Lipschitz Stability for a Stationary 2-D Inverse Problem with Unknown Polygonal Boundary

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Abstract

We consider the stability issue for the inverse problem of determining an unknown portion Σ of a two-dimensional simply connected domain from overdetermined boundary data for the Laplace equation. In this paper we study the case in which Σ is a polygonal line. We prove a Lipschitz stability estimate under further a priori geometric assumptions on Σ.

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1 Introduction

Let Ω be a bounded simply connected domain of $\mathbb{R}^2$ with a Lipschitz continuous boundary $\partial\Omega$, a closed part of which, Σ, is not known and not accessible. Let $\Gamma = \partial\Omega \backslash \Sigma$ and assume that Γ is known and accessible. Let γ be an open subset of Γ and let f be a nontrivial function belonging to the trace space $H^{1/2}(\partial\Omega)$, whose support is contained in γ. Let us consider the Dirichlet boundary value problem

$$\begin{cases} \triangle u = 0 \ , & \text{in } \Omega, \\ u = 0 \ , & \text{on } \Sigma, \\ u = f \ , & \text{on } \Gamma. \end{cases} \tag{1}$$

We consider the inverse problem of determining Σ provided that $\dfrac{\partial u}{\partial \nu}_{|\gamma}$ is known, where ν is the exterior unit normal to $\partial\Omega$.

This problem arises from non-destructive testing in corrosion detection [13], [14] and in thermal imaging [7], where in general the non stationary model is considered. In these cases Σ represents a corroded part of $\partial\Omega$ or a privileged isothermal line [6] and one would like to determine Σ from suitable inspections and measurements on the accessible portion Γ of the boundary.

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