

A Universal Dilation of Discrete Markov Evolutions

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Abstract

Given a finite state space E , we build a universal dilation for all possible discrete time Markov chains on E , homogeneous or not: we introduce a second system (an “environment”) and a deterministic invertible time-homogeneous global evolution of the system E with this environment such that any Markov evolution of E can be realized by a proper choice of the initial (random) state of the environment, which therefore determines the transition probabilities of the system. We also compare this dilation with the quantum dilations of a Quantum Dynamical Semigroup: given a Classical Markov Semigroup, we show that it can be extended to a Quantum Dynamical Semigroup for which we can find a quantum dilation to a group of $*$ -automorphisms admitting an invariant abelian subalgebra where this quantum dilation gives just our classical dilation.

KEY WORDS: Markov chain; dilation; Markov semigroup; quantum dynamical semigroup.