

# ATTRACTORS OF PHASE-FIELD SYSTEMS WITH MEMORY

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ABSTRACT. We give a systematic presentation of a series of results about the longterm behavior of certain phase-field systems with memory effects. These systems rule the evolution of the temperature  $\vartheta$  and the phase-field  $\chi$  in a material subject to phase changes. We assume that the internal energy and the heat flux depend on the past history of  $\vartheta$ . Therefore, the energy balance leads to an integro-partial differential equation for the evolution of  $\vartheta$ , coupled with a partial differential equation for  $\chi$ . The latter can be a reaction-diffusion type equation with a cubic nonlinearity or, for phase separation processes, a fourth-order equation of Cahn-Hilliard type. In both cases, by introducing the summed past history of  $\vartheta$  as an additional variable, we reformulate the equations, endowed with proper boundary conditions, as dissipative dynamical systems in suitable phase-spaces which account for the past history. Then we state theorems about the existence of finite dimensional global attractors and exponential attractors. Some stability results are also discussed.

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2000 *Mathematics Subject Classification.* 37L05, 37L25, 37L30, 45J05, 80A22.

*Key words and phrases.* Phase-field systems, memory effects, infinite-dimensional dissipative dynamical systems, global attractors, exponential attractors.

Research supported by the Italian MIUR Project *Problemi di Frontiera Libera nelle Scienze Applicate.*