MEMORY RELAXATION OF FIRST ORDER EVOLUTION EQUATIONS

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Abstract. A first order nonlinear evolution equation is relaxed by means of a time convolution operator, with a kernel obtained by rescaling a given positive decreasing function. This relaxation produces an integrodifferential equation, whose formal limit, as the scaling parameter (or relaxation time) ε tends to zero, is the original equation. In particular, if the memory kernel is the decreasing exponential, then the relaxed equation is equivalent to the widely studied hyperbolic relaxation. In this work, we establish quite general conditions which ensure that the longterm dynamics of the two evolution equations are close, in some appropriate sense, when ε is small. Namely, we prove the existence of a family of robust exponential attractors for the related dissipative dynamical systems, which is stable with respect to the singular limit $\varepsilon \to 0$. The abstract result is then applied to Allen-Cahn and Cahn-Hilliard type equations.

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