Long-time dynamics of dissipative evolutions equations with memory

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Many physical phenomena in which delay effects occur are modeled by equations with memory, where the dynamics is influenced by the past history of the variables in play through convolution integrals against suitable memory kernels. For instance, heat propagation processes are more realistically described by integro-differential equations of the form

$$u_t - \omega\Delta u - (1-\omega)\int_0^\infty k(s)\Delta u(t-s)\,\mathrm{d}s + f(u) = g, \qquad \omega \in [0,1),$$

in place of the classical Fourier equation

$$u_t - \Delta u + f(u) = g.$$

Here, the presence of the memory term accounts for the resistance of the system to a change of state. In particular, when $\omega = 0$ the equation becomes fully hyperbolic and the unphysical feature of infinite propagation speed of initial disturbances predicted by the Fourier law is removed (see e.g. [3]).

An effective way to capture the dissipativity properties of those models is translating the equations within a proper semigroup framework, where the regime behavior can be described in terms of small sets of the phase space able to eventually capture the trajectories of the underlying semigroup/process. Of particular importance are the so-called *global attractors* and the *exponential attractors* of finite fractal dimension. Our aim is the development of new theoretical tools - in the context of the Theory of Infinite Dimensional Dynamical Systems (see e.g. [4, 5]) - for the characterization of the long-term behavior of dissipative processes arising from PDEs. Applications of interest include models defined on time-dependent phase spaces such as viscoelasticity in aging materials [1, 2], population dynamics, heat flow in real conductors, phase transition.

References

[1] M. Conti, V. Danese, C. Giorgi and V. Pata, A model of viscoelasticity with time-dependent memory kernels, *Amer. J. Math.* (to appear).

[2] M. Conti, V. Danese, C. and V. Pata, Viscoelasticity with time-dependent memory kernels. Part II: asymptotic behavior of solutions. *Amer. J. Math.* (to appear).

[3] M. Conti, V. Pata, M. Squassina, Singular limit of differential systems with memory, *Indiana Univ. Math. J.* **55** (2006), 169–215.

[4] M. Conti, V. Pata, R. Temam, Attractors for processes on time-dependent spaces. Applications to wave equations, *J. Diff. Eq.* **255** (2013), 1254–1277.

[5] R. Temam, Infinite-dimensional dynamical systems in mechanics and physics, Springer, New York, 1988.