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Numerical Methods for Flow in Fractured Porous Media

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Definition

- porous medium: A material containing voids (pores), whose size is small compared to the size of the sample. The pores are typically filled with a fluid. A porous medium is characterized by macroscopic properties which are obtained with an averaging procedure.
- fracture: a break in a material with a small thickness compared to its global extension.
- fracture network: a network composed by several intersecting fractures.

1 Introduction

Fractures are ubiquitous in porous media. Here, with the term fracture we denote a void in the porous material that has the following characteristics.

i) One of its dimensions, the aperture, is orders of magnitude smaller than the other dimensions and the size of the domain of interest, but still large compared to pore size. We will indicate with extension the size of a fracture in the directions orthogonal to the aperture. The extension of fractures in a network has a distribution that is usually assumed to be governed by a power law, which implies the presence of a large variation of space scales. See Fig 1.

ii) Fractures may be either open or infilled by a porous medium, whose physical characteristics may be strongly different from those of the surrounding material.

iii) Fractures usually form networks, often highly connected. An example of fracture network is depicted in Fig 4.

With the previous definitions one can consider as fractures, depending on the scale of interest, different objects such as: tectonic faults at the scale of sedimentary basins, cracks in glaciers, as well as fractures in concrete or in rocks.

When they are empty or filled with highly permeable materials, fractures may provide a preferential path to fluid flow, but in some cases the deposits inside fractures can become nearly impermeable. We refer to the latter situation as blocking fracture.

The presence of fractures greatly alters the macroscopic properties of the material in a complex way, in particular its mechanical and flow characteristics. We will be here concerned with the second aspect, and specifically on the mathematical modelling and computing techniques that may be adopted in the presence of fractured porous media. For a more general treatment of fractures in porous media the reader may refer to the Book of P.M. Adler et al. [1].

The irregular spatial distribution of fractures and the presence of multiple scales makes it difficult and often impossible to account for fractures by deriving effective upscaled parameters, permeability for instance, by volume averaging or homogeneization techniques. Indeed, such procedures assume a strong separation of scales. Therefore, methods have been developed that model fractures explicitly, at least those crucial for the flow. These models are based on the assumption that in the porous medium flow is governed by Darcy's law, and often a similar model

is adopted for the flow taking place in the fractures as well.

We recall the main characteristic of the Darcy's model, considering, for simplicity, just the case of single-phase flow. In this mathematical framework the two main variables are the pressure p and the macroscopic velocity field $\boldsymbol{u}$, also called Darcy's velocity. The two quantities are related by Darcy's law,

$$\mu \boldsymbol{K}^{-1}\boldsymbol{u} + \nabla p = \boldsymbol{0} \quad \text{in } \Omega, \tag{1a}$$

where $\Omega \subset \mathbb{R}^d$ represents the domain occupied by the porous material and ∇ indicates the gradient. In the case where gravity effect are relevant, equation (1) may be modified by replacing the pressure term with $p - \rho g z$, where ρ is the fluid density, g the magnitude of the gravity acceleration and z is the vertical coordinate pointing upwards from the Earth surface. The main hypotheses behind the model are that fluid velocity is small, so we can neglect inertial effects, and the main model parameters are: μ, the fluid viscosity, and $\boldsymbol{K}$, the permeability tensor of the porous medium, which is a symmetric and positive definite tensor. Permeability may be heterogeneous in space and often with high variations.

The second equation expresses continuity of mass by the following differential equation,

$$c\phi\partial_t p + \nabla \cdot \boldsymbol{u} = q \quad \text{in } \Omega, \tag{1b}$$

where $\nabla\cdot$ is the divergence operator, q a source/sink term, c accounts for the medium and fluid compressibility and ϕ is the porosity. Sometimes one is interested in the steady state solution or the compressibility can be neglected, in which case $c\phi\partial_t p = 0$.

Equations (1) form a system of partial differential equations which, complemented by appropriate boundary and initial conditions, allows to describe the evolution of $(\boldsymbol{u}, p)$ in the porous medium.

2 Numerical models for fractured porous media

Numerical models for fractured porous media may be subdivided into two main categories: Continuum Fracture Models (CFMs) and Discrete Fracture Models (DFMs).
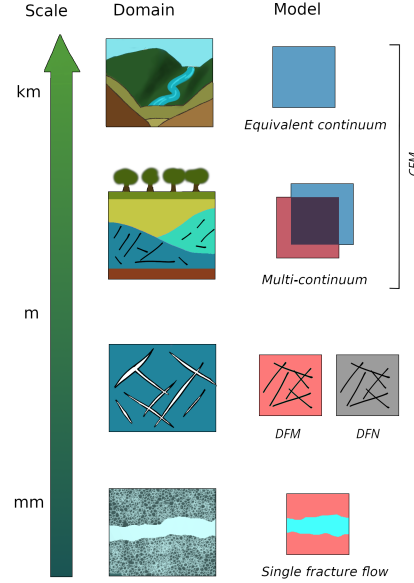

Fig. 1: Choice of the numerical model for domains at different space scales.

2.1 Continuum Fracture Models

CFMs are early models introduced in the 1960's [8], later justified mathematically in [2], and currently implemented in many industrial software. They assume a highly permeable and interconnected fracture network so that it can be modeled as a continuum superimposed to that of the porous medium.

A commonly adopted method is the dual-porosity/dual-permeability scheme. It assumes that at each point of the domain Ω we may use the Darcy's equations for flow in the fractures and in the porous medium, respectively, with a term representing the interchange of mass between them. A basic model of this type may be written as:

$$\begin{aligned} c_m\phi_m\partial_t p_m + \nabla \cdot \boldsymbol{u}_m - \alpha(p_m - p_f) = q_m \\ c_f\phi_f\partial_t p_f + \nabla \cdot \boldsymbol{u}_f + \alpha(p_m - p_f) = q_f \end{aligned} \quad \text{in } \Omega, \tag{2a}$$

where suffixes m and f refer to quantities related to the porous medium and fractures, respectively. The term $\alpha(p_m - p_f)$ represents the mass exchange between the two components, with α a rate parameter. The Darcy's velocities $\boldsymbol{u}_m$ and $\boldsymbol{u}_f$ are given by

$$\begin{aligned} \mu \boldsymbol{K}_m^{-1}\boldsymbol{u}_m + \nabla p_m = \boldsymbol{0} \\ \mu \boldsymbol{K}_f^{-1}\boldsymbol{u}_f + \nabla p_f = \boldsymbol{0} \end{aligned} \quad \text{in } \Omega. \tag{2b}$$

A simpler model, called dual-porosity/single-permeability, may be obtained formally by setting $K_m = 0$. It assumes that the porous medium acts as storage volume for flow occurring only along fractures. A further simplification, appropriate only for highly connected networks of fracture of small extension relative to the domain size, consists in the use of a single equivalent continuum with upscaled properties that account for the combined effect of porous medium and fractures.

2.2 Discrete Fracture Models

DFMs represent fractures explicitly, modeled as a network of (typically planar) surfaces Γ, immersed in the porous medium. In the fractures we typically use a Darcy-type model where some special source terms are added to account for the fluid exchange with the porous medium. The Darcy equations in the latter are also modified, with terms that act as interface conditions.

DFMs are computationally more demanding that CFMs, but also more accurate, particularly when fracture of relative large extension are present. For this reason the choice between CFM and DFM may depend on the spatial scale of interest, as illustrated in Fig. 1.

On each fracture we may identify a unit normal vector n_f, and thus a positive and a negative side of Γ, see Fig. 2. We indicate with $[\![f]\!] = f^+ - f^-$ the jump of a quantity f across the fracture.
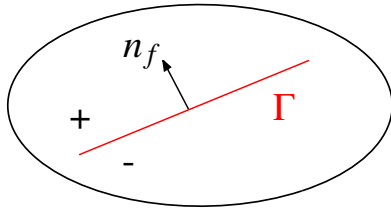

Fig. 2: Positive and negative side of Γ.

A commonly used model is derived in [7], in which the fracture permeability is split into a normal, $K_{f,n}$, and tangential, $K_{f,t}$, components to account for the fact that those quantities may be different and scale differently with the fracture aperture ϵ. In the porous medium $\Omega \setminus \Gamma$ we consider Eqs. (1), while in the fractures we have, neglecting the compressibility term,

$$
\begin{aligned}
\mu K_{f,t}^{-1} u_f + \epsilon \nabla_\tau p_f &= 0 \\
\nabla_\tau \cdot u_f - [\![u_m \cdot n_f]\!] &= q_f
\end{aligned} \quad \text{in } \Gamma, \quad (3a)
$$

where $\nabla_\tau \cdot$ and ∇_τ are the divergence and gradient operating on the tangent plane of the fractures, respectively. The jump of normal velocity across the fracture acts as a source term in the mass conservation equation and represent the net flux entering (or leaving) the fracture. This model must be complemented by appropriate boundary conditions. In the portion of the boundary of Γ that touches the boundary of Ω, the conditions are determined by the specific problem at hand: either pressure or mass flux is prescribed. In the case of a fracture tip that ends inside Ω, a zero mass flux condition is generally adopted.

A network normally exhibits intersection between fractures. A common approach is to assume that the pressure is continuous at the intersection and the net sum of fluxes is zero. More sophisticated models are available, to account for fractures with different hydraulic properties and flow along fracture intersections.

To close the problem we need also to specify interface conditions to couple Γ to the porous medium. In [7] a family of models is presented, including the following, often adopted in practice

$$
\begin{aligned}
\mu \epsilon K_{f,n}^{-1} u_m^+ \cdot n_f + 2(p_f - p_m^+) &= 0 \\
\mu \epsilon K_{f,n}^{-1} u_m^- \cdot n_f + 2(p_m^- - p_f) &= 0
\end{aligned} \quad \text{on } \Gamma. \quad (3b)
$$

These conditions can be interpreted as the application of a discrete Darcy law across the fracture, indeed they link the flux across the fracture to pressure differences.

Remark 1 If the fractures are highly permeable in the normal direction, continuity of pressure across Γ is often assumed, i.e. $p_m^+ = p_m^-$. This induces a certain simplification in the model, since it does not account for a net mass flow across the fracture, but only for porous medium-fracture exchanges.

Remark 2 For nearly impermeable porous media, a different simplification of these models, called Discrete Fracture Networks (DFNs), consists in neglecting the effect of the porous material and simulate flow just in the fracture network. They can be used in the presence of highly connected and permeable fractures.
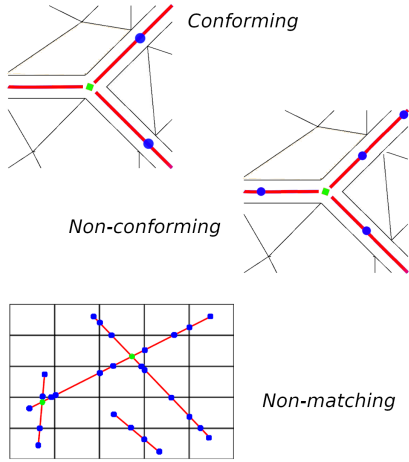
Fig. 3: Porous medium and fracture grids.

3 Discretization schemes for DFM

Numerical schemes for the approximation of the equations presented in the previous section are based on partitioning the domain Ω into a mesh of polyedral elements $\mathcal{T}_h$. The unknowns are then discretized by assuming a given variation inside each element, for instance constant or linear. The continuous solution is then replaced by the discrete values at the mesh nodes.

In the case of DFM we need to mesh both the porous medium and fracture domains and construct suitable ways to couple the two via (3b). The many discretization techniques available in the literature may be roughly subdivided into three categories, depending on the relation between porous medium and fracture grids, see Fig. 3.

i) Conforming methods. The mesh for the porous medium conforms to that adopted for the fracture network. It means that a fracture mesh element coincide (geometrically) with the faces of two porous medium mesh elements, on the positive and negative side of the fracture. Consequently, no porous medium mesh elements are cut by the fracture. This requirement poses strong constraints on the mesh generation process, which indeed can be the most time-consuming part of the simulation, particularly in the presence of complex networks. On the other hand, the implementation of (3b) is rather straightforward.

ii) Non-conforming methods. Still no elements in the porous medium grid are cut by the fractures, however the grid at the two sides of the fracture, as well as the fracture grid, are independent. The implementation of (3b) involves the set up of suit-

able operators to transfer the discrete solution in the fractures to the porous medium grid and viceversa. The so-called mortar technique, which is based on the set up of additional variables at the interface, is sometimes used to simplify the construction of the transfer operators. The process of grid generation is eased and it is simpler to avoid the generation of highly distorted elements, however it is still rather demanding in complex situations.

iii) Non-matching methods. The mesh for the porous medium and the fracture network are completely independent and the fracture grids can cut the porous medium mesh elements in an arbitrary way. This simplifies mesh generation greatly, since porous medium and fracture grids can be generated independently. It is still necessary to find the intersections, but this is simpler than generating a conforming mesh and can be done with standard geometric search tools. However, the implementation of conditions (3b) is more complex.

For every category, many different numerical schemes are at disposal. For the cases *i)* and *ii)*, it is beneficial to use techniques able to operate on arbitrary polyhedral grids, like Finite Volumes or Mimetic Finite Differences, to mention the more established one. The research in this field is, however, very active, and we mention also Gradient Schemes, the Hybrid High Order (HHO) method and the Virtual Elements Method (VEM). A reference containing examples of numerical schemes applied in a DFM context is [5].

In the case *iii)*, as already stated, the main difficulty is how to impose the interface conditions. In that respect we have two class of procedures. The first is to represent the possible jumps in the solution across the elements cut by a fracture explicitly. This is what is done in eXtended Finite Elements (XFEM), where the finite element basis functions are locally enriched to allow discontinuous solutions that can satisfy the coupling conditions (3b). Contrarily, if one accepts a less accurate representation of the solution, some manipulations of the interface conditions are possible to transform them into source terms acting both on the fracture elements and on the elements in the porous medium that are cut by the fracture. The resulting source terms have some similarity with those present in CFM techniques. The Embedded Discrete Fracture Model (EDFM), usually coupled with a simple Finite Volumes approximation, falls into this second category of methods. An reference on EDFM techniques is in [6].

The performances of many classes of methods are presented and discussed in [4] for bi-dimensional problems and [3] for three-dimensional problems.

4 An example of computational workflow

Numerical simulation of flow in fractured porous media is challenging due to the intrinsic geometrical complexity of the fractures, as well as the measurement of real fractures and their properties buried deep in the underground. These data are difficult to obtain and usually affected by large uncertainty which compromises the reliability of the numerical solutions.

Several approaches can be considered to detect fractures in the underground, from seismic inversion to outcrop interpretation, the former effective to detect big fractures few kilometres below the surface while the latter normally used as an analogue of the underground. Once the fractures are collected and digitalized by means of one of these methods, a suitable mathematical model can be adopted to perform the simulations.

We focus our attention on the case of outcrop interpretation. From highly detailed photographs of the interested region, all the fractures are collected and interpreted up to a minimum sampling scale determined by the quality of the data or by computational constraints. Smaller fractures can be accounted for by a suitable change of the porous medium properties and seen as upscaled or homogenized. What quantity defines a fracture "small" and thus not explicitly represented is still an open question, many authors consider the fracture extension a good proxy for its importance. After this operation, the digitalized version of the outcrop is thus available, see Fig. 4 on the top as an example where natural and human factors limit its exposure. The outcrop is a portion of Sotra island, near Bergen in Norway.

Another challenging aspect is the collection of the physical data, again affected by uncertainty, that are needed to run the simulation. If some data are not available, it is possible to fill the gap considering models from the literature to, for example, relate the permeability with the aperture and the latter with the fracture extension. The model can, at this point, be discretized numerically by means of one of the methods mentioned in the previous sections.

Fig. 4 reports numerical results of a simulation, in particular the pressure and concentration of a tracer (for instance a contaminant) in the interpreted outcrop. Two extreme cases are considered, if the fractures are higher or lower permeable than the surrounding porous medium. Data are homogeneous and a left to right pressure gradient is imposed. The solutions are obtained with the library PorePy, see `https://github.com/pmgbergen/porepy`.

5 Conclusion

Modeling flow in porous media in presence of fractures and fracture networks is a challenging task. Several approaches are available in literature, and a few of them implemented in specialised software. The choice depends on the scales at which the phenomenon has to be considered, on the connectivity of the fracture network and on the level of accuracy desired. Simpler continuum fracture models are suitable for highly connected and dense networks of fracture, while in presence of fracture with a larger extension and more sparse, discrete fracture models provide more accurate results. Clearly, it is also possible to use a combination of the two approaches. In this chapter we gave a rapid review of the different strategies and, for the sake of simplicity, we focused the attention on single-phase flow. The general conclusions can however be extended to the more complex situation of multi-phase flows.

Cross-References

- Lithosphere, Mechanical Properties
- Numerical Methods, Finite Element
- Sedimentary Basins

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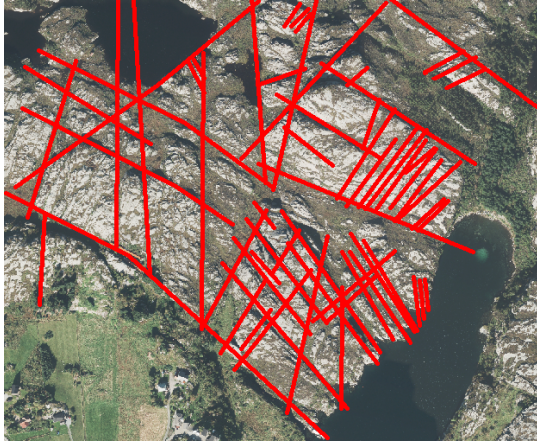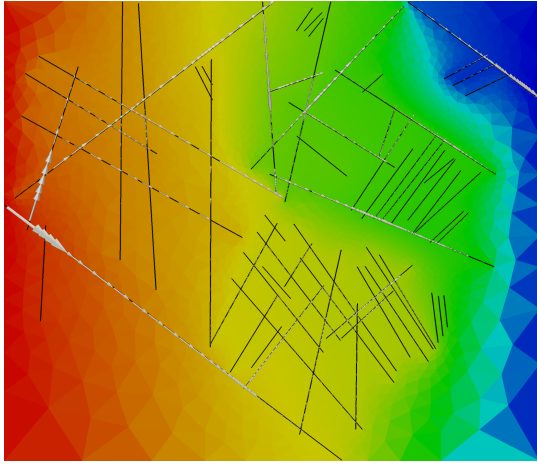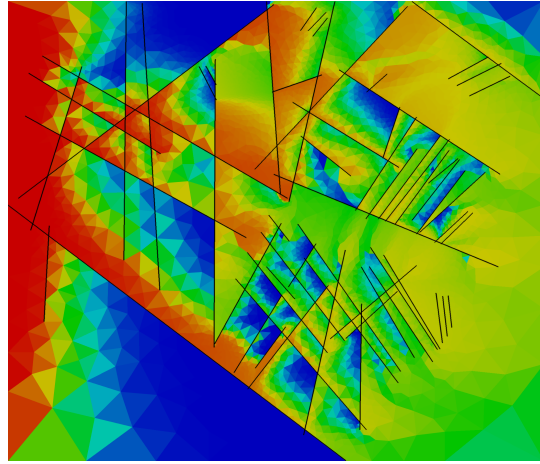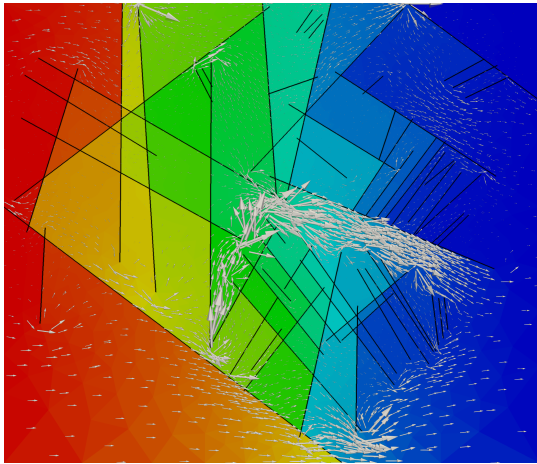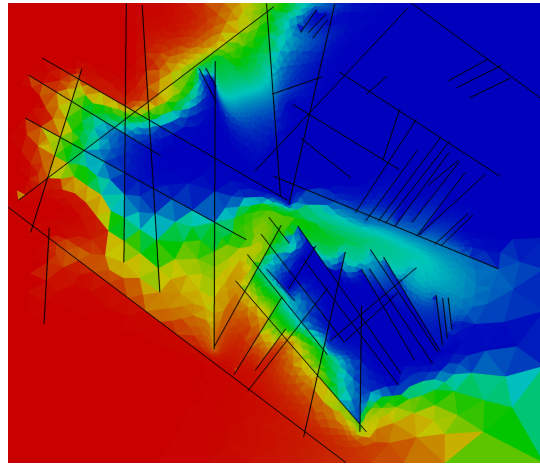
(a) Interpreted outcrop.
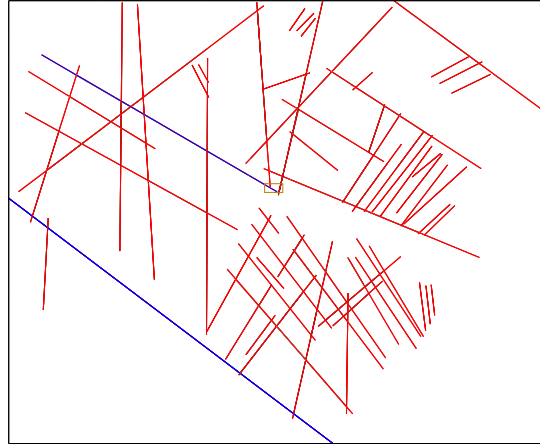
(b) Digitalized fractures.

(c) p and $\boldsymbol{u}$ with high permeable fractures.

(d) Scalar with high permeable fractures.

(e) p and $\boldsymbol{u}$ with low permeable fractures.

(f) Scalar with low permeable fractures.

Fig. 4: On the top the interpreted outcrop and digitalized fractures. The blue fractures are geometrically simplified due to software constraint. Images are taken from [4]. The others represent pressure, velocity field, and a scalar tracer at a specific time. The colour scheme spans from the lowest in blue to the highest in red. Images are taken from Fumagalli, A., Keilegavlen, E.: Dual virtual element methods for discrete fracture matrix models. Oil & Gas Science and Technology - Revue d'IFP Energies nouvelles 74(41), 1–17 (2019). under CC BY 4.0 licence.

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