

Here we will only touch a few highlights of the challenging topic of Hamiltonian chaos.

At first, from the more general point of view of dynamical systems, we notice that there is no widely accepted definition of chaos. It has been shown that there are features which chaos might be expected to have, but which are not all present in any one case; among them we recall: period doubling, Smale horseshoes, sensitive dependence on initial conditions, positive Lyapunov exponents. The conclusion is that for any given definition of chaos, there may always be some 'clearly' chaotic system which do not fall under that definition, thus making chaos a cousin to Gödel's undecidability.

In order to numerically detect chaotic orbits, some tools are used in literature: Poincaré maps, Lyapunov exponents, phase space diagrams, power spectra. Usually, when more than one of them are used, they are always based on the same time series, generated by the only chosen numerical method. Consequently, if the chosen numerical method is not reliable for Hamiltonian systems, even the used tools are not.

Conservative dynamical systems are not structurally stable against non-conservative perturbations. Actually the popular explicit Runge-Kutta methods (with both fixed and variable stepsize) introduce such kind of perturbations. Indeed mathematical models are often discretized according to algorithms that have little to do with the original problem, but computational methods should reflect known structural features of the problem under consideration, in particular they should preserve Hamiltonian for Hamiltonian problems.

Because of the result of Ge Zhong and Marsden in 1988, the literature divided into those favoring symplectic methods and those favoring energy conserving methods, plus recently a few trying to follow the way towards symplectic locally-conserving integrators.

For Hamiltonian systems, here we suggest the use of conservative integration, which guarantees the conservation of Hamiltonian, because they are simple and reliable. Then the drawback is that Hamiltonian phase-space structure will be not preserved. However for systems which are to remain essentially non-dissipative through numerical simulations (such systems in structural engineering, which are often stiff), conservative numerical methods seem to be the best choice.

Instead of two numerical tools using the same time series, it might be wiser to use the same tool using two different numerical simulations by two methods, one symplectic and the other conserving, taking into account the advantages and the disadvantages of both of them.

Indeed, by some more numerical caution for Hamiltonian systems, a lot of reported computed chaos might disappear.