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using Sparse Neural ODEs**

Botteghi, N; Brook, O; Fasel, U; Califano, F

MOX, Dipartimento di Matematica
Politecnico di Milano, Via Bonardi 9 - 20133 Milano (Italy)

mox-dmat@polimi.it

<https://mox.polimi.it>

Interconnection and Damping Assignment Passivity-Based Control using Sparse Neural ODEs

Nicolò Botteghi^a, Owen Brook^b, Urban Fasel^b, Federico Califano^c

^a*MOX – Department of Mathematics, Politecnico di Milano, Piazza Leonardo da Vinci 32, I-20133, Milano, Italy*

^b*Department of Aeronautics, Imperial College London, United Kingdom*

^c*Department of Electrical Engineering, Mathematics, and Computer Science, University of Twente, Netherlands*

Abstract

Interconnection and Damping Assignment Passivity-Based Control (IDA-PBC) is a nonlinear control technique that assigns a port-Hamiltonian (pH) structure to a controlled system using a state-feedback law. While IDA-PBC has been extensively studied and applied to many systems, its practical implementation often remains confined to academic examples and, almost exclusively, to stabilization tasks. The main limitation of IDA-PBC stems from the complexity of analytically solving a set of partial differential equations (PDEs), referred to as the *matching conditions*, which enforce the pH structure of the closed-loop system. However, this is extremely challenging, especially for complex physical systems and tasks.

In this work, we propose a novel numerical approach for designing IDA-PBC controllers without solving the matching PDEs exactly. We cast the IDA-PBC problem as the learning of a *neural ordinary differential equation*. In particular, we rely on *sparse dictionary learning* to parametrize the desired closed-loop system as a sparse linear combination of nonlinear state-dependent functions. Optimization of the controller parameters is achieved by solving a *multi-objective optimization* problem whose cost function is composed of a generic task-dependent cost and a matching condition-dependent cost. Our numerical results show that the proposed method enables (i) IDA-PBC to be applicable to complex tasks beyond stabilization, such as the discovery of periodic oscillatory behaviors, (ii) the derivation of closed-form expressions of the controlled system, including residual terms in case of approximate matching, and (iii) stability analysis of the learned controller.

1. Introduction

Port-Hamiltonian (pH) systems are nonlinear affine systems encoding a rich set of system-theoretical and behavioral properties, conveniently modeling networks of physical systems in any physical domain [1, 2]. PH systems describe the energy exchanges occurring within the modeled physical systems, such as energy routing, dissipation, and external energy transfers through e.g., control inputs or external systems. From a control-theoretic perspective, a pH system encodes desirable properties such as passivity, stability, and general energetic behaviors that can serve as performance metrics in complex tasks [3, 4]. It is these properties that motivate control methods aiming to obtain a closed-loop pH structure. Among these energy-based techniques, *Interconnection and Damping assignment Passivity-Based Control* (IDA-PBC) [5, 4] is one of the most popular, as it assigns a general pH structure to a controlled physical system using a nonlinear state-feedback control law. Despite its popularity, IDA-PBC has remained largely limited to academic examples because of two main challenges: (i) exactly solving the matching condition, and (ii) applying the method beyond simple stabilization tasks. First, most of the methods (see e.g., [6, 7, 8, 9, 10]) aim to solve the so-called *matching conditions exactly and analytically*. The matching conditions are a set of nonlinear partial differential equations (PDEs), whose feasibility determines the possibility of shaping the desired closed-loop pH structure. However, the solution of the matching conditions is often prohibitively expensive or even impossible to achieve for complex control problems and systems. Second, while the broader motivation of energy-based methods is to realize *desired behaviors* beyond mere stabilization, almost all IDA-PBC methods in the literature are limited to solving *stabilization* tasks. This is mostly because the IDA-PBC technique has *universal stabilizing* properties, i.e., it can be proven that *all* asymptotically stabilizing controllers on control affine systems can be implemented through IDA-PBC designs [4, 5].

The first mentioned problem is recently receiving attention, as IDA-PBC-inspired designs are being investigated together with machine learning approaches, aiming at using neural networks to approximate solutions of matching PDEs. In scenarios where exact matching is impossible to achieve, the use of IDA-PBC controllers and related closed-loop analysis under *approximate matching conditions* would be highly beneficial to controller designs. In particular, *physics-informed neural networks* [11, 12] have been applied to IDA-PBC [13, 14, 15]. These approaches consist of training neural networks to minimize cost functions encoding the residuals of the matching PDEs and closed-loop stability requirements, extending the applicability of IDA-PBC methods beyond examples in which matching PDEs can be solved analytically. Notably, in [14] a robustness analysis of the stability of the closed-loop equilibrium is performed using Lyapunov arguments (a strategy initially proposed in [5]), providing robust theoretical guarantees on the closed-loop system emerging from the neural design. However, the application of neural network-based approaches, although potentially introducing flexibility in the task-based objective, are still limited to stabilization tasks. The second problem is rarely addressed. While attempts have been made to use IDA-PBC controllers for tasks other than stabilization (discussed in [5, Sec.6]), these efforts have resulted in isolated, yet notable works, such as [16], where the orbital stabilization problem is solved through exact matching.

In this paper, we address the two aforementioned limitations of IDA-PBC. In particular, we present a novel way to design IDA-PBC controllers capable of tackling general tasks, going beyond mere stabilization, while achieving, at the same time, approximate system matching along the optimal trajectories. This is a key aspect for extending the applicability of IDA-PBC designs beyond academic examples that rely on analytical solutions to the matching condition PDE. To do so, we cast IDA-PBC as a multi-objective optimal control problem. The cost function of the optimization problem combines two elements: (i) a task-dependent performance metric used to identify a desired pH closed-loop structure and encode the desired energy-based behavior, and (ii) a penalty term that accounts for the feasibility of achieving that closed-loop system using the IDA-PBC controller. The multi-objective optimal control problem is solved using neural ODEs [17]. We (i) parametrize the IDA-PBC controller, i.e., the desired-system matrices that represent the closed-loop pH dynamics, then (ii) we numerically solve the neural ODE forward and evaluate the cost of the trajectory, and eventually (iii) we optimize the parameters of the controller to minimize the cost through adjoint or backpropagation techniques. Instead of using a black-box neural ODE to parametrize our controller, we rely on dictionary learning [18] to obtain a sparse and interpretable expressions of the resulting control law and the closed-loop system. We test our approach using the electromechanical system introduced in [5]) in the task of (i) regulating the position of the capacitor plate and (ii) discovering periodic oscillations of the capacitor plate. In addition, the resulting analytical expression from the sparse dictionary learning enables the computation of the state transition matrix, a fundamental tool in stability analysis of orbits.

2. Background

2.1. Port-Hamiltonian systems and IDA-PBC

PH systems [1, 19, 2] are a class of control affine systems in the form:

$$\begin{cases} \dot{x} = (J(x) - R(x))\partial_x H(x) + g(x)u \\ y = g(x)^\top \partial_x H(x) \end{cases} \quad (1)$$

where $x \in \mathcal{D} \subseteq \mathbb{R}^n$ is the state, $u \in \mathcal{U} \subseteq \mathbb{R}^m$ is the input, $g(x)$ is input matrix, $J(x) = -J(x)^\top$ and $R(x) = R(x)^\top \geq 0$ are respectively skew-symmetric and positive semi-definite symmetric matrices representing the power-preserving and the dissipative components of the system. The non-negative function $H : \mathcal{D} \rightarrow \mathbb{R}^+$, called the *Hamiltonian*, maps the state into the total energy of the system. $\partial_x H(x) \in \mathbb{R}^n$ denotes the gradient of H , represented as a column, and $\partial_x^\top H(x)$ denotes its transposed. PH systems are *passive* [19] with respect to the Hamiltonian as a storage function:

$$\dot{H} = -\partial_x H^\top R(x) \partial_x H + y^\top u \leq y^\top u. \quad (2)$$

In extreme synthesis, the motivation underlying pH methods in control are based on assessing desired energetic behaviors from the system matrices in (3) and the power balance (2), and using the Hamiltonian as a Lyapunov function for stability analysis [3]. IDA-PBC [4, 5] is a technique that regulates the behavior of

nonlinear systems assigning a pH structure to the closed-loop system, whose peculiar parametrization can be conveniently used to represent a desired target behavior. The goal of an IDA-PBC controller is to transform the nonlinear plant system

$$\dot{x} = f(x) + g(x)u \quad (3)$$

into the desired target pH system

$$\dot{x} = (J_d(x) - R_d(x))\partial_x H_d(x). \quad (4)$$

Using the state feedback $u = \beta(x)$, the desired closed loop system (4) is obtained if

$$f(x) + g(x)\beta(x) = (J_d(x) - R_d(x))\partial_x H_d(x). \quad (5)$$

While in the space spanned by the input matrix $g(x)$, (5) can be satisfied directly by means of a proper choice of the feedback law $\beta(x)$, the difficulty arises in satisfying (5) in the *non actuated* coordinates. This is seen by pre-multiplying the latter equation by the full rank left annihilator of $g(x)$, namely the full-rank matrix $g^\perp(x) \in \mathbb{R}^{(n-m) \times n}$ such that $g^\perp(x)g(x) = 0$. The following $(n-m)$ scalar equations form a nonlinear PDE in the state variables, called *matching conditions*:

$$g^\perp(x)f(x) = g^\perp(x)(J_d(x) - R_d(x))\partial_x H_d(x). \quad (6)$$

By solving the matching conditions (6) for $g^\perp$, J_d , R_d , H_d , one achieves the target closed-loop system (4), *de facto* solving the IDA-PBC problem, using the state feedback:

$$u = g^+(x)\{(J_d(x) - R_d(x))\partial_x H_d(x) - f(x)\}, \quad (7)$$

where $g^+(x)$ is the left pseudo-inverse of g .

2.2. Neural Ordinary Differential Equations

Neural ordinary differential equations (neural ODEs) [17] are a class of deep learning models in which the evolution of the hidden states is modeled as a continuous-time dynamical system, defined by an ODE:

$$\frac{dx}{dt} = f_\phi(x(t), t),$$

where the vector field f_ϕ is parametrized by a neural network of parameter ϕ . To obtain the output, we can solve an initial value problem, i.e., we integrate the neural ODE from an initial state $x(t_0)$ at time t_0 to a final time t_f using an ODE solver:

$$\begin{aligned} x(t_f) &= \text{ODESolve}(f_\phi, x(t_0), t_0, t_f) \\ &= x(t_0) + \int_{t_0}^{t_f} f_\phi(x(t), t) dt. \end{aligned}$$

Similarly to traditional neural networks, we can optimize the parameters ϕ of the neural ODE by minimizing a loss function. In the case of a scalar-valued loss function $\mathcal{L}$, for example the mean-squared error, we obtain:

$$\mathcal{L}(\phi) = \|x_{\text{ref}} - \text{ODESolve}(f_\phi, x(t_0), t_0, t_f)\|_2^2,$$

where x_{ref} is the reference output in the context of a regression task. This operation requires the calculation of the gradients of the loss with respect to ϕ and can be performed by any neural network optimizers such as SGD or ADAM. This operation is known as the adjoint sensitivity method [20]:

$$\begin{aligned} &\min_{\phi} \mathcal{L}(\phi) \\ \text{s.t. } &\frac{dx}{dt} = f_\phi(x(t), t) \quad \text{with } x_0 = x(t_0). \end{aligned}$$

2.3. Sparse Dictionary Learning

Sparse dictionary learning methods are data-driven approaches approximating nonlinear functions as sparse linear combinations of candidate dictionary functions, e.g., polynomial or trigonometric functions [18]. Sparse dictionary learning has been used by SINDy [21] for discovering governing equations of dynamical systems. In particular, given a set of N input data $X = \{x_1, \dots, x_N\}$ and labeled outputs $Y = \{y_1, \dots, y_N\}$, we want to identify the unknown function $f : \mathcal{X} \rightarrow \mathcal{Y}$ that maps x to y as $y = f(x)$. To find the best approximation of f , we construct a dictionary of candidate functions $\Theta(X) = \{\theta_1(X), \dots, \theta_D(X)\}$. Given this dictionary, we can write the input-output relation as $Y = \Theta(X)\Xi$, where Ξ is the matrix of learnable coefficients of coefficient to be found. Unlike standard neural networks, which impose no restriction on the function class being approximated, the choice of a dictionary restricts the set of possible functions. This restriction, however, reduces the number of learnable parameters to only the coefficients of the matrix Ξ .

While typically the coefficient matrix Ξ is sparsified by sequentially thresholded least squares, alternative approaches exist. One such example is the differentiable L_0 regularization method for sparsifying neural networks [22]. This approach relaxes the discrete nature of L_0 to allow efficient gradient-based optimization and can be used for sparsifying the coefficient matrix Ξ . In the context of sparse dictionary learning, using the L_0 norm is beneficial in the context of SINDy [23]. Moreover, recent work shows that the L_0 loss provides improved performance when combining dimensionality reduction with SINDy for discovering governing equations of stochastic dynamical systems [24]. Eventually, L_0 -sparse dictionary models are used in [25] to learn interpretable polynomial policies in the context of control of parametric partial differential equations with reinforcement learning. We refer the reader to [26] for the derivation of the L_0 regularization and a step-by-step code tutorial.

3. Methodology

In this section, we propose a general methodology for IDA-PBC that relies on neural ODEs optimization and sparse dictionary learning to learn controllers for nontrivial tasks in a principled and interpretable way.

Although not the most common perspective in the literature, IDA-PBC presents a *multi-objective* nature. On one side, the shaping of the desired closed-loop system (4) should be driven by high-level task metrics, a fact motivated very deeply in all pH literature, i.e., (4) encodes a desired *behavior* of the controlled system, beyond mere stability properties [3]. On the other hand, the structure of the plant (3) influences the achievable closed-loop systems, a constraint represented by the matching condition (6). In case of a stabilization task, the closed-loop Hamiltonian H_d has a known structure – a minimum at the desired equilibrium x^* . However, for a generic task, going beyond stabilization, the shape of H_d may not be known a priori. In this work, we formally present the IDA-PBC methodology in the form of a multi-objective optimization, aiming to find the best trade-off between achieving a desired behavior and solving the matching conditions. The decision variables are $\Psi = \{J_d, R_d, H_d\}$. The cost function comprises a (i) task-based term J_{task} , and (ii) a cost penalizing a deviation from the matching condition (6):

$$J_{\text{mc}} = \|g^\perp(x)(f(x) - f_d(x))\|,$$

where $f_d(x) = ((J_d(x) - R_d(x))\partial_x H_d(x))$ depends on the current guess for the decision variables Ψ . In this case the matching condition is evaluated on a $(n - m)$ -dimensional vector representing the non-actuated coordinates, and the control law (7) is determined by the minimizer of the optimization $\Psi^* = \{J_d^*, R_d^*, H_d^*\} = \text{argmin}(J_{\text{task}} + \lambda J_{\text{mc}})$. Denoting $f_d^*(x) = ((J_d^*(x) - R_d^*(x))\partial_x H_d^*(x))$, the closed-loop system results in

$$\dot{x} = f(x) + g(x)g^+(x)((f_d^*(x) - f(x)), \quad (8)$$

which can be written as:

$$\dot{x} = f_d^*(x) - \eta(x) = f_d^*(x) - (I_n - gg^+)(f_d^*(x) - f(x)). \quad (9)$$

This form suggests another way to represent the matching conditions:

$$J_{\text{mc}} = \|\eta(x)\|.$$

In this form $g^\perp$ does not appear and the matching condition is evaluated on the complete n -dimensional state space, in a form which implicitly assumes the use of the controller (7).

Therefore, we aim to solve the following optimization problem:

$$\begin{aligned} \min_{\Psi} \quad & J_{\text{task}}(x, u) + \lambda J_{\text{mc}}(x) + \gamma J_{\text{sparse}} \\ \text{s.t.} \quad & \dot{x} = (J_d(x) - R_d(x)) \frac{\partial H_d(x)}{\partial x} - \eta(x), \quad x(0) = x_0, \end{aligned} \quad (10)$$

where J_{sparse} indicates the L_0 regularization loss mentioned in Section 2.3, used to learn a sparse controller, specifically one with the least number of active terms. The control variables are $\Psi = \{J_d, R_d, H_d\}$. To account for non-perfect matching, we include the residual $\eta(x)$ in the closed-loop system dynamics, i.e., the dynamical system in the constraint is the system obtained using the IDA-PBC controller (7) with current guesses of J_d, R_d , and H_d , which do not satisfy the matching conditions exactly, unless $\eta(x) = 0$.

For complex tasks, the structure of J_d, R_d, H_d may not be known a priori. While it is possible to parametrize J_d, R_d, H_d with neural networks [13, 14, 15], here we argue that a neural network does not allow for any type of a-posteriori interpretability. Therefore, following the work in [25], we rely on a completely differentiable sparse dictionary learning approach to approximate J_d, R_d, H_d that we can use to solve our neural ODE optimization problem. We indicate the parametrization of the matrices as $J_d(x) = J_d(x; \Xi_{J_d}, Z_{J_d})$, $R_d(x) = R_d(x; \Xi_{R_d}, Z_{R_d})$, $H_d(x) = H_d(x; \Xi_{H_d}, Z_{H_d})$ where Ξ indicates the matrix of learnable coefficients and Z the sparsification mask. In this work, for the sake of simplicity, we assume J_d, R_d having the following structure:

$$J_d(x) = \begin{bmatrix} 0 & a & \beta \\ -a & 0 & c \\ -\beta & -c & 0 \end{bmatrix}, \quad R_d(x) = \begin{bmatrix} d & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & f \end{bmatrix}, \quad (11)$$

where $a = a(x; \Xi_{J_d}, Z_{J_d})$, $\beta = \beta(x; \Xi_{J_d}, Z_{J_d})$, $c = c(x; \Xi_{J_d}, Z_{J_d})$, $d = d(x; \Xi_{R_d}, Z_{R_d})$, $e = e(x; \Xi_{R_d}, Z_{R_d})$, and $f = f(x; \Xi_{R_d}, Z_{R_d})$ are parametrized state-dependent functions. However, it is worth mentioning that any differentiable parametrization of J_d, R_d, H_d allows us to solve the optimization problem in (10) using gradient-descent methods efficiently, e.g., ADAM.

4. Numerical Results

To test our approach, we study two control problems, namely (i) regulation and (ii) discovery of an oscillatory behavior, of an electrostatic microactuator, modeled as voltage-controlled capacitor with moving plate [5]. The state is $x = (q, p, Q)$, the dynamic equations in pH form are:

$$\begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{Q} \end{bmatrix} = \left(\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1/R \end{bmatrix} \right) \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \\ \frac{\partial H}{\partial Q} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/R \end{bmatrix} u, \quad (12)$$

with $H(q, p, Q) = \frac{1}{2}k(q - q_0)^2 + \frac{1}{2m}p^2 + \frac{q}{2A\epsilon}Q^2$. Here q is the airgap in the capacitor (position of the movable plate, with the position of the fixed plate at zero), q_0 the rest length of the spring attached to the moving plate, p the momentum of the movable plate, Q the charge of the capacitor, A the area of the plate, m the mass of the plate, ϵ the permittivity in the gap, k and b spring and damping coefficients, R electric resistance, and u input voltage.

4.1. Regulation of the Capacitor's Moving Plate

The goal is to find a controller that can steer the plate of the capacitor to a desired reference q^* . We define our task-dependent cost function J_{task} as:

$$J_{\text{task}}(x, u) = J_{\text{reg}}(x) + \gamma_1 J_{\text{eff}}(u), \quad (13)$$

with

$$\begin{aligned} J_{\text{reg}}(x) &= \int_0^T (q(t) - q^*)^2 dt \\ J_{\text{eff}}(u) &= \alpha_3 \int_0^T u(t)^2 dt. \end{aligned} \quad (14)$$

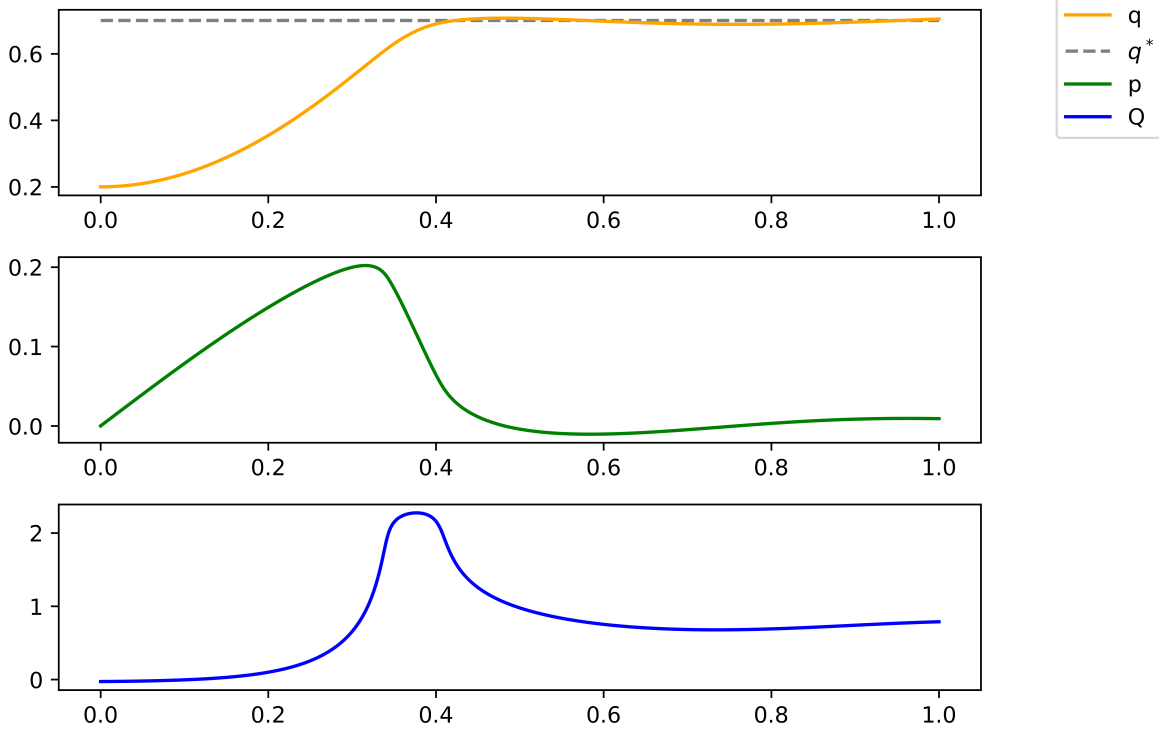


Figure 1. Evolution of the state variables over time.

Using the task-dependent cost function defined in (13), we solve the optimization problem in (10) and we show the evolution of the state variables of the closed-loop system in Figure 1. Our approach is not only capable of solving the regulation task, but in addition it provides closed-form expressions of H_d , J_d and R_d (and consequently of the matching-condition residuals). In particular, for the regulation case we obtain:

$$\begin{aligned}
H_d &= 2.5271q - 0.4659p + 4.090q^2 - 2.2953Q^2 - 1.2348p^2Q \\
a &= 0, \quad \beta = -(0.2141 - 0.7414p)Q - 4.2964qp \\
c &= 0.2217 + 1.7312pQ, \quad d = -0.1824 + 0.3306Q \\
e &= 1.6327, \quad f = 3.7035p + 6.4384qp + 6.4919pQ + 5.8333pQ^2 \\
&\quad - 1.6864qQ^3 + 4.6417pQ^3, \\
Q(0) &= -0.0275.
\end{aligned}$$

where a, β, c, d, e, f are the elements of J_d and R_d , respectively, as shown in Equation (11).

4.2. Oscillation shaping of the Capacitor's Moving Plate

We now solve a more challenging task, namely learning J_d , R_d , and H_d such that the capacitor's moving plate oscillates according to the high-level metric. To learn such oscillations, we propose a variant of the cost function introduced in [27] to learn energy-efficient oscillations for a purely mechanical system. In particular, we include an additional term $J_{\text{period}}(x)$ to promote the periodicity of the state variable Q that we call $J_{\text{period}}(x)$:

$$J_{\text{task}}(x, u) = J_{\text{mid}}(x) + \gamma_1 J_{\text{eigen}}(x) + \gamma_2 J_{\text{eff}}(u) + \gamma_3 J_{\text{period}}(x), \quad (15)$$

with:

$$\begin{aligned}
J_{\text{mid}}(x) &= \frac{\alpha_1}{2} \left(q\left(\frac{T}{2}\right) - q^* \right)^2 \\
J_{\text{eigen}}(x) &= \lambda_1 (\|q(t) - q(T-t)\|_{\infty, T} \\
&\quad + \alpha_2 \|p(t) - p(T-t)\|_{\infty, T}) + \frac{\lambda_2}{2} \|p\left(\frac{T}{2}\right)\|_2^2 \\
J_{\text{eff}}(u) &= \alpha_3 \int_0^T u(t)^2 dt, \\
J_{\text{period}}(x) &= \alpha_4 |Q(0) - Q(T)|,
\end{aligned} \tag{16}$$

where q^* is a desired position to be reached at time $t = \frac{T}{2}$, $\|\cdot\|_2$ is the 2-norm, T is the period of the oscillation, $\|\cdot\|_{\infty, T} = \max_{t \in \frac{T}{2}} (\|\cdot\|_1)$ with $\|\cdot\|_1$ indicates the 1-norm, and $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \lambda_1, \lambda_2, \lambda_3$ are scaling factors balancing the contribution of each term. The cost function J_{task} is composed of four main terms, namely J_{mid} , J_{eigen} , $J_{\text{eff}}(u)$, and $J_{\text{period}}(x)$. The first term (weakly) enforces that $q(T/2)$ to be close to a desired value q^* . The second term is derived from the eigenmanifold theory [27] and promotes symmetric trajectories for q and p with the additional constraint on $p(T/2)$ to be equal to zero. The third term considers the minimization of the control effort and penalizes aggressive and energy-inefficient controllers, while the fourth term enforces $Q(0)$ to be close to $Q(T)$. We utilize the loss function in Equation (15) to optimize the free parameters of $J_d(x)$, $R_d(x)$, $H_d(x)$ and the initial value of the variable Q , i.e., $Q(0)$ by solving the neural ODE optimization problem in (10).

In Figure 2 we show the learned oscillatory behavior achieved by numerically solving the closed-loop system with the control input, defined as in Equation (7). We highlight with red dots the initial and final values for q and p and with blue crosses the desired target values for $T/2$, with period of oscillation equal to 1 second. While we are not analytically solving the matching conditions, as it is impossible for complex systems and tasks, we are capable of keeping the norm residual of the matching conditions $\|\eta\|_2^2$ small around the optimal trajectory with an average value of 0.0009 over the first oscillation period T .

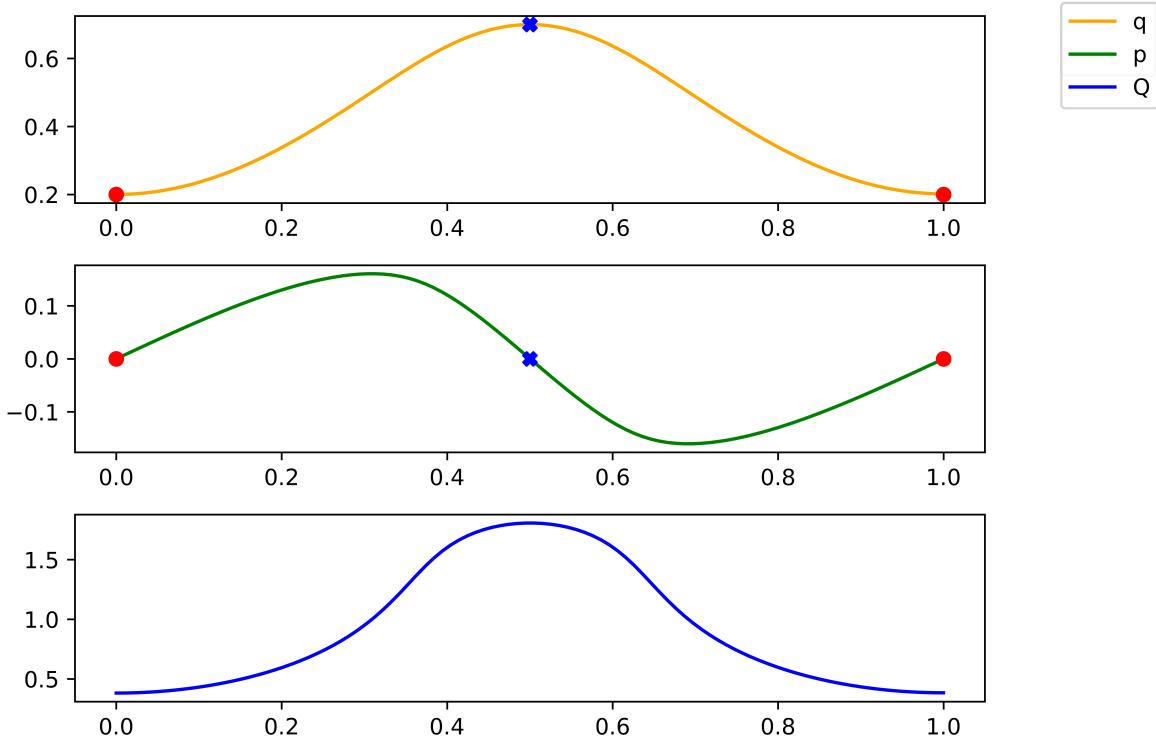


Figure 2. State evolution for one period of oscillation T .

Moreover, we report the closed-form expression for H_d , J_d , and R_d that can be used for interpretability and further analysis. In particular, we obtain:

$$\begin{aligned} H_d &= 2.0114 + 2.2373q - 1.7219Q^2 \\ J_d &= \begin{bmatrix} 0 & -0.3035 & -4.8265p \\ 0.3035 & 0 & 0.1458Q^4 \\ -4.8265p & -0.1458Q^4 & 0 \end{bmatrix} \\ R_d &= \begin{bmatrix} -0.0175 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (1.9354 + 1.5700Q + 2.6368Q^2 + 3.5828Q^3)p \end{bmatrix}. \\ Q(0) &= 0.3835. \end{aligned}$$

4.2.1. Stability of the Learned Periodic Oscillation

The use of automatic differentiation in this architecture allows us to compute the state transition matrix (STM), which describes the linearized evolution of infinitesimally small perturbations from a trajectory. We evaluate the STM along a near-periodic solution of the system across one period T . This is known as the monodromy matrix M , and its eigenvalues describe the stability of a periodic orbit [28].

We take the flow $\phi(t, x_0) : x(t_0) \rightarrow x(t)$ of a dynamical system $\dot{x} = f(x)$, and consider a small perturbation in initial condition $\delta\bar{x}_0$ from a reference trajectory $\bar{x}(t)$. The perturbation at time t from the reference is given by

$$\delta\bar{x}(t) = \phi(t; \bar{x}_0 + \delta\bar{x}_0) - \phi(t; \bar{x}_0). \quad (17)$$

Since $\delta\bar{x}_0$ is small, we take the Jacobian of the flow map

$$\delta\bar{x}(t) = \frac{\partial\phi(t; \bar{x}_0)}{\partial x_0} \delta\bar{x}_0 = \Phi(t, t_0) \delta\bar{x}_0, \quad (18)$$

which defines the STM describing the linearized evolution of infinitesimally small perturbations. Since we do not have an analytic expression for $\phi(t, x_0)$, we must find $\Phi(t, t_0)$ by numerical integration. We take the time derivative of (18), and given that the time derivative of the flow is the system dynamics $f(x)$, we consider the system

$$\dot{\Phi}(t, t_0) = \frac{\partial}{\partial x_0} \left(f(\bar{x}(t)) \right) = \frac{\partial f}{\partial \bar{x}} \frac{\partial \bar{x}(t)}{\partial x_0}. \quad (19)$$

This creates the initial value problem:

$$\dot{\Phi}(t, t_0) = Df(\bar{x}(t))\Phi(t, t_0), \quad \text{where } \Phi(t_0, t_0) = I, \quad (20)$$

which must be numerically integrated with the system dynamics to find $\Phi(t, t_0)$ at a given time.

For the controlled system (10) with initial conditions $x = [0.2; 0; 0.3835]$, which correspond to a near periodic behaviour, this gives the monodromy matrix

$$M = \begin{bmatrix} 0.9972 & 0.0233 & 0.0023 \\ -0.0615 & 0.9964 & 0.2547 \\ -0.0033 & 0.0315 & 1.0055 \end{bmatrix},$$

with eigenvalues 1.0812, 0.9990, 0.9188. For a truly periodic behaviour, we expect an eigenvalue of one, with the eigenvalue 0.9990 indicating a near-periodic behavior and a small instability associated with the eigenvalue 1.0812. The associated eigenvectors of M correspond to the local manifold directions. Computing the STM permits the use of other mathematical tools such as differential correction methods and Floquet theory, which decomposes the STM into a periodic function $P(t)$ and an exponential term with constant matrix multiplier R in the expression $\Phi(t) = P(t)e^{Rt}$ [28].

5. Conclusion

In this work, we propose a numerical approach for tackling regulation and periodic-orbit stabilization tasks using IDA-PBC methodology without solving the matching PDEs exactly. In particular, we parametrize the

desired closed-loop system, namely H_d, J_d, R_d using sparse dictionary models and optimize the learnable parameters of the neural ODE by minimizing a task-dependent cost and the residuals of the matching conditions. Results show that our approach is capable of shaping the closed-loop system to solve complex tasks, spanning from regulation to discovery of periodic behaviors, while keeping the residuals of the matching conditions minimal along the optimal trajectories. In addition, the use of automatic differentiation allows us to analyze the stability of the learned trajectory through computing the state transition matrix.

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