Robust exponential attractors for a phase-field system with memory

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Abstract. H.G. Rotstein *et al.* proposed a nonconserved phase-field system characterized by the presence of memory terms both in the heat conduction and in the order parameter dynamics. These hereditary effects are represented by time convolution integrals whose relaxation kernels k and h are nonnegative, smooth and decreasing. Rescaling k and h properly, we obtain a system of coupled partial integrodifferential equations depending on two relaxation times ε and σ. When ε and σ tend to 0, the formal limiting system is the well-known nonconserved phase-field model proposed by G. Caginalp. Assuming the exponential decay of the relaxation kernels, the rescaled system, endowed with homogeneous Neumann boundary conditions, generates a dissipative strongly continuous semigroup $S_\varepsilon(t)$ on a suitable phase space, which accounts for the past histories of the temperature as well as of the order parameter. Our main result consists in proving the existence of a family of exponential attractors $\mathcal{E}_{\varepsilon,\sigma}$ for $S_\varepsilon(t)$, with $\varepsilon, \sigma \in [0,1]$, whose symmetric Hausdorff distance from $\mathcal{E}_{0,0}$ tends to 0 in an explicitly controlled way.

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