

We study a forward-backward system of stochastic differential equations in an infinite dimensional framework and its relationships with a semilinear parabolic differential equation on a Hilbert space, in the spirit of the approach of Pardoux-Peng. We prove that the stochastic system allows to construct a unique solution of the parabolic equation in a suitable class of locally Lipschitz real functions. The parabolic equation is understood in a mild sense which requires the notion of a generalized directional gradient, that we introduce by a probabilistic approach and prove to exist for locally Lipschitz functions. The use of the generalized directional gradient allows to cover various applications to option pricing problems and to optimal stochastic control problems (including control of delay equations and reaction-diffusion equations), where the lack of differentiability of the coefficients precludes differentiability of solutions to the associated parabolic equations of Black-Scholes or Hamilton-Jacobi-Bellman type.