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Improving seismic risk protection through mathematical modeling

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1. Introduction and state of the art

Mathematical and numerical modeling can be used to better understand the physics of earthquakes, enhance seismic-risk maps and improve the design of site-strategic structures and facilities. There are two main approaches to estimate ground motion in seismic hazard analysis:

i. Ground motion prediction equations (GMPE), which are empirical relations predicting the level of ground shaking at a given site/location based on a small set of evaluation criteria such as earthquake magnitude, soil properties, source-to-site distance, and fault mechanisms;
ii. Deterministic-based numerical simulations;

As pointed out in [1], different GMPE models may lead to very different results, even if a common database is used. Moreover, GMPE strongly relies on the availability of historical earthquake records, which may be very poor for extreme earthquakes (moment magnitude bigger than 7/8 on the Richter scale) and short distance from the epicenter location. For these reasons, deterministic models have become increasingly popular because they are able to provide accurate and reliable ground-motion predictions in an affordable computational cost. By simulating a number of realistic earthquake scenarios, it is possible to obtain reliable estimates of the severity of seismic events, to quantify their possible effects on large urban areas, and establish collapse-prevention strategies for structures located in the proximity of a fault. For the above mentioned reasons, in the past twenty years, there has been an intensive research on numerical methods for linear and non-linear seismic wave propagation problems in highly heterogeneous media.

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Spectral Element (SE) methods have emerged as a powerful tool and have shown to be much more effective compared to other numerical techniques such as Finite Difference, Finite Element and Boundary Element methods. The main idea behind SE methods is that the finite dimensional space is made by high order (piecewise) polynomials sampled at Gaussian integration points. Therefore, SE methods retain the geometrical flexibility of low-order finite elements while featuring the accuracy typical of high order methods. Indeed, one of the key points of the SE method is the capability of providing an arbitrarily accurate numerical solution by simply increasing the local polynomial degree [2],[3]. In particular, for wave propagation problems, SE methods feature a very low dispersion error, yielding therefore negligible phase errors. The capabilities of SE methods have been proved successful in a wide range of phenomena governed by the elastodynamics equation, namely: i) ground and structural vibrations induced by train passage [4], [5],[6]; ii) seismic wave propagation problems [7], [8]; iii) dynamic soil-structure interaction effects related to the dynamic response of large infrastructures, such as dams and viaducts [9]. In all the above mentioned applications a key point that often represents a bottleneck for the whole simulation is the design of the computational grid that must be able to i) correctly describe the (possibly very) complex three-dimensional geometry; ii) respect the heterogeneity of the media; ii) capture all the different spatial scales that characterize the phenomenon.

For these reasons flexible strategies in modern elastodynamics codes are nowadays mandatory. In this respect, the capability of SE methods has been further improved and Discontinuous Galerkin (DGSE) methods have been recently developed and analyzed in the context of elastodynamics equations [10],[11]. The DGSE methods proposed and analyzed in [10] are based on the following domain decomposition paradigm: the computational domain is firstly partitioned into a number of nonoverlapping substructures (coherently with the – known- medium properties), then within each subdomain (continuous) spectral elements are employed, whereas across interfaces the discrete solution is discontinuous and (weak) continuity is imposed according to the DG philosophy, i.e., by penalizing the jump of the discrete displacement. Such a domain decomposition approach has been proved to be competitive for practical three-dimensional applications since the proliferation of unknowns, which is the main drawback of classical DG schemes (where the discrete space is made of elementwise discontinuous polynomials), is kept under control. Moreover, the DGSE methods proposed in [10] preserve the same accuracy of SE methods and feature low dissipation and dispersion errors, and therefore guarantee an accurate approximation of amplitudes and phases of the wave fields, which provide important information on the interior structure and consistency of soil layers. On

the other hand, DGSE methods are much more flexible than SE methods, since they can handle subdomainwise non-matching grids and variable polynomial approximation degrees, making such schemes well suited for simulations with adaptive choice of discretisation parameters. More precisely, the spatial discretization and/or the local polynomial degree can be tailored to the region of interest (e.g., buildings or civil engineering structures). Furthermore, DGSE methods enjoy a high level of intrinsic parallelism, making such a discretization technique well suited for massively parallel computations.
The DGSE methods introduced in [10] have been implemented in SPEED, a certified open-source code for the prediction of the near-fault ground motion and the seismic response of three-dimensional structures, see [12], and http://mox.polimi.it/it/progetti/speed. SPEED is developed at Politecnico di Milano by MOX (the Laboratory for Modeling and Scientific Computing) within the Department of Mathematics, jointly with the Department of Civil and Environmental Engineering. SPEED has been successfully tested in a number of realistic seismic events, including the earthquakes in L'Aquila, Italy (2009), Chile (2010), Christchurch in New Zealand (2011), and Northern Italy (2012), see [12].
In the framework of elementwise DG approximations of elastic and acoustic wave propagation problems in primal form, Interior Penalty DG methods have been proposed and analyzed in [13], [14]. These schemes have been extended to viscoelastic models in [15] and to nonlinear elastodynamics in [16]. Velocity-stress DG formulations have been proposed in [17] and [18]. A coupled elastic-acoustic wave problem approximated by DG methods has been studied in [19]. Recently, a unified framework for analyzing the stability and convergence properties of semi-discrete displacement and displacement-stress DG approximations of a general elastodynamics problem has been proposed in [20].

The rest of this paper is organized as follows. In the next section we report the governing equation modeling a visco-elastic material and its DGSEM approximation. In Section 3 we report some realistic earthquakes scenario obtained using the computational code SPEED.

2. Governing equations and DGSE discretization

We consider a visco-elastic heterogeneous medium with mass density ρ occupying an open, bounded three-dimensional region Ω with smooth boundary, and denote by $\mathbf{u}$ the medium displacement and by $\sigma(\mathbf{u})$ the Cauchy stress tensor. Given a density of body forces $\mathbf{f}$, the governing equation we consider is the following

$$\rho \mathbf{u}_{tt} + 2\rho\xi \mathbf{u}_t + \rho\xi^2 \mathbf{u} + \nabla \cdot \sigma(\mathbf{u}) = \mathbf{f} \qquad \text{in } \Omega \times [0,T] \qquad (1)$$

Here, ξ is a suitable decay factor (that scales as the inverse of time) that is used to model approximately the visco-elastic response of the medium. Notice, that a fully visco-elastic model would require in general a convolution integral between the stress and strain tensors: such an approach would be quite challenging from the computational point of view. For this reason, approximate models relying on suitable modification of the equation of motion are usually employed to model visco-elastic materials. Model (1) considered here has been proposed in [21], and in [22] it has been demonstrated that, for seismic applications, it is a sufficiently accurate approximation of the fully visco-elastic model. Equation (1) is then supplemented by suitable boundary and initial conditions. To prevent reflections at artificial boundaries, suitable absorbing boundary conditions are considered. Here, we adopt a variant of the first order approximation proposed in [23], which have been proved to be effective for seismic applications. We mention that other (and more accurate) techniques as the (Convolution) Perfectly Matched Layer, [24], [25] for example, are available to prevent spurious reflections.

To balance flexibility and computational costs, problem (1) is discretized with the DGSE method [10]. The computational domain Ω is firstly decomposed into M nonoverlapping polyhedral subdomains Ω_k, k=1,..., M, and we denote by Γ_s, s=1,....M_s, the interfaces between subdomains, cf. Fig. **1** for a representative example. Such decomposition is usually provided by Seismic Engineers or Geologists since it accounts for the material properties/soil layers. Then, within each subdomain Ω_k, we construct a grid $\mathcal{T}_h^k$ made of hexahedral elements and assign a polynomial approximation degree $N_k \geq 1$, cf. Fig. **1**. Notice that mesh generation is performed independently on each subdomain and also the local polynomial degree N_k can vary subdomainwise. We collect all the faces (here a face is the non empty interior of the intersection of two-neighboring elements) of the partition $\mathcal{T}_h^k$ that lie on the interface Γ_s, s=1,2... M_s, in the set $\mathcal{F}_s$ and adopt the following notation

$$(w,z)_{\mathcal{F}_s} = \sum_{F \in \mathcal{F}_s} (w,z)_F$$

for (regular enough) functions *w* and *z*. Problem (1) is then discretized on each subdomain Ω_k with a SEM of degree N_k and at the interfaces Γ_s the DG paradigm is employed. Then, denoting by V^h the discrete space of function that are piecewise continuous polynomials of degree N_k on each subdomain, the semi-discrete DGSEM reads as follows: for any $t \in [0,T]$, find $u^h \equiv u^h(t)$ in V_h such that

$$(\rho u_{tt}^h, v)_\Omega + 2\xi(\rho u_t^h, v)_\Omega + \xi^2(\rho u^h, v)_\Omega + \mathcal{A}(u^h, v) = \mathcal{L}(v) \qquad \forall v \in V^h, \qquad (2)$$

where $\mathcal{L}(.)$ is a suitable linear functional containing all the external loads acting on the system and $(.,.)_\Omega$ denotes the standard L^2 inner product in Ω. The bilinear form $\mathcal{A}(.,.)$ appearing in (2) is defined as follows

$$\mathcal{A}(u,v) = \sum_{k=1}^{M} (\sigma(u), \varepsilon(v))_{\Omega_k} + \sum_{s=1}^{M_s} \gamma_s([\![u]\!], [\![v]\!])_{\mathcal{F}_s} - \sum_{s=1}^{M_s} ([\![u]\!], \{\!\{\sigma(v)\}\!\})_{\mathcal{F}_s} - \sum_{s=1}^{M_s} (\{\!\{\sigma(u)\}\!\}, [\![v]\!])_{\mathcal{F}_s},$$

where the average $\{\{.\}\}$ and jump $[[.]]$ trace operators are defined as in [26], while γ_s is a suitable stabilization function, cf. [10].

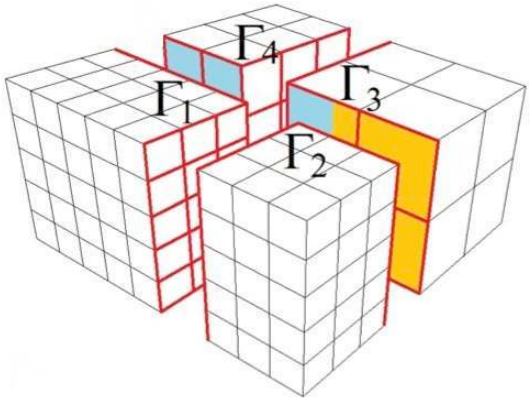

Fig. 1 Example of the partition of the computational domain Ω into M=4 nonoverlapping polyhedral subdomains and computational hexahedral grids built independently on each subdomain. The interfaces between subdomains are denoted by Γ_k, k=1,...,M.

Problem (2) reduces to a system of ordinary differential equations that we discretize by the leap-frog method.

3. Earthquakes scenarios

In the framework of earthquake scenarios SPEED has been employed for the generation of both deterministic and hybrid deterministic-stochastic ground shaking maps [27] in the following strategic areas: (i) Santiago de Chile; (ii) Christchurch, New Zealand; (iii) Wellington, New Zealand; (iv) Po Plain, North Eastern Italy. Besides a relevant interest from the economic loss exposure viewpoint, all of these sites were chosen because of sufficiently detailed information on the active faults surrounding the sites and on the shallow and deep geological structures were available. Moreover, in the Christchurch and Po Plain cases a significant amount of records were also available. Here, we focus our

attention on the generation of seismic scenarios for the Wellington area; for a detailed description of seismic hazard analysis in the other mentioned locations see [27] and [28].

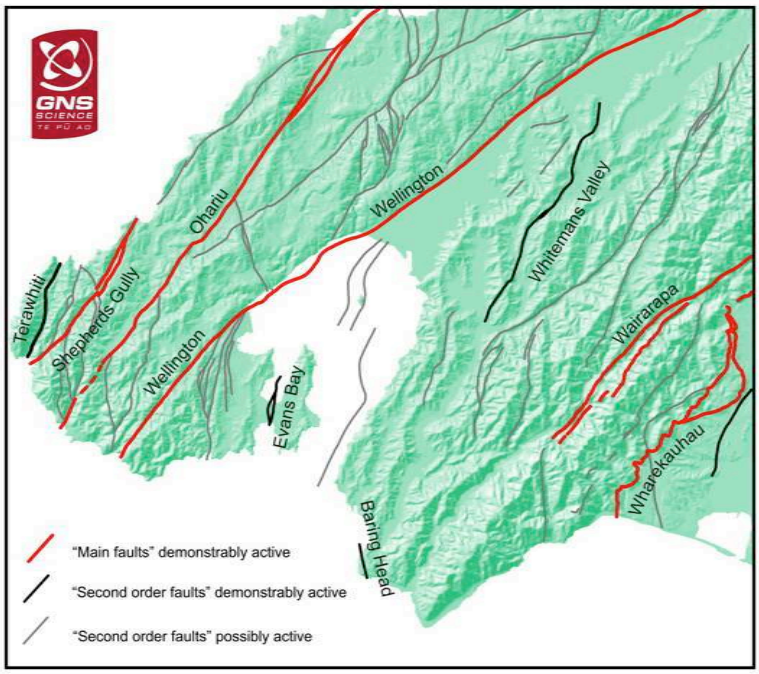

Fig. 2 Main active faults in Wellington region (courtesy of GNS institute http://www.gns.cri.nz/)

Seismic hazard in the metropolitan area of Wellington is dominated by several major active fault systems, i.e., from West to East, the Ohariu, Wellington-Hutt and Wairarapa faults, as indicated in Figure 2. The first set of seismic scenarios have been produced considering only the Wellington–Hutt fault, which is a 75-km long strike-slip fault characterized by a return period between 420 and 780 years for a magnitude between M_w 7.0 and M_w 7.8, cf. [28]. The numerical model also takes into account the most important geological features of the area, i.e., the 3D basin bedrock topography, the 3D irregular soil layers deposited over the bedrock, free-surface topography. All these features have been integrated taking into account the available geological and geophysical data (borehole, bathymetry, gravity, seismic) down to about 800 m depth [29]. To better describe such geological discontinuities, non-conforming grids have been employed to model the Wellington Valley, as depicted in Fig. **3**. The computational domain is of size

80 x 50 x 45km^3 and consists of 335496 spectral elements, yielding approximately to 30 millions of unknowns for a local polynomial approximation degree equal to 3. With such a computational grid it is possible to correctly propagate signals up to about 2 Hz. A set of 30 seismic scenarios has been generated by considering different ruptures along the Wellington-Hutt Fault. These scenarios are characterized by magnitudes ranging from 6 M_w to 7 M_w, a focal depth D varying from 5.2 Km to 10 km and several nucleations. It is relevant to underline that for these numerical simulations a linear viscoelastic soil behavior has been assumed according to equation (2).

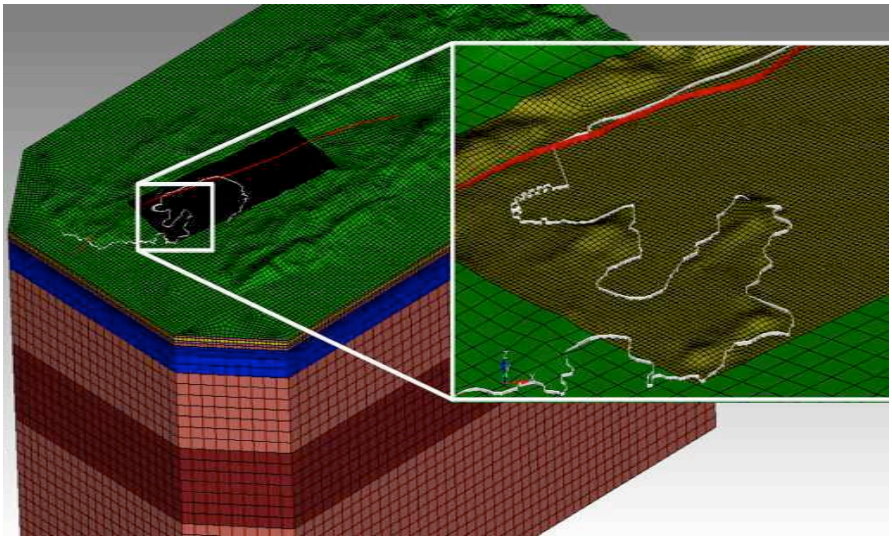

Fig. 3 Computational domain for the Wellington Valley. Zoom on the non-conforming mesh adopted for modeling the alluvial deposits (approximately 30 millions of unknowns).

As an illustrative example, Fig. 4 depicts the ground shaking maps (geometric mean of horizontal components) in terms of the Peak Ground Displacement (PGD, Fig. 4, left) and the Peak Ground Velocity (PGV, Fig. 4, right). These sets of results have been obtained for a hypothetical M_W 7 earthquake rupturing. The simulations have been performed on the FERMI BlueGene/Q cluster located at CINECA (www.hpc.cineca.it/).

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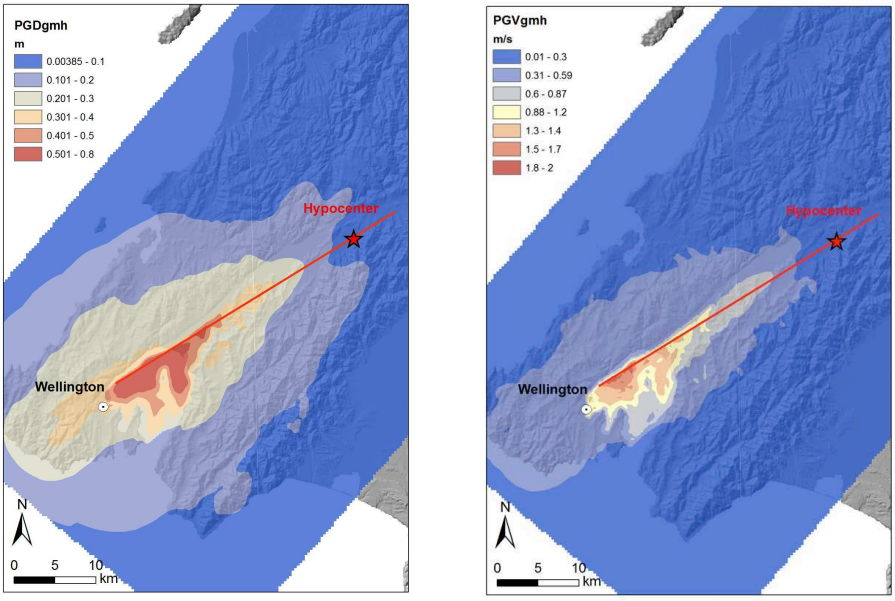

Fig. 4 Ground shaking maps for a hypothetical M_W 7 scenario along the Wellington-Hutt Fault. Peak Ground Displacement (left) and Peak Ground Velocity (right).

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