

**WELL-POSEDNESS AND LONG TIME BEHAVIOR
OF A PARABOLIC-HYPERBOLIC PHASE-FIELD
SYSTEM WITH SINGULAR POTENTIALS**

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ABSTRACT. In this article, we study the long time behavior of a phase-field parabolic-hyperbolic system arising from the phase-field theory of phase transitions. This system consists of a parabolic equation governing the (relative) temperature which is nonlinearly coupled with a weakly damped semilinear hyperbolic equation ruling the evolution of the order parameter. The latter is a singular perturbation through an inertial term of the parabolic Allen-Cahn equation and it is characterized by the presence of a singular potential, e.g., of logarithmic type, instead of the classical double-well potential. We first prove the existence and uniqueness of strong solutions when the inertial coefficient ε is small enough. Then, we construct a *robust* family of exponential attractors (as ε goes to 0).

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