Mathematical Models for Suspension Bridges

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The main goal of this research project is to obtain quantitative answers concerning the structural instability of suspension bridges in the presence or wind, that is, the analytical study of the interaction between internal resonance and aeroelastic phenomena. Skills in PDE's, ODE's, fluid-mechanics will be intensively used.

Modeling a suspension bridge requires some knowledge of elasticity [2, 6] and of higher order partial differential equations [4]. Basic facts from the Floquet theory and the Hill equation [1] are also needed to study the stability. Finally, the Navier-Stokes equations [5] will be used to model the behavior of the fluid (air) around the structure.

It is clear that in absence of wind or external loads the deck of a bridge remains still. When the wind hits a bluff body (such as the deck of a bridge) the flow is modified and goes around the body. Behind the deck, or a "hidden part" of it, the flow creates vortices which are, in general, asymmetric. This asymmetry generates a forcing lift which starts the vertical oscillations of the deck. The vortices induced by the wind increase the internal energy of the structure and generate wide longitudinal oscillations which look periodic in time. The longitudinal oscillations of the structure reach a periodic motion which is maintained in amplitude by a somehow perfect equilibrium between the input of energy from the wind and internal dissipation. If the longitudinal oscillation is sufficiently large then a structural instability appears: this is the onset of destructive torsional oscillations, see [3].

The purpose of this research project is to model with great accuracy all these steps: from the vortex formation, through the fluid-structure interaction, until the appearance of instabilities.

References

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