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Abstract

We consider an Extended Finite Elements method to handle the case of composite independent grids that lead to unfitted meshes. We detail the corresponding discrete formulation for the Poisson problem with discontinuous coefficients. We also provide some technical details for the 3D implementation. Finally, we provide some numerical examples with the aim of showing the effectiveness of the proposed formulation.

1 Introduction

The need of solving elliptic problems with unfitted meshes may arise in many applicative contexts, e.g. when the coefficients are discontinuous across an internal interface or when different overlapping meshes are composed to manage coupled problems.

For the first class of problems, conforming/matching grids at the interface are difficult to generate, since the internal interface provides a severe constraint for the mesh generation. To overcome such a problem, in [11] an unfitted strategy has been proposed for Finite Elements, based on doubling the degree of freedoms in the elements cut by the interface (eXtended Finite Element Method (XFEM)), see also [20]. This allows one to use a mesh which is completely independent of the interface position, see also [6, 2] for 3D applications, [25] for the case of a prescribed moving interface, [12, 4] for the case of solid mechanics, [14, 9] for the Stokes problem, [15] for the fluid-fluid coupling, and [1] for the fluid-structure interaction problem.
As regards composite meshes, usually we have to face with a (background) mesh on which another (foreground) mesh is overlapped, to account for example for different physical properties or even physical laws. In such a case we have that the background mesh is again non-matching the interface given by the intersection between the two meshes. To handle this problem with Finite Elements, a new strategy has been proposed in [13], see [16] for 3D applications, [21, 17] for the fluid equations, [7] for the fluid-structure interaction problem.

Both these situations lead to mesh elements with complex shape (polyhedra), and the proposed strategies guarantee to maintain the accuracy of the standard Finite Elements method. The common ingredient is the Discontinuous Galerkin (Nitsche) mortaring at the internal interface.

In this paper, starting from the two strategies mentioned above [11, 13], we introduce an XFEM formulation adapted to the case of composite meshes. This allows us to treat situations where, due to the thin thickness of the foreground mesh, possibly smaller than the background characteristic mesh size, an element of the background mesh is split into two or more disconnected subparts, thus generating two or more polyhedra with the foreground mesh in between. This situation occurs also at the corners of the overlapped region, where, independently of the thickness of the foreground mesh, it may happen that elements of the background mesh are split into disconnected polyhedra. On the other side, unlike the classical interface method of [11], our strategy allows us to use independent meshes, in particular to refine the foreground one without changing the background one.

The paper is organized as follows. In Section 2 we introduce the formulation for the Poisson equation on composite grids. In Section 3 we provide some details on the implementation of the proposed method in 3D. In Section 4 we show some 3D numerical tests to assess and validate the proposed method.

## 2 Numerical formulation

In this section, we introduce the numerical formulation of the problem we consider, i.e. the Poisson equation on composite meshes. Whereas at the continuous level the two resulting subdomains perfectly match the interface, we assume a different treatment of the corresponding meshes: the first one (background mesh) is fixed and in general does not match the interface; the second one (overlapping mesh) perfectly matches the interface.

### 2.1 Governing equations

Referring to Figure 1, we consider two domains \( \Omega_1 \) and \( \Omega_2 \) such that \( \Omega = \Omega_1 \cup \Omega_2 \subset \mathbb{R}^d, d = 2, 3, \) and \( \Sigma = \overline{\Omega}_1 \cap \overline{\Omega}_2 \) is the common interface. In particular the foreground domain \( \Omega_2 \) overlaps the background domain \( \Omega_1 \). We denote by \( \partial\Omega_1 \) and \( \partial\Omega_2 \) the boundaries of the background and foreground domains, respectively, and we define \( \Gamma_1 = \partial\Omega_1 \setminus \Sigma \) and \( \Gamma_2 = \partial\Omega_2 \setminus \Sigma \). Finally, we indicate with \( \mathbf{n}_1 \)
and \( m_2 \) the outward unit normal to the domain \( \Omega_1 \) and \( \Omega_2 \), respectively. On the interface \( \Sigma \) we have \( n_1 = -n_2 = n \).

**Figure 1:** Sketch of the background domain \( \Omega_1 \) and foreground domain \( \Omega_2 \) with the interface \( \Sigma \).

The continuous problem reads as follows: Given the functions \( f_1 : \Omega_1 \to \mathbb{R} \) and \( f_2 : \Omega_2 \to \mathbb{R} \), find the background solution \( u_1 : \Omega_1 \to \mathbb{R} \) and the foreground solution \( u_2 : \Omega_2 \to \mathbb{R} \), such that

\[
\begin{align*}
-\nabla \cdot (\mu_1 \nabla u_1) &= f_1 \quad \text{in } \Omega_1, \quad (1a) \\
u_1 &= 0 \quad \text{on } \Gamma_1, \quad (1b) \\
-\nabla \cdot (\mu_2 \nabla u_2) &= f_2 \quad \text{in } \Omega_2, \quad (1c) \\
u_2 &= 0 \quad \text{on } \Gamma_2, \quad (1d) \\
u_1 &= u_2 \quad \text{on } \Sigma, \quad (1e) \\
\mu_1 \nabla u_1 \cdot n &= \mu_2 \nabla u_2 \cdot n \quad \text{on } \Sigma, \quad (1f)
\end{align*}
\]

where \( T > 0 \), \( \mu_1 \) and \( \mu_2 \) are the diffusion parameters, and where, for the sake of simplicity, we have considered homogeneous Dirichlet conditions on \( \Gamma_1 \) and \( \Gamma_2 \).

We consider the spaces \( V_1 = H^1_{\Gamma_1}(\Omega_1) = \{ v \in H^1(\Omega_1); v|_{\Gamma_1} = 0 \} \) and \( V_2 = H^1_{\Gamma_2}(\Omega_2) = \{ v \in H^1(\Omega_2); v|_{\Gamma_2} = 0 \} \). The weak formulation of the problem given by (1) reads as follows: find \((u_1, u_2) \in V_1 \times V_2\) such that \( u_1 = u_2 \) on \( \Sigma \), and

\[
a_1(u_1, v_1) + a_2(u_2, v_2) = (f_1, v_1)_{\Omega_1} + (f_2, v_2)_{\Omega_2},
\]

for all \((v_1, v_2) \in V_1 \times V_2\) such that \( v_1|_{\Sigma} = v_2|_{\Sigma} \). We have indicated by \((\cdot, \cdot)_{\Omega_i}, i = 1, 2\), the \( L^2 \) product over \( \Omega_i \). Moreover, we have introduced the bi-linear forms \( a_1 : V_1 \times V_1 \to \mathbb{R} \) and \( a_2 : V_2 \times V_2 \to \mathbb{R} \) defined as

\[
a_1(u_1, v_1) = (\mu_1 \nabla u_1, \nabla v_1)_{\Omega_1}, \quad a_2(u_2, v_2) = (\mu_2 \nabla u_2, \nabla v_2)_{\Omega_2}.
\]

### 2.2 Space discretization

To ease the presentation, we assume that \( \Omega_1, \Omega_2 \) and \( \Sigma \) are polyhedral. We denote by \( T_{2,h} \) the foreground mesh that covers the domain \( \Omega_2 \) and is fitted to
Σ and Γ₂ and by \( \mathcal{T}_{1,h} \) the background mesh that covers the whole domain \( \Omega \) and is fitted to \( \Gamma_1 \), but in general not to \( \Sigma \). We indicate with \( h > 0 \) the space discretization step, which is a function varying among the elements \( K \) of the meshes. As a result, the foreground mesh \( \mathcal{T}_{2,h} \) overlaps the background mesh \( \mathcal{T}_{1,h} \), see Figure 2.

![Figure 2: The foreground mesh \( \mathcal{T}_{2,h} \) overlaps the background mesh \( \mathcal{T}_{1,h} \).](image)

We notice that the elements of the background mesh could be cut by the foreground mesh into several disconnected polyhedra, see Figure 3, left. To ease the presentation, in what follows we suppose that the foreground domain is a connected set and that the cut background elements are divided at most into two subparts, see [24] for the general case. In this situation, we can split \( \Omega_1 \) in the union of the three non-overlapping domains \( \Omega^0_{1,h}, \Omega^1_{1,h}, \Omega^2_{1,h} \), where \( \Omega^1_{1,h} \) and \( \Omega^2_{1,h} \) are formed by the background elements that are split into two disconnected polyhedra by the foreground mesh, whereas \( \Omega^0_{1,h} \) is formed by the remaining background elements, see Figure 3, center.

We denote by \( \mathcal{T}^0_{1,h} \) the smallest mesh composed of the elements \( K \in \mathcal{T}_{1,h} \) that covers the set \( \Omega^0_{1,h} \), i.e.,

\[
K \in \mathcal{T}^0_{1,h} \iff K \cap \Omega^0_{1,h} \neq \emptyset
\]

see Figure 3 (right). Finally, we denote by \( \mathcal{T}^i_{1,h}, \ i = 1, 2 \), the smallest background meshes that consist of all the elements of \( \mathcal{T}_{1,h} \) that covers the set \( \Omega^i_{1,h} \), \( i = 1, 2 \), i.e.,

\[
K \in \mathcal{T}^i_{1,h} \iff K \cap \Omega^i_{1,h} \neq \emptyset, \ i = 1, 2.
\]

In this way, each element \( K \in \mathcal{T}_{1,h} \) belongs either to \( \mathcal{T}^0_{h} \) or to \( \mathcal{T}^1_{1,h} \) and at the same time to \( \mathcal{T}^2_{1,h} \). This will allow us to double the degrees of freedom in the interfacial region. We observe that the set covered by \( \mathcal{T}^i_{1,h} \) is larger than the one covered by the corresponding \( \Omega^i_{1,h} \), see Figure 3 (right).

We can classify the faces in the region involved by the interface as follows:
• Faces belonging to the interface $\Sigma$, see Figure 4(a), where we impose weakly
the continuity of the unknown and of its normal derivative by means of
the DG formulation, see, e.g., [11, 8, 7, 16];

• The physical part $F_{h,\text{phy}}^i$ of the faces in $T_{1,h}^i$, $i = 1, 2$, see Figure 4(b),
where we impose weakly the continuity of the unknown and of its normal
derivative by means of the DG formulation, see, e.g., [3, 10];

• The computational faces $F_{h,\Sigma}^i$ of $T_{1,h}^i$, $i = 1, 2$, see Figure 4(c), where we
prescribe the ghost stabilization, see below and [5].

The degrees of freedom associated to the elements in $\Omega_{1,h}^1$ and $\Omega_{1,h}^2$ are du-
plicated according to the XFEM strategy: a set of degrees of freedom is used to
compute the solution over $T_{1,h}^1$, and a second set of degrees of freedom is
used to compute the solution over $T_{1,h}^2$. Notice that we use DG on the edges
$F_{h,\text{phy}}^i$, $i = 1, 2$ to have a more general implementation, since in some scenarios
it is not possible to enforce strongly the continuity of the background solution,
see [24].

We introduce the following spaces:

$$X_{1,h} = \{ v_h \in L^2(\Omega_1) : v_h \in C^0(\Omega_{1,h}^0), v_h|_K \in P_1(K), \forall K \in T_{1,h}^i \text{ for } i = 0, 1, 2 \},$$

and

$$X_{2,h} = \{ v_h \in C^0(\Omega_2) : v_h|_K \in P_1(K), \forall K \in T_{2,h} \}.$$

Notice that in the definition of $X_{1,h}$ we are requiring to take two times the degrees
of freedom in $T_{1,h} \setminus T_{1,h}^0$, according to the doubling of the d.o.f. characterizing
the XFEM method.
For the approximation of the background and foreground solutions \( u_{1,h} \) and \( u_{2,h} \) we consider the spaces

\[
V_{1,h} = \{ v_h \in X_{1,h} : v_h|_{\Gamma_1} = 0 \}, \quad V_{2,h} = \{ v_h \in X_{2,h} : v_h|_{\Gamma_2} = 0 \},
\]
respectively.

We introduce the following operators defined over an interface \( \mathcal{I} \). For functions \( q \) and \( q \), we denote by \([q]\) the jump and by \(\{q\}\) the mean across the interface \( \mathcal{I} \), defined as

\[
[q] = q_\alpha - q_\beta, \quad [q \cdot n] = q_\alpha \cdot n_\alpha - q_\beta \cdot n_\alpha, \quad \{q \cdot n\} = \frac{1}{2} (q_\alpha \cdot n_\alpha + q_\beta n_\alpha),
\]

where \(q_\alpha (q_\alpha)\) and \(q_\beta (q_\beta)\) are the traces of \(q (q)\) at the two sides of the interface.

The proposed space discretization of the problem given by (1) reads as follows: find \((u_{1,h}, u_{2,h}) \in V_{1,h} \times V_{2,h}\) such that

\[
a_1 (u_{1,h}, v_{1,h}) + a_2 (u_{2,h}, v_{2,h}) + g_h (u_{1,h}, v_{1,h}) - \frac{1}{2} (\mu_1 \nabla u_{1,h} \cdot n + \mu_2 \nabla u_{2,h} \cdot n, v_{1,h} - v_{2,h})_{\Sigma} - \frac{1}{2} (u_{1,h} - u_{2,h}, \mu_1 \nabla v_{1,h} \cdot n + \mu_2 \nabla v_{2,h} \cdot n)_{\Sigma} + \frac{\gamma_{\Sigma}}{h} (u_{1,h} - u_{2,h}, v_{1,h} - v_{2,h})_{\Sigma}
\]

\[
- \sum_{i=1,2} \sum_{F \in \mathcal{F}_{h,phy}} \left( \{\mu_1 \nabla u_{1,h} \cdot n\}_F, \|v_{1,h}\|_F \right)_F
- \sum_{i=1,2} \sum_{F \in \mathcal{F}_{h,phy}} \left( \|u_{1,h}\|_F, \{\mu_1 \nabla v_{1,h} \cdot n\}_F \right)_F
+ \sum_{i=1,2} \sum_{F \in \mathcal{F}_{h,phy}} \frac{\gamma_{phy}}{h} \left( \|u_{1,h}\|_F, \|v_{1,h}\|_F \right)_F = (f_1, v_{1,h})_{\Omega_1} + (f_2, v_{2,h})_{\Omega_2},
\]

for all \((v_{1,h}, v_{2,h}) \in V_{1,h} \times V_{2,h}\). We have indicated by \(\gamma_{\Sigma} > 0\) and \(\gamma_{phy} > 0\) the penalty parameters related to the interface \(\Sigma\) and to the faces in \(\mathcal{F}_{h,phy}\).
respectively. We have also introduced a ghost-penalty term, see [5], to guarantee robustness of the method with respect to the cut elements, defined as

\[ g_h(u_h, v_h) = \gamma_g \sum_{i=1,2} \sum_{F \in F_{h,\Sigma}} \mu_1 h \int_F [\nabla u_h \cdot \mathbf{n}]_F [\nabla v_h \cdot \mathbf{n}]_F, \]

with \( \gamma_g > 0 \).

The algebraic linear system associated with formulation (2) reads:

\[ RU = F, \]

with

\[ R = \begin{bmatrix} A_1 + G + E_{11} & E_{12} \\ E_{12}^T & A_2 + E_{22} \end{bmatrix}, \]

\[ U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \quad \text{and} \quad F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}, \]

and where \( A_i \) represent the standard mass and stiffness matrices of the Finite Element discretization, \( G \) is the matrix associated with the ghost penalty term, the matrices \( E \) contain the DG terms that couple the problems on the interface \( \Sigma \), while the matrices \( H \) contain the DG terms that ensure the weak continuity of the background solution on the faces \( F_{h,phy} \).

### 3 Implementation details

In this section, we provide some general comments on the implementation of the proposed XFEM/DG approach, in particular the specific features that characterize the method with respect to the standard Finite Element one. For an exhaustive discussion of this topic, we refer the reader to [24], where the general case of the fluid-structure interaction problem is considered.

The main steps to address for the implementation of the XFEM/DG approach for composite grids are:

1. The computation of the intersections between the two unfitted meshes. This step is necessary to identify the physical portions of the background mesh cut by the foreground mesh, since these are the parts where the integrals for assembling the matrices are computed, see Figure 5, left;

2. The generation of the background cut-mesh and sub-tetrahedralization. Once the intersection points have been computed and stored, it is possible to identify the portion of the background mesh composed by all the physical parts of the cut elements (background cut-mesh), see Figure 5, center. To compute the integrals over the background cut-mesh, we need to perform a sub-tetrahedralization of the cut elements of this mesh, see Figure 5, right. This procedure is driven by forcing the intersection points to be vertices of...
the final tetrahedralization and by imposing that the additional vertices lie inside the element. To satisfy these requirements, for each element $K$, we proceed hierarchically, by building first the one-dimensional mesh for each edge by using the intersection points that lie on the edge as vertices, and then in a similar way the two- and three-dimensional meshes corresponding to each face of $K$ and to $K$ itself, respectively;

3. The addition of the extended degrees of freedom according to the XFEM philosophy. In the case a background tetrahedron is cut into two (or more) physical elements by the foreground mesh, we have to double the associated dofs, see Figure 6. Hence, we will use a first set of dofs to compute the integrals over one physical element, and the second set of dofs to compute the integrals over the other physical element, see e.g. [11];

4. The integration over the physical parts of the background cut elements. The partially-overlapped background elements are in general complex polyhedra, so that the standard quadrature formulae are not applicable and specific techniques should be considered, see e.g. [19, 18, 22, 23]. Here, in order to exploit classic Gaussian quadrature rules available in a standard FEM implementation, we proceed by using the tetrahedralization generated inside each partially-overlapped element, by computing the integral over each sub-tetrahedron, and then by summing up all the contributions.

4 Numerical examples

In this Section, we present some numerical results with the aim of testing the reliability and accuracy of the proposed method. In Section 4.1, we consider a test case on a three-dimensional (3D) cube where the Poisson problem with
constant coefficients is solved on composite grids and we check the optimality of the method by a convergence test with the analytical solution (test I), whereas in Section 4.2 we solve the same problem with heterogeneous coefficients and we compare the solution with respect to the standard Finite Element method (FEM) (test II).

All the simulations have been run in the C++ finite element library LifeV (www.lifev.org) and P1 Finite Elements have been considered.

4.1 Poisson problem with constant coefficients

In this preliminary test case, we consider the Poisson equation on a simple 3D domain divided into two parts as reported in Figure 7 (left), i.e. we consider problem (1). We have $\Omega_2 = (0.25, 0.75) \times (0.25, 0.75) \times (0.49, 0.51)$, $\Omega_1 = (0, 1)^3 \setminus \Omega_2$, $\Sigma \equiv \partial \Omega_2$ and $f = 3\pi^2 \sin(\pi x) \sin(\pi y) \sin(\pi z)$. The analytic solution is

$$u(x) = \sin(\pi x) \sin(\pi y) \sin(\pi z).$$

We also use $\gamma_\Sigma = \gamma_{phy} = 10^3$ and $\gamma_g = 1$.

We consider five pairs of meshes with increasing refinement. In each case, the background and foreground meshes are unfitted and generated independently, so that we are in the case of composite grids, see Figure 7, right. We compute the error of the numerical solution with respect to the analytic one. In Table 1, we report for each pair of meshes the number of tetrahedra of the mesh $\mathcal{T}_{1,h}$ that covers the entire domain $\Omega_1 \cup \Omega_2$, the number of tetrahedra of the mesh $\mathcal{T}_{2,h}$ that covers the domain $\Omega_2$, and the average spatial size $h_{ave}$.
Table 1: Meshes used for the convergence test. $h_{ave}$ is the average step size of the corresponding meshes. Test I.

<table>
<thead>
<tr>
<th># tetrahedra in $T_{1,h}$</th>
<th># tetrahedra in $T_{2,h}$</th>
<th>$h_{ave}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>37k</td>
<td>0.8k</td>
</tr>
<tr>
<td>2</td>
<td>75k</td>
<td>1.2k</td>
</tr>
<tr>
<td>3</td>
<td>140k</td>
<td>1.8k</td>
</tr>
<tr>
<td>4</td>
<td>240k</td>
<td>2.6k</td>
</tr>
<tr>
<td>5</td>
<td>490k</td>
<td>4.3k</td>
</tr>
</tbody>
</table>

In Figure 8, up, we report the behavior of the $L^2$ and $H^1$ error with respect to the mesh step size $h$. We see that the error is optimal for both the norms as in the standard Finite Elements method. In Figure 8, bottom left, we plot the solution for one of the couples of meshes on the slice $z = 0.5$, whereas in Figure 8, bottom right, we plot the solution on the slice $y = 0.5$. This test case shows the reliability of the proposed method to treat unfitted meshes without loss of the order of convergence when considering cut elements and non-conforming interfaces.

### 4.2 Poisson problem with heterogeneous coefficients

In this test case, we consider the Poisson problem with discontinuous diffusion parameters $\mu_1$ and $\mu_2$ across $\Sigma$, and we compare the solution obtained with our approach with the classical FEM. We consider the same scenario as in test I with different measures for the foreground mesh, in particular $\Omega_2 = (0.25, 0.75)^3$. 
Figure 8: Up: plot of the error in $L^2$ and $H^1$ norm with respect to $h$. Bottom, left: solution on the slice $z = 0.5$. Bottom, right: solution on the slice $y = 0.5$. In yellow, a background element cut into two disconnected polyhedra by the foreground mesh. The interface is highlighted in red. Test I.
Notice that in this case the foreground mesh is not thin. This choice was due to emphasize the reliability of our method in the plots, see below. However, the doubling of the dofs are however needed in the corner of the foreground cube. We have again problem (1), with \( f = 3\pi^2 \sin(\pi x) \sin(\pi y) \sin(\pi z) \), and \( \mu(x) \) is piecewise constant in each subdomain:

\[
\mu(x) = \begin{cases} 
\mu_1 = \frac{1}{2} & \text{in } \Omega_1, \\
\mu_2 = 3 & \text{in } \Omega_2.
\end{cases}
\]

In this case, the solution presents a discontinuity of its derivative along the normal direction at the interface.

To prove the effectiveness of the proposed XFEM/DG approach with respect to classical FEM, we consider the following frameworks:

- in the XFEM/DG approach we consider two independent unfitted meshes, \( T_{1,h} \) and \( T_{2,h} \);
- in the classical FEM approach a single mesh \( T_{FEM}^h \) is used to solve the problem on the entire domain.

We use one of the pair of meshes considered for test I and we choose \( T_{FEM}^h \equiv T_{1,h} \), so that the FEM mesh is not fitting the interface \( \Sigma \). Moreover, we set \( \gamma_{\Sigma} = \gamma_{phy} = 10^3 \) and \( \gamma_g = 1 \).

In Figure 9, we compare the solutions along the plane \( z = 0.5 \) obtained with the two approaches. The value of the solution is reported both as the height of the plot and by means of colours. In the XFEM/DG case, it is clearly visible the discontinuity of the derivative of the solution along the interface \( \Sigma \), while the classical FEM approach is not able to capture such a discontinuity and it provides a smoothed solution. Of course, also the classical FEM is able to recover an accurate solution, however this is possible only by using conforming meshes fitted to the interface. The generation of such meshes is not a trivial task in general.

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Figure 9: Plot of the numerical solution (top) and a detail (bottom) on a slice at $z = 0.5$ that crosses the domain: XFEM/DG approach (left) and classical FEM (right). Test II.


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