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Does class matter more than school? Evidence from a multilevel statistical analysis on Italian junior secondary school students

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Abstract

This paper assesses the differences in educational attainments between students across classes and schools they are grouped by, in the context of Italian educational system. The purpose is to identify a relationship between pupils' reading test scores and students' characteristics, stratifying for classes, schools and geographical areas. The dataset contains detailed information about more than 500,000 students at the first year of junior secondary school in the year 2012/2013. By means of multilevel linear models, it is possible to estimate statistically significant school and class effects, after adjusting for pupil's characteristics, including prior achievement. The results show that school and class effects are very heterogeneous across macro-areas (Northern, Central and Southern Italy), and that there are substantial discrepancies between and within schools; overall, class effects on achievement tend to be larger than school ones.

1 Introduction and Motivation

The analysis of the differences in educational attainments between groups of students and across schools and classes is still attracting the attention of scholars of various disciplines. Studies on this topic are carried out in order to test and improve the educational system and to understand which variables mostly affect it (see [17], [25], [56]). In a policy perspective, the academic contributions in the field would understand whether attending a specific school makes a difference for current and future students' performances. For instance, Raudenbush & Bryk (see [44]), were among the first studying the effect exerted by attending a specific school on student achievement, by means of a multilevel model (and a recent re-analysis of traditional results of the Coleman Report's data to study the relative effects of family's background and school effects is in [27]); while the seminal studies of Card & Krueger (see [13]), and Betts (see [8]) examined the relationship between the characteristics of schools attended and subsequent earnings. In this context, the particular attention also at classroom-level phenomena is also corroborated by recent contributions that demonstrate that class-specific effects (see, for instance, [12] on class-level peer effects).

In Italy, the Italian Institute for the Evaluation of Educational System (hereafter INVALSI), founded in 2007, assesses students in reading and mathematics abilities at different stages, by means of standardized tests: at the end of the second and fifth year of primary school (when pupils are aged 7 and 10, respectively), at the end of the first and third year of lower secondary school (aged 11 and 13) and at the end of the second year of upper secondary school (aged 15).

Students are requested to answer questions (the same for everyone) with both multiple choices and open-ended questions, that test their ability in reading and mathematics. This is a way to test knowledge and reasoning that pupils should have learned in their school career. Also, they are requested to compile a questionnaire about themselves, their family, their parents' educational level and their socio-economic situation, with the aim of building and indicator about their background (namely ESCS; Economic, Social and Cultural Status). By means of this kind of information and of the use of multilevel linear model, it is possible to investigate the relationship between students' characteristics and performances and to define the school/class "impact", that is the effect exerted by attending a specific school/class on its students' achievements.

Studies on the mathematics achievements have been previously conducted (see [1]), applying multilevel linear models (see [18], [19], [39]); they allow to identify clear relationships between individual students' characteristics and achievements. For example, it emerged that females have worse average results than males, 1^{st} and 2^{nd} generation immigrant students have lower average performances than native Italian students, being early/late-enrolled students decreases the average results, students with a high level of socio-economical status have better performances than students with a lower one, and much more. Big differences exist between North, Center and South of Italy: students attending schools in the North obtain higher scores, all else equal, reinforcing the need for further exploring the differences across countries' geographical areas (see [4],

[32] and [52]) - a topic that is explicitly modeled in the present paper. Moreover, despite the institutional organization of the Italian educational system is based on strong assumptions about its equality purposes, based on the presumption that all schools/classes provide similar educational standards, these studies empirically proved that this is not true and that actually the country's educational system is characterized by a 'learning divide'.

In this paper we focus on the reading achievements and we deepen the understanding of school and class effects, with the broad objective of exploring if school and class effects are simultaneously impacting the achievement levels of students, and which of the two is eventually prevailing. The specific research questions are: (i) which is the relationship between pupils' characteristics, such as profile, socio-cultural background, household, cultural resources, and pupils' achievement? (ii) are there heterogeneous educational differences between different schools/classes and between the three geographical macro-areas of Italy (Northern, Central and Southern)? (iii) How the school/class effect is less/more pronounced for specific types of student profile? The main statistical tools employed in this kind of analysis are multilevel linear models (see [10]).

The work is organized as follows: Section 2 presents the dataset; in Section 3 we fit a three-level linear model for the reading achievement, in which pupils are nested in classes, that are nested in schools, in the three geographical areas; in Section 4 we analyze the school and class effects and we compare them; Section 5 contains discussion and conclusions.

All the analyses are made using the statistical software R (see [43]).

2 Theoretical framework and related literature

The present study deals with the general aim of identifying the effect of attending a specific school/classroom on the students' achievement, as measured through test scores. In this perspective, three streams of related literature influence our theoretical frame and empirical approach.

The first strand is the traditional statistical analysis of educational data (see [21] and [10]), which suggests the use of multilevel models for isolating the so called "school effect" from the other factors influencing the students' experience and results - typically, their (socioeconomic) background and (territorial's) contextual variables (see [45]). Many pioneering researches, in this context, did focus on data about single countries, and evidenced how variability of students' scores is much wider within schools than between them, and that the role of schools in determining such scores is lower than that attributable to students' individual characteristics. For instance, Mickelson et al. (2013, see [33]) conducted a meta-analysis of existent evidence about the racial achievement gap in US primary and secondary schools, by means of a two-levels hierarchical model, and highlighted that such gaps widen in higher grades. Thieme et al. (2013, see [54]), in a recent contribution, combined multilevel modeling techniques with frontier methods for studying the performance of a sample of Chilean fourth grade students. Their findings discuss how inadequate statistical analysis would attribute low performance due to out-of-control factors to school effects - so calling for using better methods for disentangling environment, schools and student-related factors. Sun et al. (2012, see [53]) use PISA 2006 data to explain the main factors associated with the science achievement of fifteen-years old students in Hong Kong, and while acknowledging the preeminent role of individuals' characteristics, they find how schools' SES composition and instruction time per week do play a differential role for students attending different schools. Benito et al. (2014, see [7]) present an application of the multilevel approach to an international perspective, with the aim of comparing the influence of system-level and school-level inequalities on students' performances in 16 countries' educational systems.

Following this broad area of academic research, we opted for implementing a three-levels multilevel model for studying simultaneously the role of students' characteristics, together with those of the class and school they are attending. Specifically, the idea of focusing on the classroom as the unit of analysis where a strong influence on students' results is exerted, is in line with those contributions in the educational psychology literature that emphasizes classroom-level features such as the 'climate' (see [46]) - in the same vein, an interesting paper by Martinez (2012, see [30]) shows how distorted can be those multilevel estimations of students' results that omit classroom-level mediating effects.

The second group of studies, which are directly connected to the present work, is the one inserted in the economics of education literature about the effect of specific schools' features on the students' performances, and more generally to the specification of an Educational Production Function (EPF) that can describe the process that leads some combinations of (human and material) inputs to 'produce' educational outputs (see [42]). The most noticed studies, in the field, are those that investigate whether school resources are statistically correlated with student achievements' differentials - or even cause them. In particular, many works conducted by prof. Hanushek (Stanford University) provoked a great debate among academics and practitioners, suggesting that higher levels of (school) resources are not associated with higher educational outputs (see, among many others [22], [23] and [24]). Therefore, a huge debate exists about the role of resources on education (see [5]), and some authors - criticizing Hanushek's approach - demonstrate that higher levels of resources are instead associated with better outcomes, if modeling is built in an adequate way (see, for instance, [28] on class size) - for evidence about school resources and educational output in UK, see [29], [51] and [26]; for a survey of literature until early 2000s, see [56]. A recently growing attention is being paid to the role of school principals and school practices in influencing students' results[1] (for instance, Bloom et al. 2015 (see [9]) apply a theoretical framework from management science to describe principals' managerial behavior, and show how these are associated with different school performances). In addition, some studies in the field use statistical models for testing the effects of certain policies - as an

[1] In addition to the role of principals and processes on students' results, some studies also looked at the impact on other outcomes/features, such as teachers' satisfaction - for an application of multilevel models to this latter setting, see [50].

example, Osht et al. (2013, see [35]) and Agasisti (2013, see [3]) use multilevel models for investigating whether higher levels of competition between schools are associated with higher/lower test scores and/or higher/lower variance of tests within them, while Mizala & Torche (2012, see [34]) employ a multilevel model for studying the impact of a universal school voucher policy in Chile on school-level segregation.

Our paper is related to this stream in that it attempts to 'explain' the differentials school/class effects - as estimated in a first stage through multilevel models - by means of a set of covariates measuring students' composition, ownership (i.e. private vs public), and other available administrative information. In so doing, we would understand if the differences in schools' effectiveness can be attributable to observable features, or instead to external (i.e. contextual) factors and/or unobservable features and processes for which we do not have available data, as for instance teachers' motivations, school leadership, etc.

Thirdly, previous researches have been conducted on the results of Italian students in primary and secondary schools. The available data for exploring the determinants of Italian students' results are traditionally two: (i) the sample from international exercises such as OECD PISA, IEA TIMMS and PIRLS, and (ii) the relatively recent waves of administrative data provided by INVALSI. The studies belonging to the first category tried to understand some factors associated with students' performances at primary/high school level: see Bratti et al. 2007 ([11]), who focused on territorial differences in mathematics test scores; Agasisti (2011, see [2]) or Ponzo (2011, see [40]), who both described the role of competition between schools in influencing their average academic results; and Ponzo & Scoppa (2014, see [41]), who estimated the 'effect' of the entry age on students' subsequent performances. INVALSI data are still under-utilized, given their relatively recent story - the first wave of available census data is about the academic year 2008/09, thus there are still few published papers to refer to. Among them, Sani & Grilli (2011, see [49]) illustrates the degree of variance between schools, which is higher in the South than in the North, and is driven by a (limited) number of few schools with very high (average) test scores. The chapter by Petracco-Giudici et al. (2010, see [37]) describes how the role of students' socioeconomic background as a critical factor that affects students' test scores. Agasisti & Vittadini (2012, see [4]) merge the INVALSI data with TIMSS ones, and build an argument around the role of territorial socioeconomic differences between Provinces. Paccagnella & Sestito (2014, see [36]) discuss the measured cheating in INVALSI test scores as a variable that is correlated with measures of (geographically-based) social capital.

Our paper extends this existent literature by using INVALSI data with the specific purpose of analyzing the role of classrooms' effects - not only the schools' one. A major innovation of our work is that it is among the few that exploits the longitudinal characteristics of INVALSI data, by considering the transition of students from grade 5 to grade 6, so allowing for a Value-Added formulation of the school and class effects.

Summarizing, we build on previous literature and innovate it in three main directions. First, we put a specific emphasis on classroom level of analysis, aiming at checking whether class-level or school-level effects are influencing more students' achievement. Second, we try to characterize the factors associated with class or school factors, following those studies that attempt deriving managerial and policy consequences for improving students' results. Thirdly, we enlarge the empirical evidence about the determinants of Italian students' test scores, by making use of Value-Added measures and multilevel models combined.

3 Dataset and Models

The dataset contains information about more than 500,000 students attending the first year of junior secondary school in the year 2012/2013, provided by Invalsi. It supplies the reading achievements of students and information at pupil, class and school's level.

At pupil's level, the following information is available: gender, immigrant status (Italian, first generation, second generation immigrant), if the student is early-enrolled (i.e. was enrolled for the first time when five years-old, the norm being to start the school when six years-old), or if the student is late-enrolled (this is the case when the student must repeat one grade, or if he/she is admitted at school one year later if immigrant). The dataset contains also information about the family's background: if the student lives or not with both parents (i.e. the parents are died, or are separated/divorced), and if the student has siblings or not. Also, Invalsi collects information about the socioeconomic status of the student, by deriving an indicator (called ESCS-Economic and Social Cultural Status), which is built in accordance to the one proposed in the OECD (The Organisation for Economic Co-operation and Development)-PISA framework, in other words by considering (i) parents' occupation and educational titles, and (ii) the possession of certain goods at home (for instance, the number of books). Once measured, this indicator has been standardized to have mean zero and variance one. The minimum and maximum observed values are about -3.11 and 2.67. In general, pupils with ESCS equal to or greater than 2 are very socially and culturally advantaged (high family's socioeconomic background). Among data, there are also the Invalsi scores in Reading test at grade 5 of the previous year (ranging between 0 and 100), which are used as a control in the multilevel model to specify a Value-Added estimate of the school's fixed effect. It is well known from the literature that education is a cumulative process, where achievement in the period t exerts an effect on results of the period $t + 1$. Lastly, Invalsi collects the oral and written pupils' grades at school in both reading and mathematics. The dataset also allows to explore several characteristics at class level, among which the class-level average of several individuals' characteristics (for example: class-average ESCS, the proportion of immigrant students, etc.). Of particular importance, there is a dummy for schools that use a particular schedule for lessons ("Tempo Pieno" classes comprise educational activities in the afternoon, and no lessons on Saturday, while traditional classes end at lunchtime, from Monday to Saturday). Also the variables at school level measure some school-average characteristics of students, such as the proportion of immigrants, early and late-enrolled students, etc. Two dummies are included to distinguish (i) private schools from public ones, and (ii) ”Istituti Comprensivi” which are schools that include both primary and lower-secondary schools in the same building/structure. This last variable is relevant to understand if the “continuity” of the same educational environment affects (positively or negatively) students results. Some variables about size (number of students per class, average size of classes, number of students of the school) are also included to take eventual scale effects into account. Lastly, regarding geographical location, we include two dummies for schools located in Central and Southern Italy and the district in which the school is located; some previous literature, indeed, pointed at demonstrating that students attending the schools located in Northern Italy tend to have higher achievement scores than their counterparts in other regions, all else equal (see [4]). As we have the anonymous student ID, we have also the encrypted school and class IDs that allow us to identify and distinguish schools and classes. The output (RS, i.e., the score in Reading standardized test administered by Invalsi) is expressed as ”cheating-corrected” test scores (CRS).[2] These variables take values between 0 and 100.

Unfortunately, there are lots of missing data in the score at grade 5. This kind of data may have been lost by the Ministry of Education in the passage of administrative information between primary and junior secondary schools. Since having longitudinal data is very important for this study, we omit the individuals with missing data at grade 5, loosing almost 300,000 students. Anyway, this new dataset is representative of the original one, without loss of information (see [1]). The final and reduced dataset collects 221,529 students, almost half of the initial dataset, within 16,246 classes, within 3,920 schools. Hereafter, all the analysis are made on this reduced dataset with 221,529 students, which has been proved to be statistically representative of the universe (data available for authors, see also Agasisti et al. [1]). The variables and some related descriptive statistics are presented in Table 1.

[2] Invalsi estimates the propensity-to-cheating as a percentage, based on the variability of intra-class percentage of correct answers, modes of wrong answers, etc.; the resulting estimates are used to ”deflate” the raw scores in the test.

Level	Type	Variable Name	Mean	sd
Student	-	Student ID	-	-
Student	(Y/N)	Female	49.8%	-
Student	(Y/N)	1^{st} generation immigrants	4.4%	-
Student	(Y/N)	2^{nd} generation immigrants	4.9%	-
Student	num	ESCS	0.24	1.02
Student	(Y/N)	Early-enrolled student	1.6%	-
Student	(Y/N)	Late-enrolled student	2.8%	-
Student	(Y/N)	Not living with both parents	12.6%	-
Student	(Y/N)	Student with siblings	83.3%	-
Student	%	Cheating	0.016	0.05
Student	num	Written reading grade	9.41	2.74
Student	num	Oral reading grade	6.80	1.13
Student	num	CRS5-5^{th} year Primary school reading score	74.5	13.50
Class	-	Class ID	-	-
Class	num	Mean ESCS	0.18	0.48
Class	%	Female percentage	43.7	10.07
Class	%	1^{st} generation immigrant percent	5.4	6.47
Class	%	2^{nd} generation immigrant percent	4.7	5.83
Class	%	Early-enrolled student percent	1.4	3.24
Class	%	Late-enrolled student percent	6.2	6.11
Class	%	Disable percentage	5.8	5.58
Class	count	Number of students	23	3.49
Class	(Y/N)	"Tempo pieno"	0.023%	-
School	-	School ID	-	-
School	num	Mean ESCS	0.18	0.41
School	%	Female percentage	43.3	5.46
School	%	1^{st} generation immigrant percent	5.4	4.65
School	%	2^{nd} generation immigrant percent	4.6	4.06
School	%	Early-enrolled student percent	1.5	2.23
School	%	Late-enrolled student percent	6.3	3.94
School	count	Number of students	143	76.52
School	count	Average number of students	22.6	2.94
School	count	Number of classes	6.2	3.05
School	(Y/N)	North	52%	-
School	(Y/N)	Center	18%	-
School	(Y/N)	South	30%	-
School	-	District	–	-
School	(Y/N)	Private	3.1%	-
School	(Y/N)	"Istituto comprensivo"	65.8%	-
Outcome	num	CRS-Reading Score corrected for Cheating	65	14.65

Table 1: Variables of the database

The main statistical tools requested to make this analysis are multilevel linear models, in which the outcome variable is the reading achievement. These models are developed using the R package *nlme* (see [38]). In particular, we develop three-level linear models in which pupils are nested in classes, that are in turn nested in schools. We consider only variables at student level with random effects on schools and classes. This allows us to individuate the relationships between the test results and the characteristics of student's profile and to estimate the random effects, such as school and class effects. Furthermore, the models decompose the total variability in pupils test scores into parts that vary between pupils, classes and schools. The Variance Partition Coefficient (VPC) captured by random effects is obtained as the proportion of random effects variance over the total variation

$$\frac{\sigma_R^2}{\sigma_R^2 + \sigma_\epsilon^2} \tag{1}$$

The histogram reporting the distribution of the CRS is shown in Figure 1.

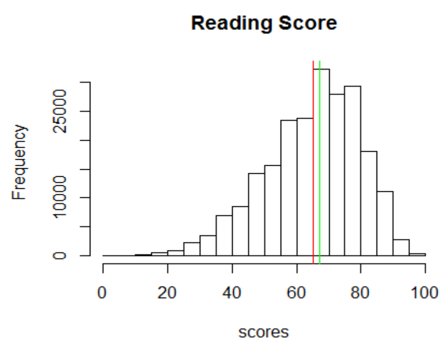

Figure 1: Histogram of Reading Score of pupils in the Invalsi database. The red line refers to the mean, the green one to the median.

Before to analyze the models and the results, there are some considerations that can be made by graphical analysis. Figure 2 shows the CRS stratified by certain students characteristics.

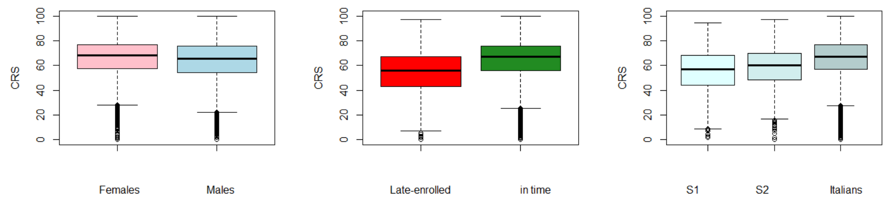

Figure 2: CRS stratified by gender, late-enrolled/in time students and immigrants/native italians.

From the first boxplots, we can assert that females have better average results than males (p-value of Wilcoxon test less than 2.2e-16). From previous studies, it can be seen that in maths this is the opposite: males have better average results than females (see [1]). From the second boxplots, it can be seen that late-enrolled students have worse average results than "in time" students (p-value of Wilcoxon test less than 2.2e-16). The last boxplots show that 1^{st} and 2^{nd} generation immigrant students have lower average performances than native italians (p-values of Wilcoxon tests less than 2.2e-16). These last two trends are similar to the ones that we obtained for maths.

Another important consideration can be made observing the CRS stratified by macro-areas (Figure 3).

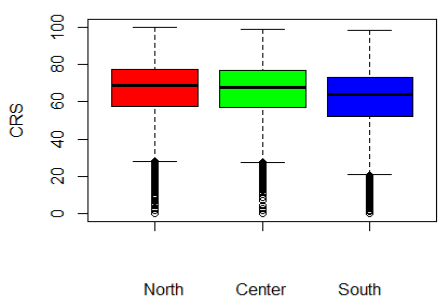

Figure 3: CRS stratified by macro-areas.

It is clear that students of the Center and especially of the South of Italy have lower average performances than students of the North (p-values of the Kruskal-Wallis tests less than 2.2e-16) - see the median and the whole distribution. In mathematics, we had the same trend and analyzing the results emerged the necessity of having three different models, one for each macro-area.

4 Three-level linear models: students nested in classes, nested in schools.

The model proposed for the empirical analysis is a three-level linear model in which students (level 1) are nested in classes (level 2), that are nested in schools (level 3). The reason for employing a three level model is that we are interested in estimating school and class effects on students' test scores simultaneously, so that we can compare their magnitude. A preliminary fixed effect model shows that there are big differences across the three macro-areas, so we fit a multilevel model for each area. The model, for pupil $i, i = 1, ..., n_{lj}^{(R)}$; $n^{(R)} = \sum_{l,j} n_{l,j}^{(R)}$, in class $l, l = 1, ..., L_j^{(R)}$; $L^{(R)} = \sum_k L_j^{(R)}$, in school $j, j = 1, ..., J^{(R)}$ can be written as:

$$y_{ilj}^{(R)} = \beta_0^{(R)} + \sum_{k=1}^{K} \beta_k^{(R)} x_{kilj} + b_j^{(R)} + u_{lj}^{(R)} + \epsilon_{ilj}^{(R)} \tag{2}$$

with

$$b_j^{(R)} \sim N(0, {\sigma_{School}}^{2(R)}), \quad u_{lj}^{(R)} \sim N(0, {\sigma_{Class}}^{2(R)}), \quad \epsilon_{ilj}^{(R)} \sim N(0, {\sigma_\epsilon}^{2(R)}) \tag{3}$$

where

$R = \{North, Center, South\}$;

y_{ilj} is the CRS of pupil i, in class l, in school j;

$\boldsymbol{\beta} = \{\beta_0, ..., \beta_K\}$ is the (K+1)-dimensional vector of parameters;

x_{kilj} is the value of the k-th predictor at student's level;

b_j is the random effect of school j;

u_{lj} is the random effect of class l, in school j;

ϵ_{ilj} is the error

and we assume b independent of ϵ and u independent of ϵ.

The estimates of model (2) are reported in Table 2.

Fixed Effects	North	Center	South	Italy
Intercept	$17.46***$	$23.28***$	$26.01***$	$20.86***$
Female	$2.15***$	$1.86***$	$2.17***$	$2.10***$
1^{st} generation immigr	$-3.48***$	$-3.27***$	$-1.59*$	$-3.39***$
2^{nd} generation immigr	$-3.38***$	$-2.98***$	$-1.18.$	$-3.14***$
Early-enrolled student	$-1.85***$	$-0.93.$	-0.31	$-0.86***$
Late-enrolled student	$-3.20***$	$-2.76***$	$-4.46***$	$-3.39***$
ESCS	$1.59***$	$2.02***$	$2.51***$	$2.01***$
not living with both parents	$-0.87***$	$-1.26***$	$-0.94***$	$-0.93***$
Student with siblings	$-0.54***$	$-0.58***$	$-0.65***$	$-0.64***$
written reading grade	0.00	0.00	-0.00	0.00
oral reading grade	$0.01***$	$0.02***$	$0.07***$	$0.03***$
CRS5	$0.64***$	$0.55***$	$0.46***$	$0.57***$
Random Effects				
σ_{School}	2.33	2.96	3.67	3.86
σ_{Class}	5.00	5.37	5.77	5.31
σ_{ϵ}	9.68	10.63	11.55	10.49
VPC_{Class}	20.1%	19.1%	18.4%	20.4%
VPC_{School}	4.4%	5.8%	7.5%	11.9%
Size				
Number of observations	$115,368$	$39,847$	$66,314$	$221,529$
Number of groups (Classes)	$7,754$	$3,066$	$5,426$	$16,246$
Number of groups (School)	$1,800$	688	$1,432$	$3,920$

Table 2: ML estimates of model (2) fitted to data of Northern, Central and Southern area and in the whole Italy. Asteriscs denote different levels of significance: . 0.01 < p-val < 0.1; * 0.001 < p-val < 0.01; ** 0.0001 < p-val < 0.001; *** p-val < 0.0001.

Looking at the coefficients of the student variables, we deduce some clear relationships. Being female increases the average result of about 2 points in all the three macro-areas, respect to being a male; while in maths males are on average better than females (see [1]), here it is the opposite. Being 1^{st} and 2^{nd} generation immigrants weighs negatively in the whole Italy, meaning that immigrants students have more difficulties than native Italian; moreover, it weighs more in the North than in the South and this is probably due to the fact that there are more immigrant students in the North than in the South. Being late/early-enrolled students or pupils not living with both parents decreases the mean test score. The ESCS is positively correlated with scores in all the three macro-areas, suggesting that pupils with a high socio-economical level are educationally advantaged, but its role is stronger in the South than in the North, suggesting that in the South the socio-cultural and familiar background influences substantially more the students' performances. The scholastic written and oral reading grades do not seem to be significant, that is, there is not a relevant correlation between INVALSI scores and scholastic grades. Lastly, the CRS5 weighs positively in the whole Italy, but more in the North than in the South, suggesting a major continuity of the students' performances in the North. In the last column, we report the coefficients estimated in the whole Italy to have a global overview of the variables at national level and to compare each area with the average national level.

In order to test if there are statistically significant differences in the coefficients of correlation between INVALSI scores and variables at student level across North and South of Italy, we compute a Fisher transformation on the two coefficients of correlation (North and South) for each variable and we make a z test. Table 3 reports the p-value of the z test, for each variable.

Variable	coef North	coef South	z	p-value
Female	0.088	0.079	1.90	0.05
1^{st} generation immigr	-0.184	-0.054	27.22	$0.00***$
2^{nd} generation immigr	-0.142	-0.022	24.73	$0.00***$
Early-enrolled student	-0.022	0.013	7.35	$0.00***$
Late-enrolled student	-0.150	-0.081	14.42	$0.00***$
ESCS	0.264	0.272	1.79	0.07
not living with both parents	-0.046	-0.040	1.23	0.21
Student with siblings	-0.044	-0.043	0.10	0.91
written reading grade	0.003	0.025	4.38	$0.00***$
oral reading grade	0.007	0.038	6.41	$0.00***$
CRS5	0.603	0.419	51.80	$0.00***$

Table 3: Test for the significance of the differences in the coefficients of correlation between INVALSI scores and students' variables across North and South of Italy. Asteriscs denote different levels of significance: . $0.01 <$ p-val < 0.1; * $0.001 <$ p-val < 0.01; ** $0.0001 <$ p-val < 0.001; *** p-val < 0.0001.

The variables that result to be statistically influential with different weights across North and South are: 1^{st} and 2^{nd} generation immigrants and late/early-enrolled, that all weigh more negatively in the North than in the South; and the INVALSI score at grade 5 and written/oral reading grades, that are more (positively) correlated with the INVALSI score in the North than in the South. These results confirm the trend that emerges by the coefficients estimated in the multilevel model showed in Table 2 and also corroborate the substantial differences of scores (and their determinants) across areas.

Regarding the random effects, in all the three macro-areas the major part of variability is explained at class level (about 20%) and a smaller part at school level (about 6%). This means that there are bigger differences within-schools (between-classes) than between-schools, so that, attending specific classes may influence the students performances more than being enrolled in a specific schools. As discussed in the later sections, this finding is very important on a practical ground. Indeed, such internal variability raises the issue of equality of opportunities not only across schools, but also within them. Moreover, the investigation of specific class and school level factors associated with achievements becomes even more important to pursue equality objectives. Also, early research suggests how important is considering classes as mediating sub-organization for school effects (see [14]).

5 An investigation of school and class effects' determinants

Now, we would like to understand how the information at school/class level is correlated with the school/class effects b_j and u_{lj}. The variables at school/class level are divided into two groups: (i) the peers effects related to the composition of the student body and (ii) managerial and structural features of the school/class. We use these variables to model the factors affecting the estimated random effects, through a simple linear model.

Regarding the school effect, the model is:

$$\hat{b}_j^{(R)} = \gamma_0^{(R)} + \sum_{k=1}^{K} \gamma_k^{(R)} z_{kj} + \eta_j^{(R)} \tag{4}$$

$$\eta_j^{(R)} \sim N(0, \sigma_\eta^2) \tag{5}$$

where

$j = 1, ..., J$ is the index of the school;

$\hat{b}_j$ is the random effect of the j-th school estimated in model (2);

z_{kj} is the value of the k-th predictor variable at school level;

$\gamma = (\gamma_0, ..., \gamma_K)$ is the (K+1)-dimensional vector of parameter;

η_j is the zero mean gaussian error.

The R^2s of the regressions are very low, suggesting that lot of variability remains unexplained. In this sense, much more detailed information about effective practices at school and class level will be necessary to extend this line of research in the next future. Moreover, the design matrices result to be affected by a high correlation among the columns, which means that it can be multicollinearity between the variables and the result can be biased. In order to address this last issue, we fit a Lasso regression model (see [55]) to the random effects estimates of each geographical area $R = \{North, Center, South\}$.

Table 4 shows the results of the three models.

Lasso Model coefficients	North	Center	South
Intercept	0.005	$-0.442*$	-0.324
Mean ESCS	$-0.695***$	-0.314	$0.687***$
Female percentage			0.016.
1^{st} generation imm perc	$0.014*$		
2^{nd} generation imm perc		$0.082***$	
Early-enrolled student perc		$-0.138**$	$-0.060**$
Late-enrolled student perc		$0.035*$	$-0.031.$
Number of classes			
Number of students	0.001.		
Average num of stud per class			
Private school	$-0.340*$		

Table 4: ML estimates of model (4) -school effects-, by macro-area, with the only variables selected by the LASSO. Asteriscs denote different levels of significance: . $0.01 <$ p-val < 0.1; * $0.001 <$ p-val < 0.01; ** $0.0001 <$ p-val < 0.001; *** p-val < 0.0001.

The average ESCS weighs in all the three macro-areas, but while it weighs negatively in the North, suggesting that there is a negative influence of the socio-economic status on the student's performances (all else equal), it weighs positively in the South, where schools with a high mean ESCS give a high positive contribution. This means that the context in which students study and the socio-cultural background of their peers are influential. In particular, in the South schools attended by socio-culturally advantaged pupils perform better than others, thus contributing to widen socio-economic initial conditions. On the other hand, in the North it seems that schools attended by students with a high average ESCS give a lower contribution to students' performances (all else equal). Other variables that seem to be significant are the ones that describe the school's composition body, such as the proportion of females, 1^{st} and 2^{nd} generation immigrant students and late/early-enrolled students. The sizes of classes and schools do not seem to be significant. The index of private school weighs only in the North and it weighs negatively.

In the same way, we estimate the class effects and we fit a linear model for each macro-area:

$$\hat{u}_{lj}^{(R)} = \alpha_0^{(R)} + \sum_{k=1}^{K} \alpha_k^{(R)} w_{ljk} + \eta_{lj}^{(R)} \tag{6}$$

$$\eta_{lj}^{(R)} \sim N(0, \sigma_{\eta}^{2(R)}) \qquad (7)$$

The coefficients selected by the Lasso regression model are reported in Table 5.

Lasso Model coefficients	North	Center	South
Intercept	$-1.70***$	$-0.54***$	$-1.03**$
Mean ESCS	$-1.19***$	$-0.59**$	$0.32**$
Female percentage			
1^{st} generation imm perc	$0.02**$	0.02	
2^{nd} generation imm perc		$0.03**$	
Early-enrolled student perc			$-0.02*$
Late-enrolled student perc	$0.03***$	$0.05***$	
Disable percentage			-0.00
Number of students	$0.06***$		$0.053***$
Tempo Pieno			

Table 5: ML estimates of model (6) - class effects- by macro-area, with the only variables selected by the LASSO. Asteriscs denote different levels of significance: . $0.01 < \text{p-val} < 0.1$; * $0.001 < \text{p-val} < 0.01$; ** $0.0001 < \text{p-val} < 0.001$; *** $\text{p-val} < 0.0001$.

The only variable relevant in all the three macro-areas is the mean ESCS of the class: in the North (coefficient -1.19) classes with a high mean ESCS give a negative contribution to students' results, instead of in the South (coefficient 0.32), where classes with a high mean ESCS give a positive value-added. Again, we have the same trend that we had in the school value-added: the average ESCS of a class is significant for the class value-added, but, while in the South classes attended by students with a higher ESCS perform better than others, in the North it is the opposite. All the analysis made until now prove that the ESCS is one of the most influential variables for the students' performances and the educational and socio-cultural context in which pupils live at home and at school can be fundamental for their education. Lastly, the percentage of immigrants is irrelevant in the South, where, however, class sizes are important, contrarily to the North. The private/public school effect disappears from all the three macroareas.

5.1 School effect vs Class effect

From the VPCs of model (2) it has been noted that the main part of the explained variability in students' test scores is explained at class level. This suggests that the main differences arise between classes, so that within schools, and not between schools. In order to clarify this aspect, figure 4 reports the histograms of the school and class effects estimated in model (2).

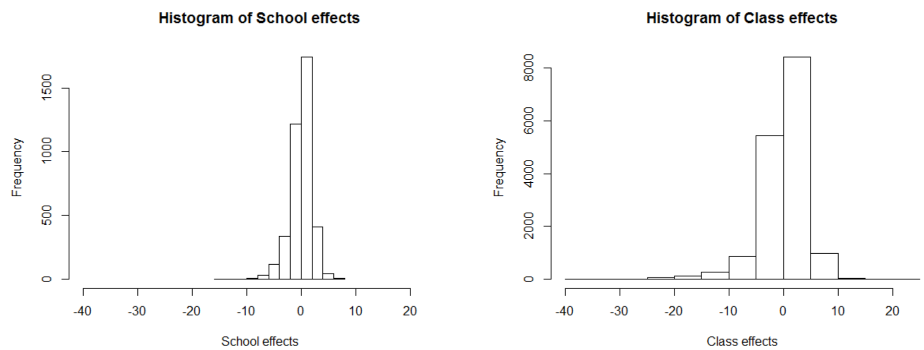

Figure 4: Histogram of school and class effects' estimates.

From previous studies (see [1]) we deduced that there are schools better than others, that it is true, but now we can assert that the main differences elapse within schools, that is between classes, that it might means between teachers and between different peer groups, if systematically different across classes.

Moreover, as we saw from the results of model (2), there are some consistent differences across macro-areas. Figure 5 shows the boxplots of school and class effects in the three macro-areas.

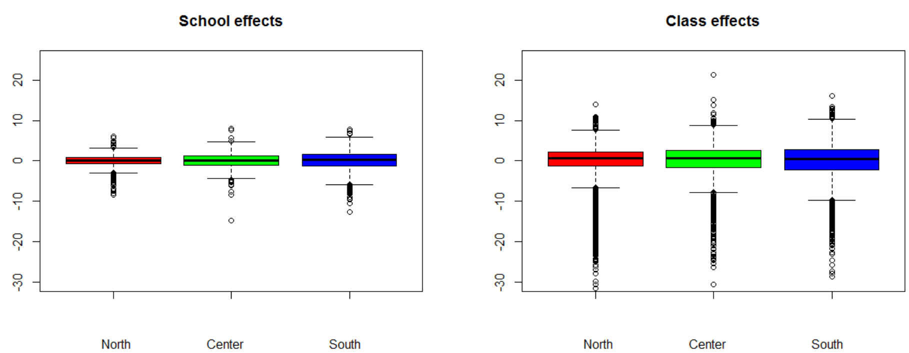

Figure 5: Boxplots of school and class effects in the three macro-areas, estimates. Colors identify macro-areas: red for the North, green for the Center and blue for the South.

In addition to confirming the importance of class effect over the school one, the boxplots show that both the class and school effects are stronger in the South than in the North. Indeed, both the variabilities of $\hat{b}_j$ and $\hat{u}_{lj}$ are higher in the South than in the North (both the p-values of the Levene's test are less than $2.2e - 16$). The between and within-schools differences are stronger in the South, where the impact of schools and classes on students' performances is higher, than in the North. In the South, students are very affected by their peers and by the context in which they study.

As a further step, we have first computed the average class effect for each school and we have then computed the correlations between school effects and contained classes effects. The main aim of such procedure is to check if school and class effects go in the same directions (i.e. negative/positive) by school or if there is some incoherence to be highlighted. Such correlations are very high in all the country: 85.2% in the North, 87.2% in the Center and 82.4% in the South. This result suggests that those schools that have a high effect on pupils' achievements usually contain classes which in turn give high effects. In particular, the positive correlations confirm that in those schools which effect is positive (negative), the average class effect tend to be positive (negative) as well. Figure 6 shows the correlation between the two effects. Therefore, there are some schools for which class effects is negative (positive) and school effects positive (negative), and this evidence highlights how the choice of a school is not guaranteing the expected results (we are discussing this point more in detail in the final section).

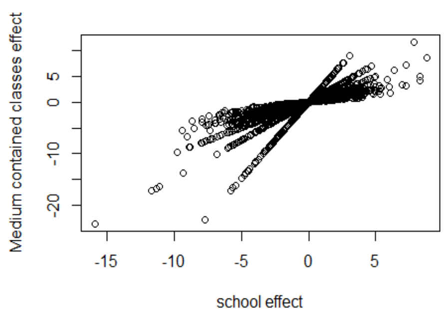

Figure 6: Correlation between school effect and contained classes average effect.

6 Concluding remarks

In this paper, we explore which are the aspects of students' profile that mostly affect their scholastic performances and which are the effects of attending specific schools and classes, the latter being this study's main aim. The empirical exercise is conducted on a sample of students at the first year of Italian junior secondary schools, scholastic year 2012/13 - and we have several variables at-hand to control for prior achievement, and student and school characteristics, and the output is a standardized test score in Reading. Coherently with some previous contributions in this field, we first observe a relationship between reading achievements and students' profile. As already pointed out with reference to test scores in mathematics (see [1]), (i) students enrolled in schools in the South of Italy have worse medium performances than students of the North, (ii) in all the three macro-areas, 1st and 2nd generation immigrants have lower medium results than native Italian students; (iii) being early/late-enrolled students decreases the medium average results, and (iv) students with a high socio-economical level are scholastically advantaged. Contrarily to results about math, females have better medium results than males in reading, as expected from international evidence on this ground (see a discussion in [20] and [47]).

Anyway, the main result of the paper focuses on random effects at school and class level, and our findings reveal that classes matter more than schools, in the sense that about the 20% of the total variability in students' achievements is explained at class level and about the 6% at school level. This means that there are more differences in students' test scores within-schools (i.e. between-classes) than between-schools, so that, attending certain classes affects students' performances more than attending certain schools. This evidence can also explain why most variables at school level turn out to be statistically uncorrelated with students' performances.

Moreover, it has been possible to compare these random effects across macro-areas, proving again that there are discrepancies within the country. Both the school and class effects are stronger in the South than in the North, suggesting that the differences between and within schools are higher in the South than in the North. This point is very relevant in a policy perspective, because it adds a piece of evidence about the role of the 'geographical achievement gap' across the country. In the past, some authors argue that differential in human capital endowment and intelligence can be called for as one factor (see the discussion in [16] and [15]), while others pointed at highlighting resources unevenly distributed (see [11]), other again discussed the mobility of teachers across the North-South directory (see [6]). Whatever the causes, our paper discusses one consequence for the geographical gap: not only the levels of (measured) cognitive skills are higher for students in the North than in the South, all else equal; but also the role of schools in influencing test scores is stronger in the South, so adding to the inequality of opportunities for those students who are enrolled in schools where the impact on achievement is negative.

We have tried then to interpret and 'explain' school/class effects using the available variables at school/class level. Whilst most of the variability of the random effects remains unexplained, some patterns of these effects' determinants can be detected. For instance, private schools seem to add lower values in terms of achievement score than the public ones, despite the higher (raw) level of achievement scores. Moreover, the other relevant variables at school and class level are the mean ESCS: in particular, in the South Italy school and class-mean ESCS positively influences the value-added of school/class, tending to increase the inequalities between more disadvantaged and advantaged students - i.e. for more advantaged students (with higher ESCS, and who attend institutions with more advantaged classmates), schools tend emphasize the positive role of background on achievement. This peer effect, in turn, can be another channel through which education reinforces gaps between students of different background (on peer effects, see [48]). A policy implication that is very direct is that information about the relative size of class and school effects should be probably disclosed (in an aggregate fashion) to parents, as the idea that choosing the school is the only critical factor is misleading - given the observed, substantial effect of specific classrooms.

Current developments are ongoing in order to clarify the relationships between the random effects (school and class effects) for reading and mathematics simultaneously, and to point out if they are coherent or not (i.e. to investigate whether schools/classes that add more value to one subject also do so in the other) (see [31]). Lastly, future research should be able to propose new school-level indicators, related to teaching and managerial practices, that can help in understanding more in detail the differences between (and within) schools' effectiveness.

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