A REACTION-DIFFUSION EQUATION WITH MEMORY

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Abstract. We consider a one-dimensional reaction-diffusion type equation with memory, originally proposed by W.E. Olmstead *et al.* to model the velocity u of certain viscoelastic fluids. More precisely, the usual diffusion term u_{xx} is replaced by a convolution integral of the form $\int_0^\infty k(s)u_{xx}(t-s)ds$, whereas the reaction term is the derivative of a double-well potential. We first reformulate the equation, endowed with homogeneous Dirichlet boundary conditions, by introducing the integrated past history of u. Then we replace k with a time-rescaled kernel k_ε, where $\varepsilon > 0$ is the relaxation time. The obtained initial and boundary value problem generates a strongly continuous semigroup $S_\varepsilon(t)$ on a suitable phase-space. The main result of this work is the existence of the global attractor for $S_\varepsilon(t)$, provided that ε is small enough.

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