EXPONENTIAL ATTRACTORS FOR A PHASE-FIELD MODEL WITH MEMORY AND QUADRATIC NONLINEARITIES

Stefania Gatti, Maurizio Grasselli, Vittorino Pata

To Professor Giovanni Prouse on his seventieth birthday

Abstract. We consider a phase-field system with memory effects. This model consists of an integrodifferential equation of parabolic type describing the evolution of the (relative) temperature ϑ, and depending on its past history. This equation is nonlinearly coupled through a function λ with a semilinear parabolic equation governing the order parameter χ. The state variables ϑ and χ are subject to Neumann homogeneous boundary conditions. The model becomes an infinite-dimensional dynamical system in a suitable phase-space by introducing an additional variable η accounting for the (integrated) past history of the temperature. The evolution of η is thus ruled by a first-order hyperbolic equation. Giorgi, Grasselli, and Pata proved that the obtained dynamical system possesses a universal attractor $\mathcal{A}$, which has finite fractal dimension provided that the coupling function λ is linear. Here we prove, as main result, the existence of an exponential attractor $\mathcal{E}$ which entails, in particular, that $\mathcal{A}$ has finite fractal dimension when λ is nonlinear with quadratic growth. Since the so-called squeezing property does not work in our framework, we cannot use the standard technique to construct $\mathcal{E}$. Instead, we take advantage of a recent result due to Efendiev, Miranville, and Zelik. The present paper contains, to the best of our knowledge, the first example of exponential attractor for an infinite-dimensional dynamical system with memory effects. Also, the approach introduced here can be adapted to other dynamical systems with similar features.

Keywords: phase-field models, memory effects, infinite-dimensional dissipative dynamical systems, invariant absorbing sets, universal attractor, exponential attractors, fractal dimension.

AMS (MOS) Classification: 35B40, 35B41, 35Q99, 37L25, 37L30, 45K05, 80A22.