We consider an optimal stochastic control problem, assuming Lipschitz conditions and allowing degeneracy of the diffusion coefficient, under some structural constraint on the state equation. We formulate the problem in the strong form, i.e. we fix the probability space. We relate the value function and the feedback law to a forward-backward stochastic differential system. We prove existence and uniqueness of a global solution to the latter and deduce existence and, in some cases, uniqueness of an optimal control. To solve the (coupled) forward-backward system we use a priori estimates which follow from its control-theoretic interpretation.