

Integration by parts on the law of the reflecting Brownian motion

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Abstract

We prove an integration by parts formula on the law of the reflecting Brownian motion $X := |B|$ in the positive half line, where B is a standard Brownian motion. In other terms, we consider a perturbation of X of the form $X^\epsilon = X + \epsilon h$ with h smooth deterministic function and $\epsilon > 0$ and we differentiate the law of X^ϵ at $\epsilon = 0$. This infinitesimal perturbation changes drastically the set of zeros of X for any $\epsilon > 0$. As a consequence, the formula we obtain contains an infinite dimensional generalized functional in the sense of Schwartz, defined in terms of Hida's renormalization of the squared derivative of B and in terms of the local time of X at 0. We also compute the divergence on the Wiener space of a class of vector fields not taking values in the Cameron-Martin space.