Einstein metrics and Ricci solitons

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The fundamental problem of capturing the topological properties of a manifold by its metric structure opened, in the last decades, extremely fruitful areas of mathematics. From this perspective, there has been an increasing interest in the study of Riemannian manifolds endowed with metrics satisfying special structural equations, possibly involving curvatures and vector fields. These structures arise naturally not only in several different mathematical frameworks, but also in Physics, in particular in General Relativity and String Theory. More precisely, the main objective of the thesis will be the study of geometric and analytic properties of well known special Riemannian structures such as Einstein metrics [1], i.e. Riemannian manifold (M, g) satisfying

$$Ric_g = \lambda\, g$$

for some constant $\lambda \in \mathbb{R}$, and Ricci solitons [2], i.e. Riemannian manifold (M, g) satisfying

$$Ric_g + \nabla^2_g f = \lambda\, g$$

for some constant $\lambda \in \mathbb{R}$ and some potential function f on M.

More in general, we are interested in Riemannian metrics satisfying structural conditions, involving their Ricci curvature, scalar curvature and globally defined vector fields. In particular, the aim is to prove rigidity, triviality and classification results, both in the compact and in the non-compact case for these type of structures. See, for instance [3, 4, 5, 6].

References

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