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A reduced model for flow and transport in fractured porous media with non-matching grids

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Abstract

In this work we focus on a model reduction approach for the treatment of fractures in a porous medium, represented as interfaces embedded in a n-dimensional domain, in the form of a $(n-1)$-dimensional manifold, to describe fluid flow and transport in both domains. We employ a method that allows for non-matching grids, thus very advantageous if the position of the fractures is uncertain and multiple simulations are required. To this purpose we adopt an XFEM approach to represent discontinuities of the variables at the interfaces, which can arbitrarily cut the elements of the grid. The method is applied to the numerical solution of the Darcy problem, and advection-diffusion problems in porous media.

1 Introduction

Subsurface flows are strongly influenced by the heterogeneities of the porous medium and in particular by the presence of fractures, faults and discontinuities between different layers. While micro-fractures can be accounted for by means of homogenization, large fractures and faults can act as preferential paths or barriers for the flow, and should be resolved by the grid. Since the characteristic width of these features is usually very small compared to the typical mesh size one possibility to address this problem in a computationally efficient way is to use a reduced model in which the fractures are represented as interfaces immersed in the porous medium, with proper coupling conditions between the fracture and the medium. The reduced model for the single phase Darcy problem was first introduced in [1] and extended in [2, 8]. In [4, 6] the authors extended the work of [8] allowing for non matching grids between the porous domain and the fracture, increasing the flexibility of the method: an important advantage of non-matching grids is indeed the possibility to run multiple simulations with different fractures configuration, without meshing each time the domain. In the present work we derive, in the same framework, a reduced model for the problem of the advection and diffusion of a tracer in a fractured porous medium, with the aim of providing a flexible and efficient tool to simulate realistic problems such as groundwater contamination. We obtain the advection field solving a Darcy problem, formulated as in [4], and employ the same space discretization with non-matching grids to approximate the transport problem.

The paper is structured as follows. In Section 2 the governing equations of single phase Darcy flow and passive transport-diffusion in a porous medium are briefly presented. In Section 3 the reduced model for the transport problem is derived, and its numerical approximation is described in Section 4. In Section 5 two numerical test are illustrated. Section 6 is devoted to conclusions.

2 Governing equations

We consider the problem of a passive scalar (tracer) transported by an external field $\boldsymbol{u}$ in a porous medium. The external field is, in the case of our interest, obtained solving a Darcy problem in the porous medium.

We are interested in the case of domains crossed by faults or large fractures characterized by a permeability tensor $\boldsymbol{K}$ that differs significantly from the porous matrix. Let us consider a regular domain $\Omega \in \mathbb{R}^n$, with boundary $\overline{\Gamma} = \overline{\Gamma}_N \cup \overline{\Gamma}_D$ and outward unit normal $\boldsymbol{n}_\Gamma$, cut by a thin region $\Omega_f \subset \Omega$ of thickness d representing the fracture , as shown in Figure 1, such that $\overline{\Omega} = \overline{\Omega}_1 \cup \overline{\Omega}_f \cup \overline{\Omega}_2$ and $\mathring{\Omega}_i \cap \mathring{\Omega}_j = \emptyset$ for $i \neq j$. The Darcy

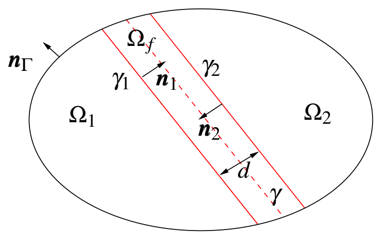


Figure 1: Domain divided in two sub-domains Ω_1 and Ω_2 by a thin region Ω_f.

flow is described by the following system for $i = 1, 2, f$ and $j = 1, 2$

$$\begin{cases} \nabla \cdot \boldsymbol{u}_i = q_i \\ \boldsymbol{u}_i = -\boldsymbol{K}_i \left(\nabla p_i - \boldsymbol{q}_i \right) \end{cases} \text{in } \Omega_i, \qquad \begin{cases} \boldsymbol{u}_j \cdot \boldsymbol{n}_j = \boldsymbol{u}_f \cdot \boldsymbol{n}_j \\ p_j = p_f \end{cases} \text{on } \gamma_j, \tag{1}$$

where the subscript i denotes the quantity in each sub-domain i and $\gamma_j \in \mathbb{R}^{n-1}$ is the interface between Ω_j and Ω_f with unit normal $\boldsymbol{n}_j$. We impose to (1) boundary conditions $p_i = \overline{p}_i$ on Γ_N^p and $\boldsymbol{u}_i \cdot \boldsymbol{n}_\Gamma = \overline{g}_i$ on Γ_D^p.
Moving to the advection-diffusion problem, we indicate with c the concentration of the passive scalar, defined as the volume fraction of tracer in the porosity, the total flux $\boldsymbol{\chi} := -\boldsymbol{D}\nabla c + \boldsymbol{u}c$ and we denote with $\boldsymbol{D} \in [L^\infty(\Omega)]^{n\times n}$ the molecular diffusion tensor which is symmetric and positive definite. Introducing the interval of time $\mathscr{I}_T := (0, T)$ and the domain $Q_i := \Omega_i \times \mathscr{I}_T$ then the advection-diffusion problem in mixed form, with suitable boundary and initial conditions for $i = 1, 2, f$ and $j = 1, 2$ reads

$$\begin{cases} \Phi_i \dfrac{\partial c_i}{\partial t} + \nabla \cdot \boldsymbol{\chi}_i = g \\ \boldsymbol{\chi}_i = -\boldsymbol{D}_i \nabla c_i + \boldsymbol{u}_i c_i \end{cases} \text{in } Q_i, \qquad \begin{cases} \boldsymbol{\chi}_j \cdot \boldsymbol{n}_j = \boldsymbol{\chi}_f \cdot \boldsymbol{n}_j \\ c_j = c_f \end{cases} \text{on } \gamma_j \times \mathscr{I}_T. \tag{2}$$

Here $\Phi_i \in L^\infty(\Omega)$ denotes the porosity and g is a source term. We impose to (2) boundary conditions $c_i = \overline{c}$ on $\Gamma_N^c \times \mathscr{I}_T$ and $\boldsymbol{\chi}_i \cdot \boldsymbol{n}_\Gamma$ on $\Gamma_D^c \times \mathscr{I}_T$, furthermore we impose initial condition $c_i = c_0$ in $\Omega_i \times \{0\}$.

3 Reduced model for the advection-diffusion problem

We want to derive a reduced model for advection and diffusion in the presence of fractures, replacing the region Ω_f with a $n-1$ dimensional interface $\gamma \approx \gamma_j$ with unit normal $\boldsymbol{n} \approx \boldsymbol{n}_1 \approx -\boldsymbol{n}_2$, as shown in Figure 2. In [8] a reduced model for Darcy is derived yield-

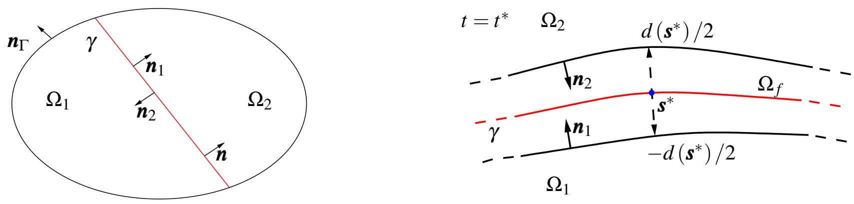


Figure 2: Left: domain cut by an $1D$ interface γ that replaces Ω_f. Right: the reducing process.

ing two coupled problems for the flow in the fracture and in the porous matrix. We report the main result for readers convenience. Given a function $a : \Omega \to \mathbb{R}^m$, $m = 1$ or n, let us set

$$[\![a]\!]_\gamma := a_1 - a_2 \quad \text{and} \quad \{\{a\}\}_\gamma := \frac{a_1 + a_2}{2} \quad \text{with} \quad a(\boldsymbol{x}) = \lim_{\varepsilon \to 0^\pm} a(\boldsymbol{x} - \varepsilon \boldsymbol{n}).$$

We also define the projection matrix $\boldsymbol{N} := \boldsymbol{n} \otimes \boldsymbol{n}$. Indicating with $\hat{\ }$ the reduced variables defined in γ, following [8] we suppose $\boldsymbol{K}_f = K_{f,\boldsymbol{n}} \boldsymbol{N} + K_{f,\boldsymbol{\tau}}(\boldsymbol{I} - \boldsymbol{N})$, then the Darcy problem for $i=1,2$ can be written as

$$\begin{cases} \nabla\cdot \boldsymbol{u}_i = q_i & \text{in } \Omega_i, \\ \boldsymbol{u}_i = -\boldsymbol{K}_i\left(\nabla p_i - \boldsymbol{q}_i\right) & \text{in } \Omega_i, \\ p_i = \overline{p}_i & \text{on } \Gamma_N^p, \\ \boldsymbol{u}_i\cdot \boldsymbol{n}_\Gamma = \overline{g}_i & \text{on } \Gamma_D^p, \end{cases} \qquad \begin{cases} \nabla_{\boldsymbol{\tau}}\cdot \hat{\boldsymbol{u}} = \hat{q} + \{\{\boldsymbol{u}\cdot\boldsymbol{n}\}\}_\gamma & \text{in } \gamma, \\ \hat{\eta}\hat{\boldsymbol{u}} + \nabla_{\boldsymbol{\tau}}\hat{p} = \hat{\boldsymbol{q}} & \text{in } \gamma, \\ \hat{p} = \hat{\overline{p}} & \text{on } \partial\gamma\cap\Gamma_N^p, \\ \hat{\boldsymbol{u}}\cdot\boldsymbol{n}_\Gamma = \hat{\overline{g}} & \text{on } \partial\gamma\cap\Gamma_D^p, \end{cases} \tag{3}$$

where $\hat{\eta} := d/K_{f,\boldsymbol{\tau}}$ and, given $a:\Omega\to\mathbb{R}$ and $\boldsymbol{a}:\Omega\to\mathbb{R}^n$, we have defined

$$\nabla_{\boldsymbol{\tau}} a := \nabla a - \boldsymbol{N}\nabla a\,, \qquad \nabla_{\boldsymbol{\tau}}\cdot\boldsymbol{a} := \nabla\cdot\boldsymbol{a} - \boldsymbol{N}:\nabla\boldsymbol{a}\,.$$

The coupling conditions, derived in [8] for $\boldsymbol{q}\equiv 0$, become in the more general case

$$\begin{cases} \xi_0\eta_\gamma[\![\boldsymbol{u}\cdot\boldsymbol{n}]\!]_\gamma + \dfrac{d}{4}[\![\boldsymbol{q}\cdot\boldsymbol{n}]\!]_\gamma = \{\{p\}\}_\gamma - \hat{p} \\ \eta_\gamma\{\{\boldsymbol{u}\cdot\boldsymbol{n}\}\}_\gamma = [\![p]\!]_\gamma + d\{\{\boldsymbol{q}\cdot\boldsymbol{n}\}\}_\gamma \end{cases} \quad \text{on } \gamma,$$

where $\xi_0\in(0,0.25]$ is a shape parameter and $\eta_\gamma := 1/\left(dK_{f,\boldsymbol{n}}\right)$, see [5] for details. We want to derive in an analogous way a reduced model for the time dependent problem of the advection-diffusion of a tracer. To this purpose we define the reduced flux and the mean concentration in the fracture, see Figure 2, as

$$\hat{\boldsymbol{\chi}}\left(\boldsymbol{s},t\right) := \int_{-\frac{d}{2}}^{\frac{d}{2}} \boldsymbol{\chi}_{f,\boldsymbol{\tau}}\left(\boldsymbol{r},t\right)d\boldsymbol{r} \quad \text{and} \quad \hat{c}\left(\boldsymbol{s},t\right) := \frac{1}{d}\int_{-\frac{d}{2}}^{\frac{d}{2}} c_f\left(\boldsymbol{r},t\right)d\boldsymbol{r}\,,$$

with $\boldsymbol{s}\in\gamma$ and $d=d\left(\boldsymbol{s}\right)$. Note that, for a function $\boldsymbol{a}_f:\Omega_f\to\mathbb{R}^n$ $\boldsymbol{a}_{f,\boldsymbol{\tau}} := \boldsymbol{a}_f - \boldsymbol{N}\boldsymbol{a}_f$. Projecting the conservation equation on the tangential space of γ and integrating in each section of the fracture, the reduced conservation equation becomes

$$d\Phi_f\frac{\partial\hat{c}}{\partial t} + \nabla_{\boldsymbol{\tau}}\cdot\hat{\boldsymbol{\chi}} = \hat{g} + [\![\boldsymbol{\chi}\cdot\boldsymbol{n}]\!]_\gamma \quad \text{on } Y\,, \tag{4}$$

where $\hat{g}$ is the reduced scalar source and $Y := \gamma\times\mathscr{I}_T$. We have assumed that Φ_f is constant in each transversal section of the fracture. Projecting the second equation of (2) on the tangential space of γ and integrating in each section of the fracture, the reduced flux equation becomes

$$\hat{\beta}\hat{\boldsymbol{\chi}} + \nabla_{\boldsymbol{\tau}}\hat{c} - d\hat{\beta}\hat{\boldsymbol{u}}\hat{c} = \boldsymbol{0} \quad \text{on } Y\,, \tag{5}$$

where $\hat{\beta} := d/D_{f,\boldsymbol{\tau}}$ and with the assumptions

$$\int_{-\frac{d}{2}}^{\frac{d}{2}} \boldsymbol{u}_{f,\boldsymbol{\tau}}c_f\,d\boldsymbol{r} \approx d\hat{\boldsymbol{u}}\hat{c} \quad \text{and} \quad \int_{-\frac{d}{2}}^{\frac{d}{2}} \boldsymbol{u}_f\cdot\boldsymbol{n}c_f\,d\boldsymbol{r} \approx 0\,.$$

We then integrate the second equation in (2) along the normal direction in Ω_f, apply the trapezium quadrature rule and exploit the continuity on γ_1 and γ_2 to obtain

$$\beta_\gamma\{\{\boldsymbol{\chi}\cdot\boldsymbol{n}\}\}_\gamma = [\![c]\!]_\gamma \quad \text{on } Y\,, \tag{6}$$

where $\beta_\gamma := 1/(dD_{f,\boldsymbol{n}})$. To close the reduced system we need another relation to model the variation of the concentration and total flux across the fracture. From a Taylor expansion in the centre of Ω_f (see [5]) we find

$$\hat{c} = \{\{c\}\}_\gamma - \beta_\gamma \xi_0 [\![\boldsymbol{\chi} \cdot \boldsymbol{n}]\!]_\gamma \quad \text{on } Y\,, \tag{7}$$

where $\xi_0 \in (0, 0.25]$ accounts for different concentration profiles in Ω_f.
Using (4) and (5) we can now write a system for the advection-diffusion problem in the porous matrix, for $i = 1, 2$, and in the fracture

$$\begin{cases} \Phi_i \dfrac{\partial c_i}{\partial t} + \nabla \cdot \boldsymbol{\chi}_i = g_i \\ \boldsymbol{\chi}_i = -\boldsymbol{D}_i \nabla c_i + \boldsymbol{u}_i c_i \end{cases} \text{in } Q_i\,, \quad \begin{cases} d\Phi_f \dfrac{\partial \hat{c}}{\partial t} + \nabla_{\boldsymbol{\tau}} \cdot \hat{\boldsymbol{\chi}} = \hat{g} + [\![\boldsymbol{\chi} \cdot \boldsymbol{n}]\!]_\gamma \\ \hat{\beta} \hat{\boldsymbol{\chi}} + \nabla_{\boldsymbol{\tau}} \hat{c} - d\hat{\beta} \hat{\boldsymbol{u}} \hat{c} = \boldsymbol{0} \end{cases} \text{in } Y\,, \tag{8}$$

coupled with the interface conditions (6) and (7) on the fracture and complemented with the initial and boundary conditions

$$\begin{cases} c_i = \overline{c}_i & \text{on } \Gamma_N^c \times \mathscr{I}_T, \\ \boldsymbol{\chi}_i \cdot \boldsymbol{n}_\Gamma = \overline{\chi}_i & \text{on } \Gamma_D^c \times \mathscr{I}_T, \\ c_i = c_{0,i} & \text{in } \Omega_i \times \{0\}, \end{cases} \qquad \begin{cases} \hat{c} = \hat{\overline{c}}_f & \text{on } \partial\gamma_N^c \times \mathscr{I}_T, \\ \hat{\boldsymbol{\chi}} \cdot \boldsymbol{n}_\Gamma = \hat{\overline{\chi}}_f & \text{in } \partial\gamma_D^c \times \mathscr{I}_T, \\ \hat{c} = \hat{c}_{0,f} & \text{on } \gamma \times \{0\}. \end{cases}$$

4 Numerical Discretization

We choose to adopt the same space discretization for the Darcy problem and for the advection-diffusion problem. In particular, we employ the mixed finite element method with the lowest order Raviart-Thomas finite elements $\mathbb{RT}_0$ while time stepping is performed, for the time dependent problem, by the implicit Euler method, as in [10]. The same discretization strategy is employed for the transport problem in the bulk medium and in the fracture. Mixed finite element are a valuable choice in problems concerning flow in porous media thanks to their local mass conservation property. Moreover, they have been successfully applied to problems of transport in porous media, see [3, 10].
We are interested in the case of domains crossed by interfaces that are non-conforming with the grid. More precisely, the triangulation $\mathscr{T}_h$ of the domain Ω and that of the interface γ are completely independent and non-matching, as shown in Figure 3. To

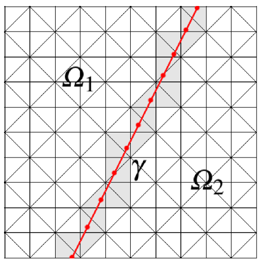


Figure 3: The triangulation of Ω and γ. The mesh elements cut by γ ad highlighted.

this purpose, we adopt an Extended Finite Element (XFEM) approach [9], enriching the classical Raviart-Thomas finite element basis on the elements cut by the fracture with discontinuous functions. In particular, we follow the approach proposed in [7], and proceed, for the transport problem in the fractured medium, as proposed in [4] for the Darcy problem. We consider discrete fluxes $\boldsymbol{\chi}_h \in \boldsymbol{V}_h$ and concentration $c_h \in Q_h$ made of two components, associated to Ω_i, for $i = 1,2$, where

$$
\begin{aligned}
&\boldsymbol{V}_h := \boldsymbol{V}_{1,h} \times \boldsymbol{V}_{2,h} \quad \text{ and } \quad Q_h := Q_{1,h} \times Q_{2,h} \quad \text{ with}\\
&\boldsymbol{V}_{i,h} := \left\{ \boldsymbol{v}_h \in \boldsymbol{H}_{\mathrm{div}}(\Omega_i) : \; \boldsymbol{v}_h|_{K_i} \in \mathbb{RT}_0(K_i), K_i \in \mathscr{T}_h \right\},\\
&Q_{i,h} := \left\{ q_h \in L^2(\Omega_i) : \; q_h|_{K_i} \in \mathbb{P}_0(K_i), K_i \in \mathscr{T}_h \right\},
\end{aligned}
$$

where, for any $K_i \in \mathscr{T}_h \cap \Omega_i$, $\mathbb{RT}_0(K_i)$ and $\mathbb{P}_0(K_i)$ are the restrictions to K_i of the standard $\mathbb{RT}_0$ and $\mathbb{P}_0$ local functions. The discrete variables can thus be discontinuous on γ, being defined on each part K_i of a cut element K by independent functions.
The global coupled system, discretized in space and time, reads

$$
\begin{bmatrix}
\boldsymbol{A} & \boldsymbol{B}^\top & \boldsymbol{0} & \boldsymbol{E}\\
\boldsymbol{B} & \boldsymbol{M} & \boldsymbol{0} & \boldsymbol{0}\\
\boldsymbol{0} & \boldsymbol{0} & \hat{\boldsymbol{A}} & \hat{\boldsymbol{B}}^\top\\
\boldsymbol{E}^\top & \boldsymbol{0} & \hat{\boldsymbol{B}} & \hat{\boldsymbol{M}}
\end{bmatrix}
\begin{bmatrix}
\boldsymbol{\chi}_h\\
\boldsymbol{c}_h^{k+1}\\
\hat{\boldsymbol{\chi}}_h\\
\hat{\boldsymbol{c}}_h^{k+1}
\end{bmatrix}
=
\begin{bmatrix}
\boldsymbol{0}\\
-\boldsymbol{g} + \boldsymbol{M}\boldsymbol{c}^k\\
\boldsymbol{0}\\
-\hat{\boldsymbol{g}} + \hat{\boldsymbol{M}}\hat{\boldsymbol{c}}^k
\end{bmatrix}
$$

where the blocks $\boldsymbol{E}$ and $\boldsymbol{E}^\top$ account for the coupling between the two problems and for the interpolation between the bulk mesh and the fracture mesh, furthermore $\boldsymbol{M}$ and $\hat{\boldsymbol{M}}$ are the mass matrices which include the time step Δt. We point out that in the relevant case of advection dominated problems a stabilization has to be applied, see [11, 10] for stabilization techniques in mixed finite elements.

5 Results

5.1 Test case 1

Let $\Omega = [0,1]^2$, $\Gamma = \{(x,y) \in \Omega : \; y = 2x - 0.4\}$, $\Gamma_D = \{0,1\} \times [0,1]$, and $\Gamma_N = [0,1] \times \{0,1\}$. The bulk flow and the flow in the fracture are described by equations (3), with $q = \hat{q} = 0$, $\boldsymbol{q} = \hat{\boldsymbol{q}} = 0$, $\overline{p} = 1 - y$, and $d = 0.01$. We consider full Neumann boundary conditions $\hat{p} = 1 - y$ on $\partial\gamma_N$, coupled by the interface conditions with $\xi_0 = 1/8$. The permeability tensor of the medium is isotropic, $\boldsymbol{K}_m = \boldsymbol{I}$ while the fracture is characterized by a high permeability in the normal and tangential directions, $\boldsymbol{K}_f = 100\boldsymbol{I}$. We want to solve (2) where the advection field is the computed Darcy velocity which is higher in the fracture than in the porous matrix. We set $\overline{c} = 0$, $\overline{\chi} = 0$ and $c_0(x,y) = 1$ if $(x-0.5)^2 + (y-0.2)^2 < 0.03$. The diffusion tensor is isotropic and constant, $\boldsymbol{D} = 0.05\boldsymbol{I}$ everywhere. We first solve this test case with the standard mixed FEM and a refined mesh that is able to resolve the fracture and compare the results with the reduced model and the XFEM approach. The time step is $\Delta t = 5 \cdot 10^{-3}$ and $T = 0.2$. In Figure 4 the

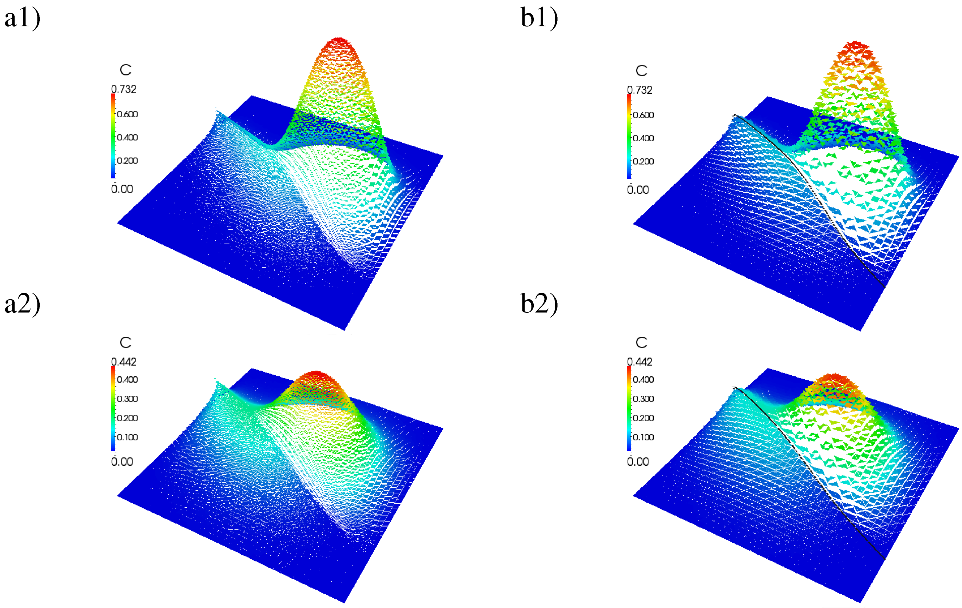
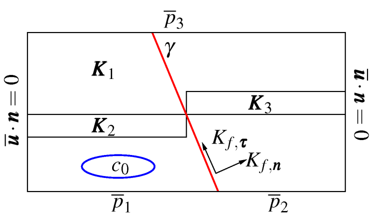


Figure 4: In a1), a2) the concentration of the tracer computed with the standard FEM and the fine grid at time $t = 0.1$ and $t = 0.2$ respectively. In b1) and b2) the solution obtained at the same time with the reduced approach and a coarse grid. The black line represents the concentration along γ.

solutions are compared at two different times. In both cases the tracer is advected upwards and flows preferably along the fracture where the fluid velocity is higher. The two methods produce results that are qualitatively in good agreement, even if a grid of only 3200 triangles and 100 segments for the fracture is used with the XFEM method and a much more refined grid ~13000 triangles is needed with the standard approach.

5.2 Test case 2

We now present a realistic example for the flow and transport problem. The domain, cut by a fracture, is sketched in Figure 5 and has spatial dimensions $6Km \times 3Km$ while the end time is $T = 10^{13}s \simeq 0.95Ma$.



$K_1 = 10^{-15}m^2$	$K_2 = 10^{-18}m^2$	$K_3 = 10^{-17}m^2$
$K_{f,\boldsymbol{\tau}} = 10^{-13}m^2$	$K_{f,\boldsymbol{n}} = 10^{-17}m^2$	$\overline{p}_1 = 34.53MPa$
$\overline{p}_2 = 29.53MPa$	$\overline{p}_3 = 0.1MPa$	$\boldsymbol{q} = -9.91\boldsymbol{e}_2$
$d = 10m$	$\xi_0 = 0.25$	$q = 0$
$\Delta t = 5 \cdot 10^{10}s$	$c_0 = 0.3$	$D = 10^{-8}m^2/s$

Figure 5: Left: computational domain showing. Right: data for problems (3) and (8).

The data for (3) and (8) are reported in Table 5. Note that the permeability is isotropic in the medium while the fracture acts as a preferential path for the flux in the tangential direction and as a barrier in the normal direction. The molecular diffusion of the tracer is homogeneous and isotropic in the whole domain, thus in the fracture we set $D_{f,\boldsymbol{n}} = Dd$ and $D_{f,\boldsymbol{\tau}} = D/d$. We impose homogeneous essential conditions on the left and the right part of Γ and natural conditions on the top and the bottom of Γ. In Figure 6 we present the concentration of the tracer at two different times comparing the results obtained accounting for the fracture and neglecting it. We can notice that the solutions are extremely different confirming the necessity to handle the fractures in an efficient and accurate way .

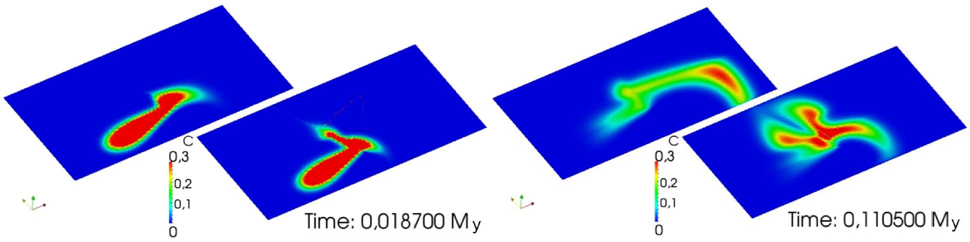


Figure 6: Comparison, at two different times, between the solution obtained without the fracture, (top), and the solution with the fracture and the reduced model (bottom).

6 Conclusions

In this paper we presented an original model for transport problems in fractured porous media. Following the approach present in literature for the single phase Darcy flow we derived a numerical model for the coupled problem of advection and diffusion in the porous medium and in the fractures, and compared the results with the traditional approach. Thanks to its moderate computational cost the method proves to be effective for cases with realistic parameters. Our future work will focus on the assessment of the theoretical properties of the method and the inclusion in this framework of suitable stabilization techniques for advection dominated problems.

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