \pi R^{1+2k}\right]$

$\frac{\left(1 + 2k\right) \sqrt{\left(3 + 2k\right)^2 \pi R^{1 + 2k}}}{4}$

prodscalH2[f3[k], f3[k]]

$$\frac{\left(1 + 2k\right) R^{1+2k} \left(-81\pi + 234k\pi - 232k^2\pi + 208k^3\pi - 80k^4\pi + 32k^5\pi + 72\pi \cos[2k\pi] - 176k\pi \cos[2k\pi] + 96k^2\pi \cos[2k\pi] - 64k^3\pi \cos[2k\pi] - 72\sin[2k\pi] + 32k\sin[2k\pi] - 32k^2\sin[2k\pi] + 32\sin[4k\pi]\right)}{4(-1 + 2k)(9 - 4k + 4k^2 - 8\cos[2k\pi])}$$

Simplify[%]

$$\frac{(1 + 2k) R^{1+2k} \left((-9 + 22k - 12k^2 + 8k^3)\pi - 8\sin[2k\pi]\right)}{-4 + 8k}$$

TeXForm $\left[\frac{(1 + 2k) R^{1+2k} \left((-9 + 22k - 12k^2 + 8k^3)\pi - 8\sin[2k\pi]\right)}{-4 + 8k}\right]$

$\frac{\left(1 + 2k\right) \sqrt{\left(3 + 2k\right)^2 \pi R^{1 + 2k}} \sqrt{\left(-9 + 22k - 12k^2 + 8k^3\right) \pi - 8\sin\left(2k\pi\right)}}{-4 + 8k}$

prodscalH2[f4[k], f4[k]]

$$\left(1 + 2k\right) R^{1+2k} \left(-81\pi + 234k\pi - 232k^2\pi + 208k^3\pi - 80k^4\pi + 32k^5\pi - 72\pi \cos[2k\pi] + 176k\pi \cos[2k\pi] - 96k^2\pi \cos[2k\pi] + 64k^3\pi \cos[2k\pi] + 72\sin[2k\pi] - 32k\sin[2k\pi] + 32k^2\sin[2k\pi] + 32\sin[4k\pi]\right) / \left(4(-1 + 2k)(9 - 4k + 4k^2 + 8\cos[2k\pi])\right)$$

Simplify[%]

$$\frac{(1+2k) R^{1+2k} ((-9+22k-12k^2+8k^3)\pi+8\sin[2k\pi])}{-4+8k}$$

TeXForm $\left[\frac{(1+2k) R^{1+2k} ((-9+22k-12k^2+8k^3)\pi+8\sin[2k\pi])}{-4+8k} \right]$

$$\frac{\left(\left(1+2k \right) R^{\left(1+2k \right)} \left(\left(-9+22k-12k^2+8k^3 \right) \pi + 8 \sin \left(2k \pi \right) \right) \right)}{-4+8k}$$

Factor[-9+22k-12k^2+8k^3]

$$(-1+2k)(9-4k+4k^2)$$

PAY ATTENTION :

from now non h labels a generic index different from k (not the label of f1, f2, , ...)
Sin (h - k) Pi always stans for 0 since
h and k are integers.

prodscalH2[f1[k], f1[h]]

$$\frac{(3+8h+4h^2)(3+8k+4k^2)R^{1+h+k}\sin[(h-k)\pi]}{4(h+h^2-k(1+k))}$$

Simplify[%]

$$\frac{(3+8h+4h^2)(3+8k+4k^2)R^{1+h+k}\sin[(h-k)\pi]}{4(h+h^2-k(1+k))}$$

prodscalH2[f2[k], f2[h]]

$$\frac{(3+8h+4h^2)(3+8k+4k^2)R^{1+h+k}\sin[(h-k)\pi]}{4(h+h^2-k(1+k))}$$

Simplify[%]

$$\frac{(3+8h+4h^2)(3+8k+4k^2)R^{1+h+k}\sin[(h-k)\pi]}{4(h+h^2-k(1+k))}$$

prodscalH2[f3[k], f3[h]]

$$\frac{1}{4(h-k)(-1+h+k)(1+h+k)} \left((1+2h)(1+2k)R^{1+h+k} \right. \\ \left(-8 \cos \left[\frac{1}{2}(-1+2k)\pi \right] \sin \left[\frac{1}{2}(-1+2h)\pi \right] + 16h \cos \left[\frac{1}{2}(-1+2k)\pi \right] \sin \left[\frac{1}{2}(-1+2h)\pi \right] + \right. \\ \left. 8 \cos \left[\frac{1}{2}(-1+2h)\pi \right] \sin \left[\frac{1}{2}(-1+2k)\pi \right] - 16k \cos \left[\frac{1}{2}(-1+2h)\pi \right] \sin \left[\frac{1}{2}(-1+2k)\pi \right] - \right. \\ \left. \sin \left[\frac{1}{2}(-5+2h)\pi - \frac{1}{2}(-5+2k)\pi \right] + 3h \sin \left[\frac{1}{2}(-5+2h)\pi - \frac{1}{2}(-5+2k)\pi \right] - \right. \\ \left. 2h^2 \sin \left[\frac{1}{2}(-5+2h)\pi - \frac{1}{2}(-5+2k)\pi \right] + 3k \sin \left[\frac{1}{2}(-5+2h)\pi - \frac{1}{2}(-5+2k)\pi \right] - \right. \\ \left. 8hk \sin \left[\frac{1}{2}(-5+2h)\pi - \frac{1}{2}(-5+2k)\pi \right] + 4h^2k \sin \left[\frac{1}{2}(-5+2h)\pi - \frac{1}{2}(-5+2k)\pi \right] - \right. \\ \left. 2k^2 \sin \left[\frac{1}{2}(-5+2h)\pi - \frac{1}{2}(-5+2k)\pi \right] + 4hk^2 \sin \left[\frac{1}{2}(-5+2h)\pi - \frac{1}{2}(-5+2k)\pi \right] \right) \left. \right)$$

Simplify[%]

$$\frac{(1+2h)(1+2k)R^{1+h+k}((-9+11k-2k^2+h^2(-2+4k)+h(11-8k+4k^2))\sin[(h-k)\pi]+8(-h+k)\sin[(h+k)\pi])}{4(h-k)(-1+h+k)(1+h+k)}$$

prodscalH2[f4[k], f4[h]]

$$\frac{1}{4 (h - k) (-1 + h + k) (1 + h + k)} \left((1 + 2 h) (1 + 2 k) R^{1+h+k} \right. \\ \left. \begin{aligned} & \left(-8 \cos\left[\frac{1}{2} (-1 + 2 k) \pi\right] \sin\left[\frac{1}{2} (-1 + 2 h) \pi\right] + 16 k \cos\left[\frac{1}{2} (-1 + 2 k) \pi\right] \sin\left[\frac{1}{2} (-1 + 2 h) \pi\right] + \right. \\ & 8 \cos\left[\frac{1}{2} (-1 + 2 h) \pi\right] \sin\left[\frac{1}{2} (-1 + 2 k) \pi\right] - 16 h \cos\left[\frac{1}{2} (-1 + 2 h) \pi\right] \sin\left[\frac{1}{2} (-1 + 2 k) \pi\right] - \\ & \sin\left[\frac{1}{2} (-5 + 2 h) \pi - \frac{1}{2} (-5 + 2 k) \pi\right] + 3 h \sin\left[\frac{1}{2} (-5 + 2 h) \pi - \frac{1}{2} (-5 + 2 k) \pi\right] - \\ & 2 h^2 \sin\left[\frac{1}{2} (-5 + 2 h) \pi - \frac{1}{2} (-5 + 2 k) \pi\right] + 3 k \sin\left[\frac{1}{2} (-5 + 2 h) \pi - \frac{1}{2} (-5 + 2 k) \pi\right] - \\ & 8 h k \sin\left[\frac{1}{2} (-5 + 2 h) \pi - \frac{1}{2} (-5 + 2 k) \pi\right] + 4 h^2 k \sin\left[\frac{1}{2} (-5 + 2 h) \pi - \frac{1}{2} (-5 + 2 k) \pi\right] - \\ & \left. \left. 2 k^2 \sin\left[\frac{1}{2} (-5 + 2 h) \pi - \frac{1}{2} (-5 + 2 k) \pi\right] + 4 h k^2 \sin\left[\frac{1}{2} (-5 + 2 h) \pi - \frac{1}{2} (-5 + 2 k) \pi\right] \right) \right) \end{aligned} \right)$$

Simplify[%]

$$\left((1 + 2 h) (1 + 2 k) R^{1+h+k} \left((-9 + 11 k - 2 k^2 + h^2 (-2 + 4 k) + h (11 - 8 k + 4 k^2)) \sin[(h - k) \pi] + \right. \right. \\ \left. \left. 8 (h - k) \sin[(h + k) \pi] \right) \right) / (4 (h - k) (-1 + h + k) (1 + h + k))$$

prodscalH2[f1[k], f2[h]]

0

prodscalH2[f1[k], f3[h]]

$$- \frac{(-1 + 4 h^2) (3 + 8 k + 4 k^2) R^{1+h+k} \sin[(h - k) \pi]}{4 (2 + h - h^2 + 3 k + k^2)}$$

Simplify[%]

$$- \frac{(-1 + 4 h^2) (3 + 8 k + 4 k^2) R^{1+h+k} \sin[(h - k) \pi]}{4 (2 + h - h^2 + 3 k + k^2)}$$

prodscalH2[f1[k], f4[h]]

0

prodscalH2[f2[k], f4[h]]

$$- \frac{(-1 + 4 h^2) (3 + 8 k + 4 k^2) R^{1+h+k} \sin[(h - k) \pi]}{4 (2 + h - h^2 + 3 k + k^2)}$$

Simplify[%]

$$- \frac{(-1 + 4 h^2) (3 + 8 k + 4 k^2) R^{1+h+k} \sin[(h - k) \pi]}{4 (2 + h - h^2 + 3 k + k^2)}$$

prodscalH2[f3[k], f4[h]]

0

(* HESSIANI DELLE fj[k] *)

dx[f1[k]]

$$\begin{aligned}
& -\frac{1}{r} \left(\sin[t] \left(-\left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right) t\right] \sin[t] - \right. \right. \\
& \quad \left. \left(-\frac{3}{2} - k \right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right) t\right] \sin[t] - \left(-\frac{3}{2} - k \right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] + \right. \\
& \quad \left. \left. \left(-\frac{3}{2} - k \right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] \right) \right) + \\
& \cos[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right) t\right] - \right. \\
& \quad \left. \left(-\frac{3}{2} - k \right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] \right)
\end{aligned}$$

Simplify[%]

$$\frac{1}{4} (3 + 8k + 4k^2) r^{-\frac{1}{2}+k} \cos\left[\frac{1}{2} (t - 2kt)\right]$$

$$\text{TeXForm}\left[\frac{1}{4} (3 + 8k + 4k^2) r^{-\frac{1}{2}+k} \cos\left[\frac{1}{2} (t - 2kt)\right]\right]$$

```
\frac{\left( 3 + 8k + 4k^2 \right) \, r^{\left( -\frac{1}{2} \right) + k} \, \cos\left(\frac{t - 2k t}{2}\right)}{4}
```

dy[f1[k]]

$$\begin{aligned}
& \cos[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right) t\right] \sin[t] + \right. \\
& \quad \left. \left(-\frac{3}{2} - k \right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] - \frac{1}{r} \left(\sin[t] \right. \right. \\
& \quad \left. \left(\left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right) t\right] + \left(-\frac{3}{2} - k \right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right) t\right] - \right. \right. \\
& \quad \left. \left. \left(-\frac{3}{2} - k \right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] + \left(-\frac{3}{2} - k \right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] \right) \right)
\end{aligned}$$

Simplify[%]

$$\frac{1}{4} (3 + 8k + 4k^2) r^{-\frac{1}{2}+k} \sin\left[\frac{1}{2} (t - 2kt)\right]$$

$$\text{TeXForm}\left[\frac{1}{4} (3 + 8k + 4k^2) r^{-\frac{1}{2}+k} \sin\left[\frac{1}{2} (t - 2kt)\right]\right]$$

```
\frac{\left( 3 + 8k + 4k^2 \right) \, r^{\left( -\frac{1}{2} \right) + k} \, \sin\left(\frac{t - 2k t}{2}\right)}{4}
```

dy[f1[k]]

$$\begin{aligned}
& \sin[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right) t\right] \sin[t] + \right. \\
& \quad \left. \left(-\frac{3}{2} - k \right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] + \frac{1}{r} \left(\cos[t] \right. \right. \\
& \quad \left. \left(\left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right) t\right] + \left(-\frac{3}{2} - k \right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right) t\right] - \right. \right. \\
& \quad \left. \left. \left(-\frac{3}{2} - k \right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] + \left(-\frac{3}{2} - k \right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] \right) \right)
\end{aligned}$$

Simplify[%]

$$-\frac{1}{4} (3 + 8k + 4k^2) r^{-\frac{1}{2}+k} \cos\left[\frac{1}{2} (t - 2kt)\right]$$

TeXForm $\left[-\frac{1}{4} (3 + 8 k + 4 k^2) r^{-\frac{1}{2}+k} \cos\left[\frac{1}{2} (t - 2 k t)\right]\right]$

$\frac{-\left(\left(3 + 8k + 4k^2\right) r^{-\left(\frac{1}{2}\right) + k}\right) \cos\left(\frac{t - 2k}{2}\right)}{4}$

dx[f2[k]]

$\cos[t] \left(-\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right) t\right] \sin[t] + \left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right) t\right]\right) - \frac{1}{r} \left(\sin[t] \left(-\left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right) t\right] + \left(\frac{3}{2} + k\right)^2 r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right) t\right] - \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] - \left(-\frac{3}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right) t\right]\right)\right)$

Simplify[%]

$-\frac{1}{4} (3 + 8 k + 4 k^2) r^{-\frac{1}{2}+k} \sin\left[\frac{1}{2} (t - 2 k t)\right]$

TeXForm $\left[-\frac{1}{4} (3 + 8 k + 4 k^2) r^{-\frac{1}{2}+k} \sin\left[\frac{1}{2} (t - 2 k t)\right]\right]$

$\frac{-\left(\left(3 + 8k + 4k^2\right) r^{-\left(\frac{1}{2}\right) + k}\right) \sin\left(\frac{t - 2k}{2}\right)}{4}$

dx[f2[k]]

$-\frac{1}{r} \left(\sin[t] \left(-\left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right) t\right] \sin[t] + \left(\frac{3}{2} + k\right)^2 r^{\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right) t\right] \sin[t] + \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] + \left(-\frac{3}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right) t\right]\right)\right) + \cos[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right) t\right] + \left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right) t\right]\right)$

Simplify[%]

$\frac{1}{4} (3 + 8 k + 4 k^2) r^{-\frac{1}{2}+k} \cos\left[\frac{1}{2} (t - 2 k t)\right]$

TeXForm $\left[\frac{1}{4} (3 + 8 k + 4 k^2) r^{-\frac{1}{2}+k} \cos\left[\frac{1}{2} (t - 2 k t)\right]\right]$

$\frac{\left(\left(3 + 8k + 4k^2\right) r^{-\left(\frac{1}{2}\right) + k}\right) \cos\left(\frac{t - 2k}{2}\right)}{4}$

dy[f2[k]]

$\frac{1}{r} \left(\cos[t] \left(-\left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right) t\right] \sin[t] + \left(\frac{3}{2} + k\right)^2 r^{\frac{1}{2}+k} \cos\left[\left(\frac{3}{2} + k\right) t\right] \sin[t] + \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right) t\right] + \left(-\frac{3}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(\frac{3}{2} + k\right) t\right]\right)\right) + \sin[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \cos\left[\left(\frac{3}{2} + k\right) t\right] + \left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \sin[t] \sin\left[\left(\frac{3}{2} + k\right) t\right]\right)$

Simplify[%]

$$\frac{1}{4} (3 + 8k + 4k^2) r^{-\frac{1}{2}+k} \sin\left[\frac{1}{2} (t - 2kt)\right]$$

TeXForm $\left[\frac{1}{4} (3 + 8k + 4k^2) r^{-\frac{1}{2}+k} \sin\left[\frac{1}{2} (t - 2kt)\right]\right]$

$$\frac{\left(\left(\left(3 + 8k + 4k^2\right)\right)\right) \, r^{\left(-\left(\frac{1}{2}\right) + k\right)}}{\sin\left(\frac{t - 2kt}{2}\right)}\{4\}$$
dx[x][f3[k]]

$$\begin{aligned} & -\frac{1}{r} \left(\sin[t] \left(-\left(\frac{1}{2} - k\right) \left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[\left(-\frac{1}{2} + k\right) t\right] \sin[t] - \right. \right. \\ & \quad \left. \left(\frac{3}{2} + k \right) r^{\frac{1}{2}+k} \cos\left[\left(-\frac{1}{2} + k\right) t\right] \sin[t] - \left(\frac{1}{2} - k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] + \right. \\ & \quad \left. \left. \left(\frac{1}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right) \right) + \\ & \cos[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \cos\left[\left(-\frac{1}{2} + k\right) t\right] - \right. \\ & \quad \left. \left(\frac{1}{2} - k\right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \sin[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right) \end{aligned}$$

Simplify[%]

$$\frac{1}{4} (1 + 2k) r^{-\frac{1}{2}+k} \left((-1 + 2k) \cos\left[\left(\frac{5}{2} - k\right) t\right] + 4 \cos\left[\left(-\frac{1}{2} + k\right) t\right] \right)$$

TeXForm $\left[\frac{1}{4} (1 + 2k) r^{-\frac{1}{2}+k} \left((-1 + 2k) \cos\left[\left(\frac{5}{2} - k\right) t\right] + 4 \cos\left[\left(-\frac{1}{2} + k\right) t\right] \right)\right]$

$$\frac{\left(\left(\left(1 + 2k\right)\right)\right) \, r^{\left(-\left(\frac{1}{2}\right) + k\right)}}{\left(\left(-1 + 2k\right)\right) \cos\left(\frac{5t}{2} - kt\right) + 4 \cos\left(-\left(\frac{1}{2}\right) + kt\right)}\{4\}$$
dx[y][f3[k]]

$$\begin{aligned} & \cos[t] \left(\left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \cos\left[\left(-\frac{1}{2} + k\right) t\right] \sin[t] + \right. \\ & \quad \left. \left(\frac{1}{2} - k\right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] - \frac{1}{r} \left(\sin[t] \right. \right. \\ & \quad \left. \left. \left(\left(\frac{1}{2} - k\right) \left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(-\frac{1}{2} + k\right) t\right] + \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[\left(-\frac{1}{2} + k\right) t\right] - \right. \right. \right. \\ & \quad \left. \left. \left(\frac{1}{2} - k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] + \left(\frac{1}{2} - k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[\left(-\frac{1}{2} + k\right) t\right] \right) \right) \end{aligned}$$

Simplify[%]

$$\frac{1}{4} (-1 + 2k) (1 + 2k) r^{-\frac{1}{2}+k} \sin\left[\left(\frac{5}{2} - k\right) t\right]$$

TeXForm $\left[\frac{1}{4} (-1 + 2k) (1 + 2k) r^{-\frac{1}{2}+k} \sin\left[\left(\frac{5}{2} - k\right) t\right]\right]$

$$\frac{\left(\left(-1 + 2k\right)\right) \left(\left(1 + 2k\right)\right) \, r^{\left(-\left(\frac{1}{2}\right) + k\right)}}{\sin\left(\frac{5t}{2} - kt\right)}\{4\}$$

dy[f3[k]]

$$\begin{aligned} & \sin[t] \left(\left(\frac{1}{2} + k \right) \left(\frac{3}{2} + k \right) r^{-\frac{1}{2}+k} \cos\left[-\frac{1}{2} + k\right] t \right) \sin[t] + \\ & \left(\frac{1}{2} - k \right) \left(\frac{1}{2} + k \right) r^{-\frac{1}{2}+k} \cos[t] \sin\left[-\frac{1}{2} + k\right] t \right) + \frac{1}{r} \left(\cos[t] \right. \\ & \left. \left(\left(\frac{1}{2} - k \right) \left(-\frac{1}{2} + k \right) r^{\frac{1}{2}+k} \cos[t] \cos\left[-\frac{1}{2} + k\right] t \right) + \left(\frac{3}{2} + k \right) r^{\frac{1}{2}+k} \cos[t] \cos\left[-\frac{1}{2} + k\right] t \right) - \\ & \left. \left(\frac{1}{2} - k \right) r^{\frac{1}{2}+k} \sin[t] \sin\left[-\frac{1}{2} + k\right] t \right) + \left(\frac{1}{2} - k \right) \left(\frac{3}{2} + k \right) r^{\frac{1}{2}+k} \sin[t] \sin\left[-\frac{1}{2} + k\right] t \right) \end{aligned}$$

Simplify[%]

$$-\frac{1}{4} (1 + 2k) r^{-\frac{1}{2}+k} \left((-1 + 2k) \cos\left[\left(\frac{5}{2} - k\right) t\right] - 4 \cos\left[-\frac{1}{2} + k\right] t \right)$$

$$\text{TeXForm}\left[-\frac{1}{4} (1 + 2k) r^{-\frac{1}{2}+k} \left((-1 + 2k) \cos\left[\left(\frac{5}{2} - k\right) t\right] - 4 \cos\left[-\frac{1}{2} + k\right] t \right)\right]$$

```
\frac{-\left( \left( 1 + 2\,k \right) \, r^{\left( \frac{1}{2} \right) + k} \, \left( \left( -1 + 2\,k \right) \, \cos\left( \left( \frac{5}{2} - k \right) t \right) - 4 \cos\left( -\frac{1}{2} + k \right) t \right) \right)}{4} \cos\left( \left( -\frac{1}{2} \right) + k \right) t \right)
```

dx[f4[k]]

$$\begin{aligned} & \cos[t] \left(-\left(-\frac{1}{2} + k\right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \cos\left[-\frac{1}{2} + k\right] t \right) \sin[t] + \\ & \left(\frac{1}{2} + k \right) \left(\frac{3}{2} + k \right) r^{-\frac{1}{2}+k} \cos[t] \sin\left[-\frac{1}{2} + k\right] t \right) - \frac{1}{r} \left(\sin[t] \right. \\ & \left. -\left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[-\frac{1}{2} + k\right] t \right) + \left(-\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \cos\left[-\frac{1}{2} + k\right] t \right) - \\ & \left. \left(\frac{1}{2} - k \right) \left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \sin[t] \sin\left[-\frac{1}{2} + k\right] t \right) - \left(\frac{3}{2} + k \right) r^{\frac{1}{2}+k} \sin[t] \sin\left[-\frac{1}{2} + k\right] t \right) \end{aligned}$$

Simplify[%]

$$\frac{1}{4} (1 + 2k) r^{-\frac{1}{2}+k} \left((1 - 2k) \sin\left[\left(\frac{5}{2} - k\right) t\right] + 4 \sin\left[-\frac{1}{2} + k\right] t \right)$$

$$\text{TeXForm}\left[\frac{1}{4} (1 + 2k) r^{-\frac{1}{2}+k} \left((1 - 2k) \sin\left[\left(\frac{5}{2} - k\right) t\right] + 4 \sin\left[-\frac{1}{2} + k\right] t \right)\right]$$

```
\frac{\left( \left( 1 + 2\,k \right) \, r^{\left( -\frac{1}{2} \right) + k} \, \left( \left( 1 - 2\,k \right) \, \sin\left( \left( \frac{5}{2} - k \right) t \right) + 4 \sin\left( -\frac{1}{2} + k \right) t \right) \right)}{4} \sin\left( \left( -\frac{1}{2} \right) + k \right) t \right)
```

dx[f4[k]]

$$\begin{aligned} & -\frac{1}{r} \left(\sin[t] \left(-\left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[-\frac{1}{2} + k\right] t \right) \sin[t] + \right. \\ & \left. \left(-\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos\left[-\frac{1}{2} + k\right] t \right) \sin[t] + \left(\frac{1}{2} - k\right) \left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \right. \\ & \left. \cos[t] \sin\left[-\frac{1}{2} + k\right] t \right) + \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \cos[t] \sin\left[-\frac{1}{2} + k\right] t \right) + \\ & \cos[t] \left(\left(-\frac{1}{2} + k\right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \cos[t] \cos\left[-\frac{1}{2} + k\right] t \right) + \\ & \left. \left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \sin[t] \sin\left[-\frac{1}{2} + k\right] t \right) \end{aligned}$$

Simplify[%]

$$\frac{1}{4} (-1 + 2k) (1 + 2k) r^{-\frac{1}{2}+k} \cos\left[\left(\frac{5}{2} - k\right) t\right]$$

TeXForm $\left[\frac{1}{4} (-1 + 2k) (1 + 2k) r^{-\frac{1}{2}+k} \text{Cos}\left[\left(\frac{5}{2} - k\right) t\right]\right]$

$\frac{1}{4} (-1 + 2k) (1 + 2k) r^{-\frac{1}{2}+k} \text{Cos}\left[\left(\frac{5}{2} - k\right) t\right]$

dy $[f4[k]]$

$\frac{1}{r} \left(\text{Cos}[t] \left(-\left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \text{Cos}\left[\left(-\frac{1}{2} + k\right) t\right] \text{Sin}[t] + \left(-\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \text{Cos}\left[\left(-\frac{1}{2} + k\right) t\right] \text{Sin}[t] + \left(\frac{1}{2} - k\right) \left(-\frac{1}{2} + k\right) r^{\frac{1}{2}+k} \text{Cos}[t] \text{Sin}\left[\left(-\frac{1}{2} + k\right) t\right] + \left(\frac{3}{2} + k\right) r^{\frac{1}{2}+k} \text{Cos}[t] \text{Sin}\left[\left(-\frac{1}{2} + k\right) t\right] \right) + \text{Sin}[t] \left(\left(-\frac{1}{2} + k\right) \left(\frac{1}{2} + k\right) r^{-\frac{1}{2}+k} \text{Cos}[t] \text{Cos}\left[\left(-\frac{1}{2} + k\right) t\right] + \left(\frac{1}{2} + k\right) \left(\frac{3}{2} + k\right) r^{-\frac{1}{2}+k} \text{Sin}[t] \text{Sin}\left[\left(-\frac{1}{2} + k\right) t\right] \right) \right)$

Simplify $[%]$

$\frac{1}{4} (1 + 2k) r^{-\frac{1}{2}+k} \left((-1 + 2k) \text{Sin}\left[\left(\frac{5}{2} - k\right) t\right] + 4 \text{Sin}\left[\left(-\frac{1}{2} + k\right) t\right] \right)$

TeXForm $\left[\frac{1}{4} (1 + 2k) r^{-\frac{1}{2}+k} \left((-1 + 2k) \text{Sin}\left[\left(\frac{5}{2} - k\right) t\right] + 4 \text{Sin}\left[\left(-\frac{1}{2} + k\right) t\right] \right)\right]$

$\frac{1}{4} (1 + 2k) r^{-\frac{1}{2}+k} \left((-1 + 2k) \text{Sin}\left[\left(\frac{5}{2} - k\right) t\right] + 4 \text{Sin}\left[\left(-\frac{1}{2} + k\right) t\right] \right)$

(10.40),(10.41) Evaluation of squared hessian jump for candidates W , Φ along jump set

$\text{dx}[u_] := \text{Cos}[t] * \text{D}[u, r] - \text{Sin}[t] * \text{D}[u, t] / r$
 $\text{dy}[u_] := \text{Sin}[t] * \text{D}[u, r] + \text{Cos}[t] * \text{D}[u, t] / r$
 $\text{dxx}[u_] := \text{dx}[\text{dx}[u]]$
 $\text{dyy}[u_] := \text{dy}[\text{dy}[u]]$
 $\text{dxy}[u_] := \text{dx}[\text{dy}[u]]$
 $\text{dyx}[u_] := \text{dy}[\text{dx}[u]]$
 $\text{lap}[u_] := \text{dxx}[u] + \text{dyy}[u]$
 $\text{hes2}[u_] := \text{dxx}[u]^2 + \text{dyy}[u]^2 + 2 * \text{dxy}[u]^2$

$W[r_, t_] := r^{(3/2)} * \text{Sqrt}[\alpha / (193 * \text{Pi})] * \left(\text{Sqrt}[21] * \left(\text{Sin}[t/2] - 5/3 * \text{Sin}[3 * t/2] \right) + \left(\text{Cos}[t/2] - 7/3 * \text{Cos}[3 * t/2] \right) \right)$

```

ϕ[r_, t_] := r^(3/2) * Sqrt[α / (193 * Pi)] *
             ( Sqrt[21] * ( Sin[t/2] - 5/3 * Sin[3 * t/2] ) -
               ( Cos[t/2] - 7/3 * Cos[3 * t/2] ) )

```

```
Simplify[hess2[W[r, π]]]
```

$$\frac{420 \alpha}{193 \pi r}$$

```
Simplify[hess2[W[r, -π]]]
```

$$\frac{420 \alpha}{193 \pi r}$$

```
Simplify[hess2[ϕ[r, π]]]
```

$$\frac{420 \alpha}{193 \pi r}$$

```
Simplify[hess2[ϕ[r, -π]]]
```

$$\frac{420 \alpha}{193 \pi r}$$

How to evaluate modes and their derivatives and hessian along jump set

```
u /. t -> -Pi
```

```
u /. t -> Pi
```

```
Simplify[dy[u]] /. t -> Pi
```

```
Simplify[dy[u]] /. t -> -Pi
```

```
Simplify[dy[dy[u]]] /. t -> -Pi
```

```
Simplify[dy[dy[u]]] /. t -> Pi
```

```
Simplify[dy[dy[dy[u]]] + 2 * dx[dx[dy[u]]]] /. t -> Pi
```

```
Simplify[dy[dy[dy[u]]] + 2 * dx[dx[dy[u]]]] /. t -> -Pi
```

```
Simplify[hess2[u]] /. t -> Pi
```

```
Simplify[hess2[u]] /. t -> -Pi
```

About Mumford-Shah functional

```

u := r^(1/2) * Sin[t/2]
Simplify[lp[u]]
Simplify[dy[u]] /. t -> -Pi
Simplify[dy[u]] /. t -> Pi
(dx[u]^2 + dy[u]^2) * r
Integrate[%, {r, 0, 1}, {t, -Pi, Pi}]
Simplify[(dx[u]^2 + dy[u]^2) * Cos[t] -
2 * dx[u] * D[u, r]]
Integrte[r * %, {t, -Pi, Pi}]
Simplify[(dx[u]^2 + dy[u]^2) * (dx[φ] + dy[ψ]) - 2 *
(dx[u] * (dx[u] * dx[φ] + dy[u] * dx[ψ]) +
dy[u] * (dx[u] * dy[φ] + dy[u] * dy[ψ]))]
Integrate[% * r, {t, -Pi, Pi}, {r, 0, 1}]
ParametricPlot3D[{r * Cos[t], r * Sin[t], u},
{r, 0, 1}, {t, -Pi, Pi}, AspectRatio -> 1/2,
ViewPoint -> {1.051, -3.180, 1.105}]

```

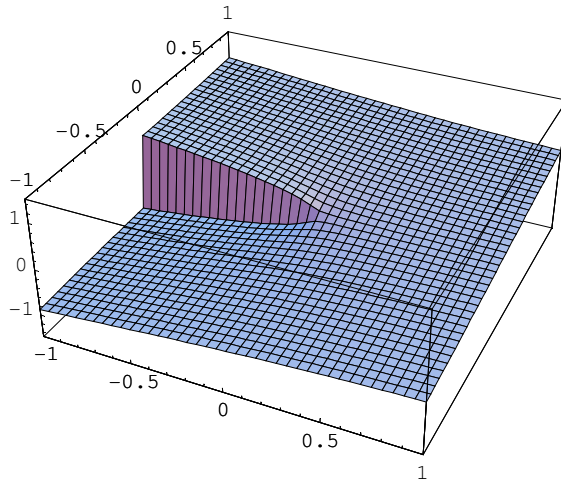
Graph of candidate for Mumford Shah functional

```

candMS[s_, t_] := If[x > 0, Im[Sqrt[2 (x + I y) / Pi]] ,
Im[Sqrt[2 (x + I y) / Pi]] ] /. {x -> s, y -> t}

```

```
Plot3D[candMS[x, y], {x, -1, 1}, {y, -1, 1},
PlotPoints -> 40, PlotRange -> {-1.5, 1.5},
ViewPoint -> {1.196, -2.581, 1.833}]
```



- SurfaceGraphics -

-SurfaceGraphics-

Computations for generic crack-tip conditions (generic vector fields η in Theorem 5.5)

```
 $\varphi := (8 * r * \text{Cos}[t] + 1) * (1 - r^2)^6$ 
```

```
 $\psi := (r * \text{Cos}[t] + 1) * (1 - r^2)^6$ 
```

```
 $\varphi := (1 - r^2)^6$ 
```

```
 $\psi = 0$ 
```

0

```
Simplify[-(hes2[ $\sigma$ ] * r * Cos[t] + 2 *
dx[ $\sigma$ ] *
(dy[lp[ $\sigma$ ]] * r * Sin[t] + dx[lp[ $\sigma$ ]] * r * Cos[t])
- 2 * r * Cos[t] * (dxx[ $\sigma$ ]^2 + dxy[ $\sigma$ ]^2) - 2 * r *
Sin[t] * (dxx[ $\sigma$ ] * dxy[ $\sigma$ ] + dxy[ $\sigma$ ] * dyy[ $\sigma$ ])) *  $\varphi$  +
2 * dx[ $\sigma$ ] * ((dxx[ $\sigma$ ] * r * Cos[t] + dxy[ $\sigma$ ] * r * Sin[t]) *
dx[ $\varphi$ ] +
(dxy[ $\sigma$ ] * r * Cos[t] + dyy[ $\sigma$ ] * r * Sin[t]) * dy[ $\varphi$ ])]
```

0

Integrate[%, {t, -Pi, Pi}]

Integrate[0, {t, -π, π}]

% /. r -> 0

Simplify[%]

Integrate[0, {t, -π, π}]

$\sigma = r^4 * \text{Cos}[4 * t] - r^2$

$-r^2 + r^4 \text{Cos}[4 t]$

Global Euler equation (Theorem 5.1 without g, test functions φ e ψ are needed)

Symbolic computations about Section 5

$\varphi := (8 * r * \text{Cos}[t] + 1) * (R^2 - r^2)^3$

$\psi := (r * \text{Cos}[t] + 1) * (R^2 - r^2)^3$

$\varphi := (1 - r^2)^6$

$\psi := 0$

φ

ψ


```

Integrte[ r * Simplify[hes2[ $\omega$ ] * (dx[ $\varphi$ ] + dy[ $\psi$ ])
  - 4 *
  dxy[ $\omega$ ] * (dxx[ $\omega$ ] * dy[ $\varphi$ ] + dxy[ $\omega$ ] * dx[ $\varphi$ ] +
    dx[ $\omega$ ] * dxy[ $\varphi$ ] + dxy[ $\omega$ ] * dy[ $\psi$ ] +
    dyy[ $\omega$ ] * dx[ $\psi$ ] + dy[ $\omega$ ] * dxy[ $\psi$ ])
  - 2 *
  dxx[ $\omega$ ] * (dxx[ $\omega$ ] * dx[ $\varphi$ ] + dxx[ $\omega$ ] * dx[ $\varphi$ ] +
    dx[ $\omega$ ] * dxx[ $\varphi$ ] + dxy[ $\omega$ ] * dx[ $\psi$ ] +
  dxy[ $\omega$ ] * dx[ $\psi$ ] + dy[ $\omega$ ] * dxx[ $\psi$ ])
  - 2 *
  dyy[ $\omega$ ] * (dxy[ $\omega$ ] * dy[ $\varphi$ ] + dxy[ $\omega$ ] * dy[ $\varphi$ ] +
    dx[ $\omega$ ] * dyy[ $\varphi$ ] + dyy[ $\omega$ ] * dy[ $\psi$ ] +
  dyy[ $\omega$ ] * dy[ $\psi$ ] + dy[ $\omega$ ] * dyy[ $\psi$ ])
], {t, -Pi, Pi}, {r, 0, R}]

```

```
dx[ $\varphi$ ]
```

```
Integrte[dx[ $\varphi$ ], {r, 0, R}] /. t -> Pi
```

```
 $\omega$ 
```

```
Simplify[%]
```

Integration by parts (Theorem 5.2 without g , test functions φ e ψ are needed)
Symbolic computations about Section 5

```
u = r^4 * Cos[t]^4 + r^4 * Cos[t]^2 * Sin[t]^2
```

```
 $\omega = u$ 
```

```
Simplify[lp[u]]
```

```
Simplify[lp[%]]
```

```
 $\varphi = r^2 * (Cos[t]^2) + r^3 * Sin[t]^3$ 
```

```
 $\psi = (r * Cos[t] + 1) * (1 + r^2)$ 
```

```
 $\varphi := (1 - r^2)^6$ 
```

$\psi := 0$

```
Simplify[hes2[ $\omega$ ] * (dx[ $\varphi$ ] + dy[ $\psi$ ])
- 4 *
  dxy[ $\omega$ ] * (dxx[ $\omega$ ] * dy[ $\varphi$ ] + dxy[ $\omega$ ] * dx[ $\varphi$ ] +
    dx[ $\omega$ ] * dxy[ $\varphi$ ] + dxy[ $\omega$ ] * dy[ $\psi$ ] +
    dyy[ $\omega$ ] * dx[ $\psi$ ] + dy[ $\omega$ ] * dxy[ $\psi$ ])
- 2 *
  dxx[ $\omega$ ] * (dxx[ $\omega$ ] * dx[ $\varphi$ ] + dxx[ $\omega$ ] * dx[ $\varphi$ ] +
    dx[ $\omega$ ] * dxx[ $\varphi$ ] +
  dxy[ $\omega$ ] * dx[ $\psi$ ] + dxy[ $\omega$ ] * dx[ $\psi$ ] + dy[ $\omega$ ] * dxx[ $\psi$ ])
- 2 *
  dyy[ $\omega$ ] * (dxy[ $\omega$ ] * dy[ $\varphi$ ] + dxy[ $\omega$ ] * dy[ $\varphi$ ] +
    dx[ $\omega$ ] * dyy[ $\varphi$ ] +
  dyy[ $\omega$ ] * dy[ $\psi$ ] + dyy[ $\omega$ ] * dy[ $\psi$ ] + dy[ $\omega$ ] * dyy[ $\psi$ ])
]
```

```
Integrte[ $\%$  * r, {r, 0, R}, {t, -Pi, Pi}]
```

```
-2 * Simplify[lp[lp[u]] * ( $\varphi$  * dx[u] +  $\psi$  * dy[u])] ]
```

```
1 = Integrte[ $\%$  * r, {t, -Pi, Pi}, {r, 0, 1}]
```

```
Simplify[hes2[u] * ( $\varphi$  * Cos[t] +  $\psi$  * Sin[t]) -
  2 *  $\varphi$  * (dxx[u] * dxx[u] * Cos[t] +
    dxx[u] * dxy[u] * Sin[t] + dxy[u] * dxy[u] * Cos[t] +
    dxy[u] * dyy[u] * Sin[t]) - 2 *  $\psi$  *
  (dxx[u] * dxy[u] * Cos[t] + dxy[u] * dxy[u] * Sin[t] +
    dxy[u] * dyy[u] * Cos[t] + dyy[u] * dyy[u] * Sin[t]) +
  2 *  $\varphi$  * dx[u] * (dx[lp[u]] * Cos[t] + dy[lp[u]] * Sin[t]) +
  2 *  $\psi$  * dy[u] * (dx[lp[u]] * Cos[t] + dy[lp[u]] * Sin[t])] ]
```

```
2 = Integrte[ $\%$ , {t, -Pi, Pi}]
```

```

Simplify[
  -2 * dx[u] * dx[φ] * (dxx[u] * Cos[t] + dxy[u] * Sin[t]) -
  2 * dy[φ] * dx[u] * (dxy[u] * Cos[t] + dyy[u] * Sin[t]) -
  2 * dy[u] * dx[ψ] * (dxx[u] * Cos[t] + dxy[u] * Sin[t]) -
  2 * dy[ψ] * dy[u] * (dxy[u] * Cos[t] + dyy[u] * Sin[t])]
3 = Integrte[%, {t, -Pi, Pi}]

```

*Integration by parts over half-disk with generic vector field η
(Theorem 5.2 without g , test functions φ e ψ are needed)*

$$u = r^4 * \text{Cos}[t]^4 - 2 * r^3 * \text{Cos}[t]^2 * \text{Sin}[t]$$

$$u = r^4 * \text{Cos}[t]^4 + r^4 * \text{Cos}[t]^2 * \text{Sin}[t]^2$$

$$\omega = u$$

```
Simplify[lp[u]]
```

```
Simplify[lp[%]]
```

$$\varphi = r^2 * (\text{Cos}[t]^2) - 3 * r^3 * \text{Sin}[t]^3$$

$$\psi = (r * \text{Cos}[t] + 1) * (1 + r^2)$$

$$\varphi := (1 - r^2)^6$$

$$\psi = r^2 - 7 * r^5 * \text{Sin}[t]^5$$

```

LHS = Integrate[ r * Simplify[hes2[ω] * (dx[φ] + dy[ψ])
  - 4 *
  dxy[ω] * (dxx[ω] * dy[φ] + dxy[ω] * dx[φ] +
    dx[ω] * dxy[φ] + dxy[ω] * dy[ψ] +
    dyy[ω] * dx[ψ] + dy[ω] * dxy[ψ])
  - 2 *
  dxx[ω] * (dxx[ω] * dx[φ] + dxx[ω] * dx[φ] +
    dx[ω] * dxx[φ] + dxy[ω] * dx[ψ] +
  dxy[ω] * dx[ψ] + dy[ω] * dxx[ψ])
  - 2 *
  dyy[ω] * (dxy[ω] * dy[φ] + dxy[ω] * dy[φ] +
    dx[ω] * dyy[φ] + dyy[ω] * dy[ψ] +
  dyy[ω] * dy[ψ] + dy[ω] * dyy[ψ])
], {r, 0, R}, {t, 0, Pi}]

```

```

1 = Integrate[
  -2 * Simplify[lp[lp[u]] * (φ * dx[u] + ψ * dy[u])] * r,
  {r, 0, R}, {t, 0, Pi}]

```

```

2 = R *
  Integrate[Simplify[hes2[u] * (φ * Cos[t] + ψ * Sin[t]) -
    2 * φ * (dxx[u] * dxx[u] * Cos[t] +
      dxx[u] * dxy[u] * Sin[t] + dxy[u] * dxy[u] *
      Cos[t] + dxy[u] * dyy[u] * Sin[t]) -
    2 * ψ * (dxx[u] * dxy[u] * Cos[t] + dxy[u] *
      dxy[u] * Sin[t] + dxy[u] * dyy[u] * Cos[t] +
      dyy[u] * dyy[u] * Sin[t]) + 2 * φ * dx[u] *
      (dx[lp[u]] * Cos[t] + dy[lp[u]] * Sin[t]) + 2 * ψ *
      dy[u] * (dx[lp[u]] * Cos[t] + dy[lp[u]] * Sin[t])],
  {t, 0, Pi}] /. r -> R

```

$$3 = \text{Integrte}[\text{Simplify}[\text{hes2}[u] * (-\psi) - 2 * \varphi * (-\text{dxx}[u] * \text{dxy}[u] - \text{dxy}[u] * \text{dyy}[u]) - 2 * \psi * (-\text{dxy}[u] * \text{dxy}[u] - \text{dyy}[u] * \text{dyy}[u]) + 2 * \varphi * \text{dx}[u] * (-\text{dy}[lp[u]]) + 2 * \psi * \text{dy}[u] * (-\text{dy}[lp[u]])], \{r, 0, R\}] /. t \rightarrow 0$$

$$4 = \text{Integrte}[\text{Simplify}[\text{hes2}[u] * (-\psi) - 2 * \varphi * (-\text{dxx}[u] * \text{dxy}[u] - \text{dxy}[u] * \text{dyy}[u]) - 2 * \psi * (-\text{dxy}[u] * \text{dxy}[u] - \text{dyy}[u] * \text{dyy}[u]) + 2 * \varphi * \text{dx}[u] * (-\text{dy}[lp[u]]) + 2 * \psi * \text{dy}[u] * (-\text{dy}[lp[u]])], \{r, 0, R\}] /. t \rightarrow \text{Pi}$$

$$5 = R * \text{Integrte}[\text{Simplify}[-2 * \text{dx}[u] * \text{dx}[\varphi] * (\text{dxx}[u] * \text{Cos}[t] + \text{dxy}[u] * \text{Sin}[t]) - 2 * \text{dy}[\varphi] * \text{dx}[u] * (\text{dxy}[u] * \text{Cos}[t] + \text{dyy}[u] * \text{Sin}[t]) - 2 * \text{dy}[u] * \text{dx}[\psi] * (\text{dxx}[u] * \text{Cos}[t] + \text{dxy}[u] * \text{Sin}[t]) - 2 * \text{dy}[\psi] * \text{dy}[u] * (\text{dxy}[u] * \text{Cos}[t] + \text{dyy}[u] * \text{Sin}[t])], \{t, 0, \text{Pi}\}] /. r \rightarrow R$$

$$6 = \text{Integrte}[\text{Simplify}[-2 * \text{dx}[u] * \text{dx}[\varphi] * (-\text{dxy}[u]) - 2 * \text{dy}[\varphi] * \text{dx}[u] * (-\text{dyy}[u]) - 2 * \text{dy}[u] * \text{dx}[\psi] * (-\text{dxy}[u]) - 2 * \text{dy}[\psi] * \text{dy}[u] * (-\text{dyy}[u])], \{r, 0, R\}] /. t \rightarrow 0$$

$$7 = \text{Integrte}[\text{Simplify}[-2 * \text{dx}[u] * \text{dx}[\varphi] * (-\text{dxy}[u]) - 2 * \text{dy}[\varphi] * \text{dx}[u] * (-\text{dyy}[u]) - 2 * \text{dy}[u] * \text{dx}[\psi] * (-\text{dxy}[u]) - 2 * \text{dy}[\psi] * \text{dy}[u] * (-\text{dyy}[u])], \{r, 0, R\}] /. t \rightarrow \text{Pi}$$

$$\text{RHS} = \text{Simplify}[1 + 2 + 3 + 4 + 5 + 6 + 7]$$

$$\text{Simplify}[\text{LHS} - \text{RHS}]$$