Long time behavior of solutions to the Caginalp system with singular potential

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Abstract

We consider a nonlinear parabolic system which governs the evolution of the (relative) temperature ϑ and of an order parameter χ. This system describes phase transition phenomena like, e.g., melting-solidification processes. The equation ruling χ is characterized by a singular potential W which forces χ to take values in the interval $[-1, 1]$. We provide reasonable conditions on W which ensure that, from a certain time on, χ stays uniformly away from the pure phases 1 and -1. Combining this separation property with the Łojasiewicz-Simon inequality, we show that any smooth and bounded trajectory uniformly converges to a stationary state and we give an estimate of the decay rate.

Key words: phase-field models, maximal monotone operators, comparison principle, asymptotic behavior, Łojasiewicz-Simon inequality

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