

Direct and inverse inequalities for the isotropic Lamé system with variable coefficients and applications to an inverse source problem

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According to the linear theory of elasticity, we consider a bounded, compressible, and isotropic body whose mechanical behavior is described by the Lamé system with density and Lamé coefficients depending on the space variables. Assuming null surface displacement on the whole boundary, we first prove an estimate of the surface traction in terms of the energy of the solution and the body force. Then, under suitable restrictions on the density and the Lamé coefficients, we show that, in the absence of body forces, the elastic energy can be controlled by the surface traction exerted on a suitable sub-boundary, provided that the final observability time is sufficiently large. The latter condition is related with the density, the Lamé coefficients, and the geometry of the body. These inequalities are then applied to an inverse source problem for the Lamé system.