DIFFERENTIAL CALCULUS ON D-POSETS WITH APPLICATIONS TO TU GAMES

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"Set functions", that is real functions defined on a family of sets, are often encountered in mathematics and its applications, in particular in mathematical economics and game theory. For example, the key concept of "transferable utility game" in cooperative game theory is formalized as a non-additive set function defined on a family of admissible sets (coalitions).
Despite to the large applications, the attempts to develop a systematic theory of calculus and subcalculus for set functions are quite recent. In particular Marinacci and Epstein introduced the concept of differentiable set function (at a set) and used this concept for proving that, under certain hypotheses, the determination of the "core" of a game is "reduced to calculus".
Moreover, from the point of view of the economic realism, it is reasonable to think of situations where not all coalitions are admissible; indeed lack of communication and information among traders or political reasons can condition the coalition formation. It becomes than natural trying to extend the results to more general structures.
In this paper, the notion of differential is introduced for functions defined on a more general structure than a σ-algebra. It is proved that, though the ambient is poorer , the main properties of the differential stay true. At the same time the result on the core of a TU game is extended to this more general situation.

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