

Finite time blow-up and global solutions
for semilinear parabolic equations
with initial data at high energy level

Filippo GAZZOLA

Dipartimento di Matematica del Politecnico - Piazza L. da Vinci - 20133 Milano (Italy)

Tobias WETH

Mathematisches Institut - Universität Giessen - Arndtstr. 2 - 35392 Giessen (Germany)

Abstract

We consider the parabolic equation $u_t - \Delta u = |u|^{p-1}u$ ($1 < p < \frac{n+2}{n-2}$) on a bounded domain Ω under Dirichlet boundary conditions. We analyze the behavior of the solutions when the initial data varies in the phase space $H_0^1(\Omega)$. We obtain both global solutions and finite time blow-up solutions. Our main tools are the comparison principle and variational methods. Particular attention is paid for initial data at high energy level; to this end, a basic new idea is to exploit the weak dissipativity (resp. antidissipativity) of the semiflow inside (resp. outside) the Nehari manifold.

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