NONLINEAR MARKOV SEMIGROUPS, NONLINEAR DIRICHLET FORMS AND APPLICATIONS TO MINIMAL SURFACES

FABIO CIPRIANI AND GABRIELE GRILLO

ABSTRACT. We introduce the notions of nonlinear Markov semigroups and nonlinear Dirichlet forms on a Hilbert space $L^2(X,m)$. Dirichlet forms are meant to be convex lower semicontinuous functionals on $L^2(X,m)$, enjoying contraction properties w.r.t. projections onto suitable closed and convex sets. We prove the one–to–one correspondence between these two classes of objects, by establishing Beurling–Deny–like criteria which characterize separately the non–expansion in $L^\infty(X,m)$ and the order preserving properties of the semigroup. Γ–limits of functionals enjoying the suitable contraction properties are nonlinear Dirichlet forms, and in particular this holds for relaxed functionals. Examples include elliptic, subelliptic and subriemannian p–Laplacians on Riemannian manifolds, possibly with measurable, non necessarily uniformly elliptic coefficients, nonlinear operators constructed from derivations in Hilbert C^*–modules, and convex functionals of the gradient including the area and the perimeter functionals. We apply the theory to construct Markov evolutions which approach minimal surfaces with given boundary contour, as well as Markov evolutions which converge to the solution of a Dirichlet problem with given boundary data. We also characterize, by similar methods, the domination property $|T_t u| \le S_t|u|$ between two semigroups in terms of contraction properties involving the corresponding functionals.

DIPARTIMENTO DI MATEMATICA, POLITECNICO DI MILANO, PIAZZA LEONARDO DA VINCI 32, 20133 MILANO, ITALY
E-mail address: `fabcip@mate.polimi.it`

DIPARTIMENTO DI MATEMATICA, POLITECNICO DI TORINO, CORSO DUCA DEGLI ABRUZZI 24, 10129 TORINO, ITALY
E-mail address: `grillo@calvino.polito.it`

Key words and phrases. Nonlinear Dirichlet forms, nonlinear Markov semigroups, lower semicontinuous functionals, Γ–convergence, Hilbert projections, minimal surfaces, Dirichlet problem.